Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.1-Sine/65-4.1.0-a-sin-^m-b-trg-^n

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [538]. This is test number [65].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (538)	0.00 (0)
Mathematica	100.00 (538)	0.00 (0)
Maple	82.90 (446)	17.10 (92)
Fricas	81.60 (439)	18.40 (99)
Mupad	46.10 (248)	53.90 (290)
Maxima	44.98 (242)	55.02 (296)
Giac	39.59 (213)	60.41 (325)
Sympy	18.96 (102)	81.04 (436)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

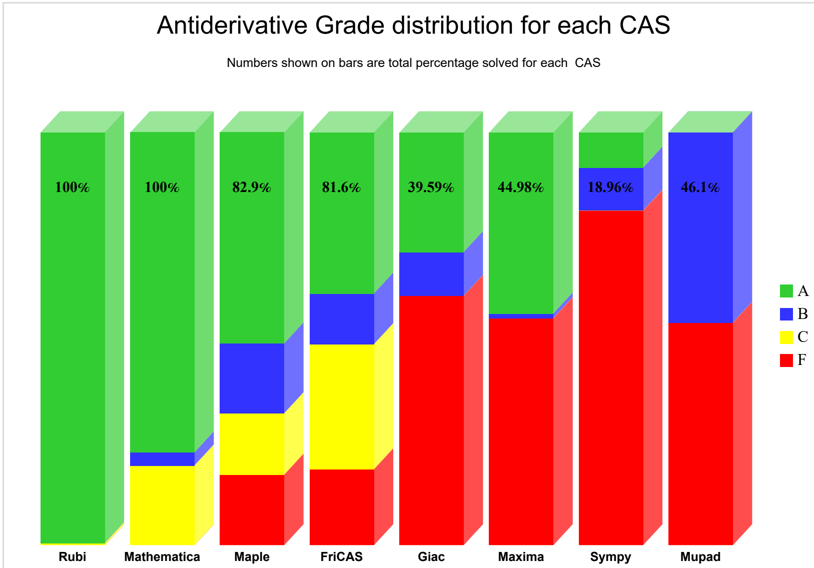
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

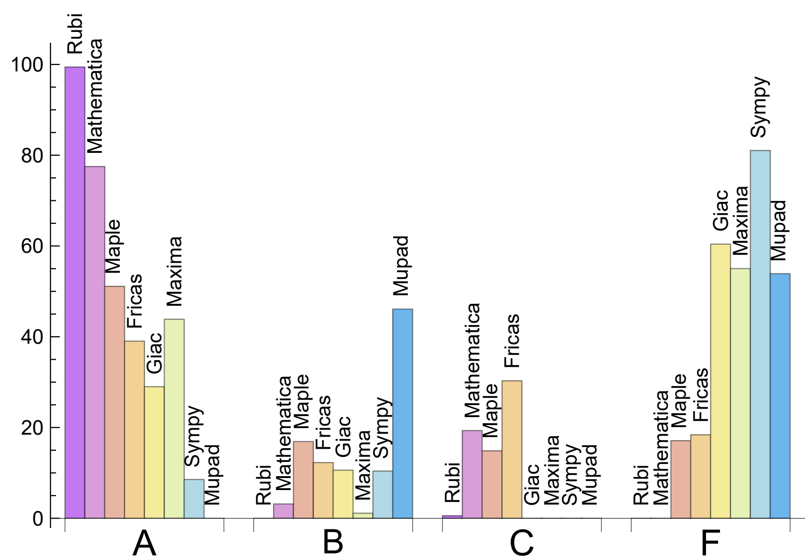
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.442	0.000	0.558	0.000
Mathematica	77.509	3.160	19.331	0.000
Maple	51.115	16.914	14.870	17.100
Maxima	43.866	1.115	0.000	55.019
Fricas	39.033	12.268	30.297	18.401
Giac	28.996	10.595	0.000	60.409
Sympy	8.550	10.409	0.000	81.041
Mupad	0.000	46.097	0.000	53.903

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	92	100.00	0.00	0.00
Fricas	99	100.00	0.00	0.00
Mupad	290	0.00	100.00	0.00
Maxima	296	98.65	1.35	0.00
Giac	325	98.15	1.85	0.00
Sympy	436	53.67	45.87	0.46

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.05
Maxima	0.21
Fricas	0.26
Giac	0.33
Mathematica	0.48
Mupad	0.87
Sympy	2.12
Maple	5.33

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	50.53	1.02	40.50	0.96
Mupad	52.19	1.14	38.50	0.92
Mathematica	61.22	0.94	57.00	0.88
Giac	74.90	1.60	55.00	1.10
Rubi	78.71	1.00	68.00	1.00
Fricas	146.34	1.57	80.00	1.28
Maple	191.12	2.25	89.50	1.40
Sympy	238.61	5.58	66.00	1.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

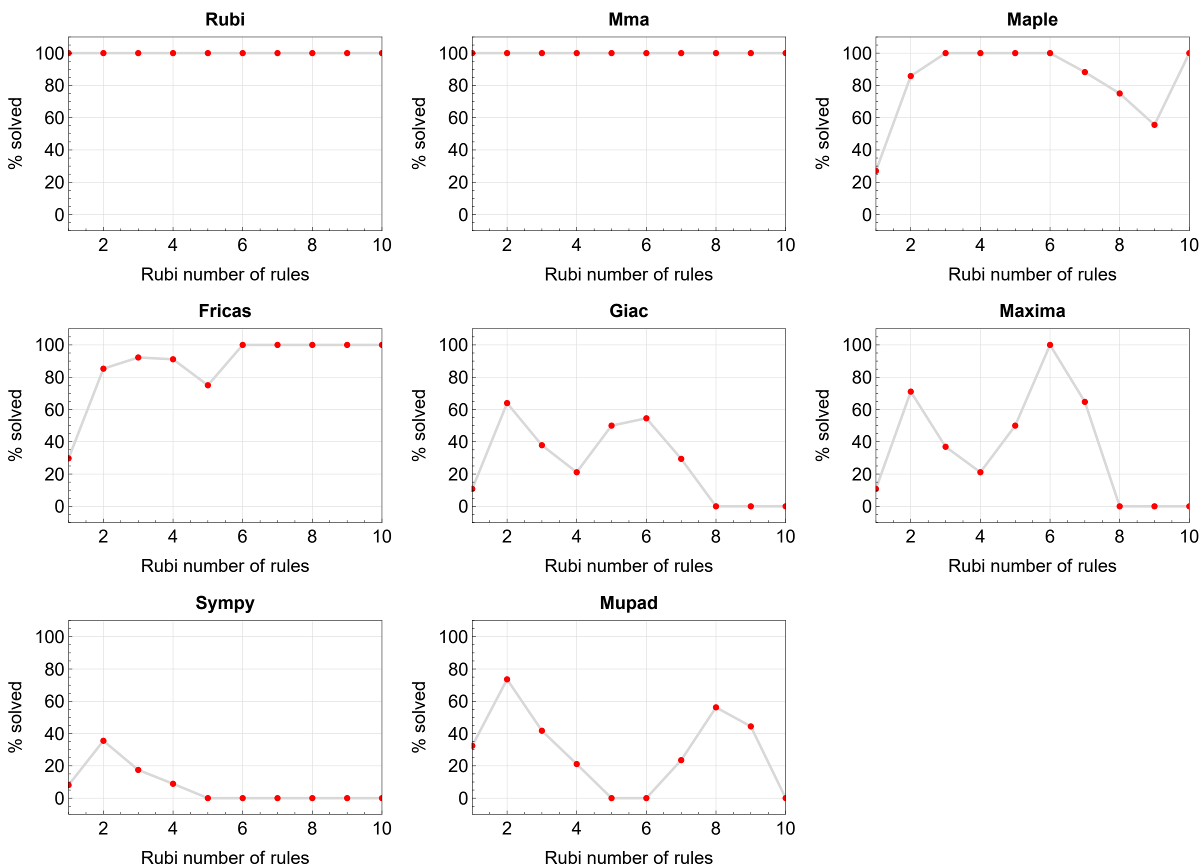


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

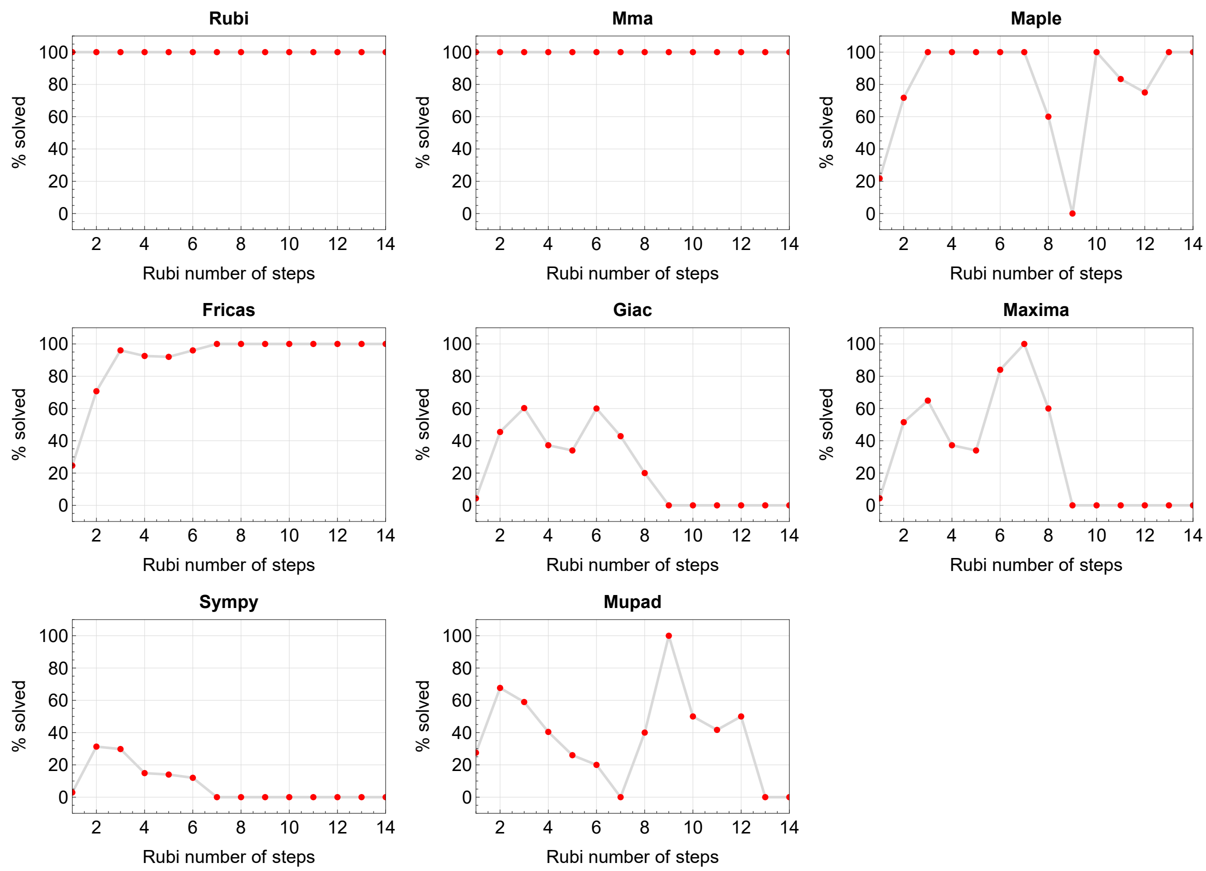


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

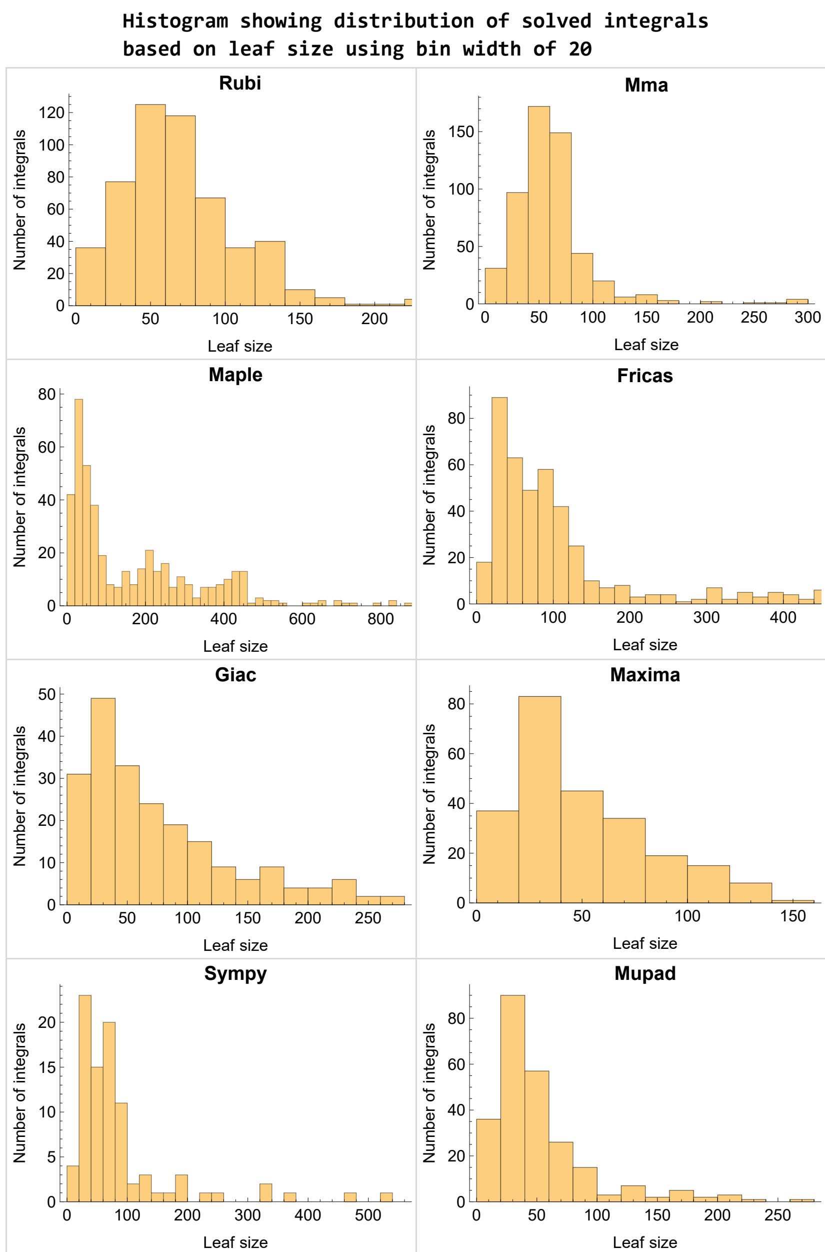


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

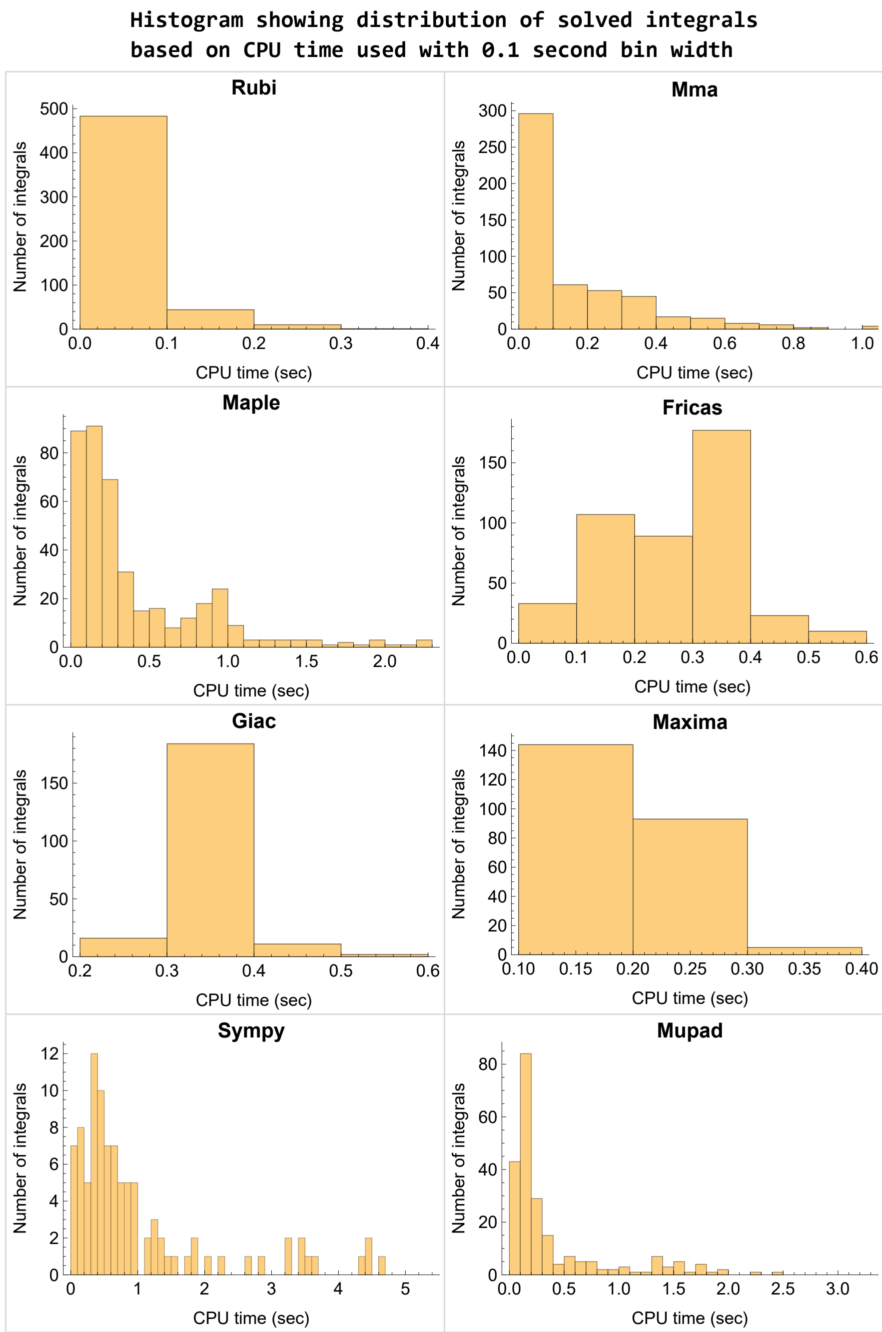


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

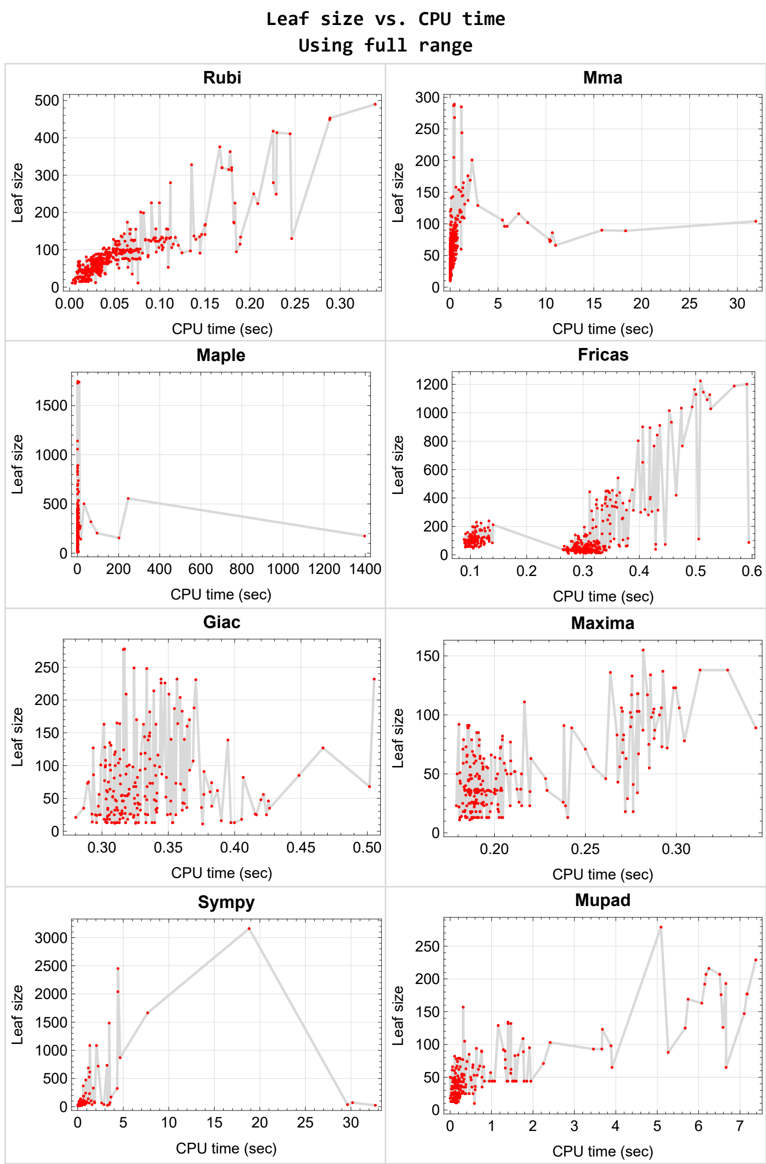


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{488, 489, 490, 491\}$

Maple $\{267, 272, 283, 291, 456, 473, 478\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

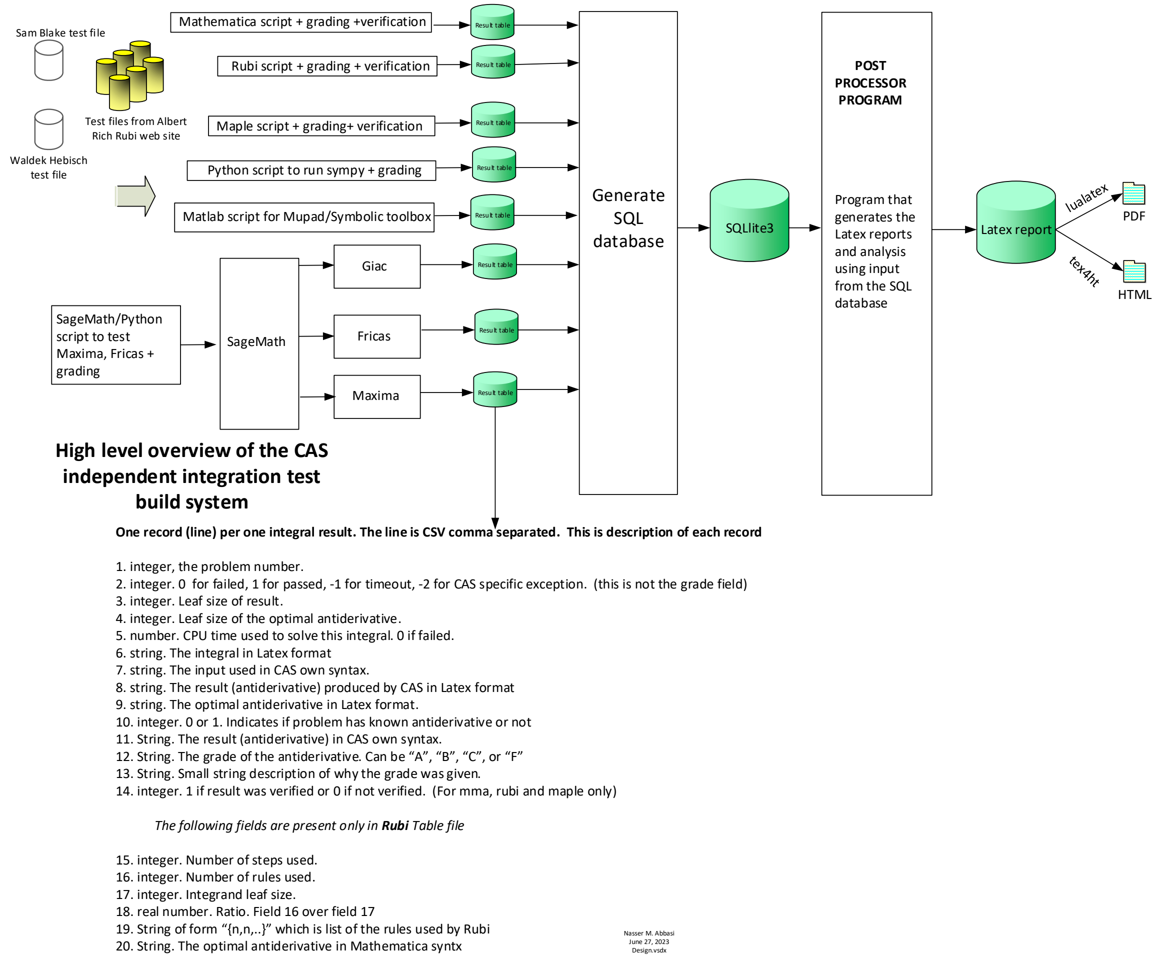
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade { }

C grade { 35, 36, 37 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 143, 145, 146, 147, 148, 149, 151, 152, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade { 44, 87, 88, 125, 126, 150, 153, 155, 176, 178, 183, 210, 221, 359, 360, 366, 496 }

C grade { 35, 36, 37, 138, 140, 142, 144, 162, 166, 168, 170, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 243, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 428, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 221, 232, 234, 239, 240, 252, 253, 254, 255, 256, 262, 263, 264, 265, 266, 268, 269, 270, 271, 273, 274, 275, 282, 284, 285, 286, 288, 292, 293, 294, 295, 296, 297, 298, 299, 340, 341, 342, 355, 356, 357, 374, 388, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 450, 451, 452, 453, 454, 455, 458, 459, 460, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 492, 493, 494 }

B grade { 10, 111, 196, 198, 199, 200, 201, 202, 203, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 272, 276, 277, 278, 279, 280, 281, 283, 287, 289, 290, 300, 301, 302, 303, 371, 372, 373, 375, 376, 377, 385, 386, 387, 389, 390, 397, 398, 399, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 457, 461, 462, 463, 464, 465, 466 }

C grade { 57, 136, 143, 160, 163, 167, 169, 267, 291, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 456, 478, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 155, 157, 158, 159, 160, 161, 163, 165, 167, 169, 170, 171, 172, 173, 174, 175, 183, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 225, 231, 242, 251, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 324, 327, 328, 329, 332, 333, 334, 335, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 385, 386, 387, 388, 397, 398, 399, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 453, 454, 455, 468, 469, 470, 471, 492, 493, 494 }

B grade { 53, 54, 63, 83, 86, 104, 111, 126, 127, 128, 140, 150, 152, 153, 154, 156, 162, 164, 166, 168, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 224, 226, 227, 228, 229, 230, 243, 244, 245, 246, 247, 248, 249, 250, 325, 326, 330, 331, 375, 376, 377, 389, 390, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 476 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 260, 261, 262, 263, 269, 270, 271, 272, 280, 281, 282, 283, 287, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 458, 459, 460, 464, 465, 466, 467, 472, 473, 474, 475, 480, 481, 482, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 257, 258, 259, 267, 268, 276, 277, 278, 279, 291, 292, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 456, 457, 461, 462, 463, 477, 478, 479, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 400, 401, 402, 403, 410, 411, 412, 413, 414, 415, 416, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 492, 493, 494 }

B grade { 75, 79, 111, 113, 117, 126 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 239, 240, 241, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 140, 238, 313, 323 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 110, 118, 123, 125, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 149, 151, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 177, 178, 179, 185, 186, 188, 189, 204, 205, 226, 227, 247, 248, 252, 253, 254, 255, 256, 342, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 401, 402, 403, 410, 411, 412, 414, 415, 416, 424, 425, 426, 428, 429, 430, 437, 438, 439, 441, 442, 443 }

B grade { 98, 99, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 124, 126, 127, 128, 129, 130, 131, 132, 138, 145, 146, 147, 148, 150, 152, 153, 154, 155, 156, 162, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 340, 341, 355, 356, 400, 413, 427, 440 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 464, 465, 468, 469, 470, 471 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 207, 208, 209, 210, 221, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 287, 288, 289, 290, 297, 298, 299, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 355, 356, 357, 374, 381, 388, 399, 400, 413, 427, 440, 453, 454, 455, 468, 469, 470, 471, 476, 492, 493, 494, 511 }

C grade { }

F normal fail { }

F(-1) timedout fail { 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 291, 292, 293, 294, 295, 296, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 5, 7, 42, 43, 44, 49, 50, 51, 52, 66, 68, 70, 80, 81, 82, 83, 98, 99, 100, 101, 103, 104, 149, 151, 157, 158, 159, 160, 165, 175, 176, 178, 179, 186, 187, 188, 204, 205, 206, 207, 208, 252, 254, 255 }

B grade { 2, 4, 6, 8, 58, 59, 60, 61, 62, 67, 69, 89, 90, 91, 92, 102, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 150, 161, 162, 163, 164, 171, 172, 173, 174, 177, 185, 189, 190, 191, 192, 253, 256, 340, 341, 342, 355, 356, 357 }

C grade { }

F normal fail { 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 63, 64, 71, 84, 85, 94, 126, 127, 128, 129, 130, 131, 132, 140, 141, 142, 143, 144, 152, 153, 154, 155, 156, 166, 167, 168, 169, 170, 180, 181, 182, 183, 184, 198, 199, 226, 227, 228, 237, 238, 239, 247, 248, 249, 259, 260, 262, 263, 268, 271, 272, 289, 292, 293, 296, 297, 300, 301, 304, 305, 306, 307, 311, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 329, 330, 331, 336, 337, 338, 339, 343, 344, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 362, 363, 364, 365, 367, 368, 369, 370, 374, 375, 376, 377, 379, 380, 381, 382, 383, 388, 389, 394, 407, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 458, 463, 464, 466, 467, 474, 479, 485, 486, 487, 488, 489, 490, 491, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 9, 17, 25, 56, 57, 65, 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 93, 95, 96, 97, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 193, 194, 195, 196, 197, 200, 201, 202, 203, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 240, 241, 242, 243, 244, 245, 246, 250, 251, 257, 258, 261, 264, 265, 266, 267, 269, 270, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 294, 295, 298, 299, 302, 303, 308, 309, 310, 312, 313, 314, 319, 327, 328, 332, 333, 334, 335, 345, 346, 360, 361, 366, 371, 372, 373, 378, 384, 385, 386, 387, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 424, 425, 426, 437, 438, 439, 449, 450, 451, 454, 455, 456, 457, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 484, 492, 493, 497, 503, 508, 515, 516, 517, 518, 523 }

F(-2) exception fail { 72, 106 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11
N.S.	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00
time (sec)	N/A	0.076	0.008	1.902	0.186	0.305	0.055	0.376	0.162

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	24	23	46	18	18
N.S.	1	1.00	0.92	0.76	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.007	0.026	0.065	0.189	0.292	0.081	0.304	0.146

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	22	22	37	25	24
N.S.	1	1.00	1.07	0.81	0.81	0.81	1.37	0.93	0.89
time (sec)	N/A	0.009	0.016	0.158	0.189	0.322	0.107	0.314	0.046

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	31	33	36	95	32	50
N.S.	1	1.00	0.72	0.67	0.72	0.78	2.07	0.70	1.09
time (sec)	N/A	0.016	0.038	0.177	0.193	0.309	0.150	0.310	0.113

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	32	34	34	60	38	32
N.S.	1	1.00	1.05	0.76	0.81	0.81	1.43	0.90	0.76
time (sec)	N/A	0.010	0.019	0.171	0.187	0.316	0.199	0.297	0.055

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	48	47	139	46	43
N.S.	1	1.00	0.67	0.66	0.72	0.70	2.07	0.69	0.64
time (sec)	N/A	0.027	0.038	0.215	0.190	0.305	0.308	0.316	0.170

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	42	44	44	80	50	43
N.S.	1	1.00	1.09	0.78	0.81	0.81	1.48	0.93	0.80
time (sec)	N/A	0.011	0.016	0.180	0.186	0.287	0.431	0.324	0.057

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	55	59	56	184	60	90
N.S.	1	1.00	0.62	0.62	0.67	0.64	2.09	0.68	1.02
time (sec)	N/A	0.032	0.049	0.293	0.189	0.293	0.640	0.382	0.758

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	84	0	73	0	0	34
N.S.	1	1.00	0.75	1.40	0.00	1.22	0.00	0.00	0.57
time (sec)	N/A	0.015	0.063	0.214	0.000	0.097	0.000	0.000	0.155

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	118	0	68	0	0	34
N.S.	1	1.00	0.85	2.88	0.00	1.66	0.00	0.00	0.83
time (sec)	N/A	0.009	0.035	0.109	0.000	0.108	0.000	0.000	0.100

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	60	0	0	34
N.S.	1	1.00	0.80	1.76	0.00	1.46	0.00	0.00	0.83
time (sec)	N/A	0.008	0.031	0.088	0.000	0.102	0.000	0.000	0.080

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	77	0	55	0	0	15
N.S.	1	1.00	1.11	4.05	0.00	2.89	0.00	0.00	0.79
time (sec)	N/A	0.004	0.022	0.160	0.000	0.104	0.000	0.000	0.071

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	57	0	47	0	0	15
N.S.	1	1.00	1.11	3.00	0.00	2.47	0.00	0.00	0.79
time (sec)	N/A	0.004	0.023	0.066	0.000	0.105	0.000	0.000	0.088

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	110	0	81	0	0	34
N.S.	1	1.00	0.86	2.97	0.00	2.19	0.00	0.00	0.92
time (sec)	N/A	0.009	0.044	0.073	0.000	0.114	0.000	0.000	0.151

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	97	0	0	34
N.S.	1	1.00	0.80	1.76	0.00	2.37	0.00	0.00	0.83
time (sec)	N/A	0.010	0.043	0.068	0.000	0.111	0.000	0.000	0.178

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	132	0	132	0	0	34
N.S.	1	1.00	0.85	2.20	0.00	2.20	0.00	0.00	0.57
time (sec)	N/A	0.015	0.041	0.075	0.000	0.109	0.000	0.000	0.173

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	104	0	87	0	0	42
N.S.	1	1.00	0.79	1.49	0.00	1.24	0.00	0.00	0.60
time (sec)	N/A	0.024	0.082	0.106	0.000	0.108	0.000	0.000	0.136

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	142	0	80	0	0	42
N.S.	1	1.00	0.94	3.02	0.00	1.70	0.00	0.00	0.89
time (sec)	N/A	0.013	0.059	0.074	0.000	0.119	0.000	0.000	0.106

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	88	0	72	0	0	42
N.S.	1	1.00	0.85	1.87	0.00	1.53	0.00	0.00	0.89
time (sec)	N/A	0.011	0.027	0.047	0.000	0.124	0.000	0.000	0.097

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	91	0	63	0	0	18
N.S.	1	1.00	1.14	4.33	0.00	3.00	0.00	0.00	0.86
time (sec)	N/A	0.005	0.013	0.099	0.000	0.118	0.000	0.000	0.057

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	69	0	55	0	0	18
N.S.	1	1.00	1.14	3.29	0.00	2.62	0.00	0.00	0.86
time (sec)	N/A	0.005	0.014	0.056	0.000	0.115	0.000	0.000	0.073

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	132	0	99	0	0	42
N.S.	1	1.00	0.91	3.07	0.00	2.30	0.00	0.00	0.98
time (sec)	N/A	0.010	0.057	0.047	0.000	0.090	0.000	0.000	0.141

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	88	0	115	0	0	42
N.S.	1	1.00	0.91	1.87	0.00	2.45	0.00	0.00	0.89
time (sec)	N/A	0.011	0.079	0.044	0.000	0.112	0.000	0.000	0.193

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	55	160	0	158	0	0	42
N.S.	1	1.00	0.79	2.29	0.00	2.26	0.00	0.00	0.60
time (sec)	N/A	0.019	0.207	0.048	0.000	0.104	0.000	0.000	0.185

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	80	108	0	105	0	0	0
N.S.	1	1.00	0.78	1.05	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.037	0.126	0.236	0.000	0.107	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	152	0	101	0	0	0
N.S.	1	1.00	0.88	2.03	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.026	0.088	0.136	0.000	0.106	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	97	0	81	0	0	0
N.S.	1	1.00	0.83	1.29	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.024	0.046	0.096	0.000	0.125	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	98	0	67	0	0	36
N.S.	1	1.00	0.98	2.28	0.00	1.56	0.00	0.00	0.84
time (sec)	N/A	0.013	0.025	0.165	0.000	0.110	0.000	0.000	0.073

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	74	0	62	0	0	36
N.S.	1	1.00	0.98	1.72	0.00	1.44	0.00	0.00	0.84
time (sec)	N/A	0.012	0.029	0.090	0.000	0.100	0.000	0.000	0.124

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	141	0	108	0	0	0
N.S.	1	1.00	0.74	1.93	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.021	0.045	0.084	0.000	0.101	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	55	105	0	127	0	0	0
N.S.	1	1.00	0.71	1.36	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.025	0.064	0.092	0.000	0.110	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	68	168	0	170	0	0	0
N.S.	1	1.00	0.65	1.60	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.038	0.126	0.104	0.000	0.109	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	517	58	55	0	0	0	0	0	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	58	55	0	0	0	0	0	0
N.S.	1	0.23	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	56	53	0	0	0	0	0	0
N.S.	1	0.21	0.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	54
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.86
time (sec)	N/A	0.010	0.034	0.000	0.000	0.000	0.000	0.000	0.337

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	76	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	24	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.47	1.60	0.87
time (sec)	N/A	0.012	0.003	0.065	0.184	0.315	0.144	0.296	0.035

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.47	0.87	0.87
time (sec)	N/A	0.012	0.003	0.053	0.191	0.330	0.099	0.303	0.026

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	13	19	13	28
N.S.	1	1.00	2.47	0.93	0.87	0.87	1.27	0.87	1.87
time (sec)	N/A	0.007	0.013	0.027	0.188	0.306	0.072	0.400	0.117

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	18	14	0	18	16
N.S.	1	1.00	1.00	1.00	1.50	1.17	0.00	1.50	1.33
time (sec)	N/A	0.029	0.007	0.056	0.184	0.331	0.000	0.319	0.138

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	12	12	0	12	20
N.S.	1	1.00	1.00	1.10	1.20	1.20	0.00	1.20	2.00
time (sec)	N/A	0.006	0.007	0.048	0.188	0.301	0.000	0.325	0.140

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	0	13	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.00	0.87	0.87
time (sec)	N/A	0.012	0.009	0.061	0.187	0.324	0.000	0.397	0.072

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	0	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.00	0.87	0.87
time (sec)	N/A	0.012	0.007	0.079	0.184	0.302	0.000	0.338	0.113

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	47	46	53	88	54	45
N.S.	1	1.00	0.77	0.77	0.75	0.87	1.44	0.89	0.74
time (sec)	N/A	0.026	0.124	0.217	0.187	0.347	0.912	0.317	0.078

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	36	36	43	66	54	36
N.S.	1	1.00	0.80	0.78	0.78	0.93	1.43	1.17	0.78
time (sec)	N/A	0.022	0.063	0.132	0.192	0.315	0.450	0.319	0.041

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	26	33	44	26	26
N.S.	1	1.00	0.87	0.84	0.84	1.06	1.42	0.84	0.84
time (sec)	N/A	0.020	0.036	0.090	0.189	0.341	0.206	0.321	0.094

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	21	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.40	1.33	0.87	0.87
time (sec)	N/A	0.011	0.003	0.047	0.190	0.321	0.101	0.311	0.015

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	19	18	31	0	18	14
N.S.	1	1.00	1.64	1.36	1.29	2.21	0.00	1.29	1.00
time (sec)	N/A	0.006	0.006	0.057	0.272	0.284	0.000	0.405	0.095

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	29	0	13	13
N.S.	1	1.00	1.00	1.47	0.87	1.93	0.00	0.87	0.87
time (sec)	N/A	0.018	0.005	0.082	0.240	0.273	0.000	0.340	0.072

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	56	42	26	39	0	26	25
N.S.	1	1.00	1.81	1.35	0.84	1.26	0.00	0.84	0.81
time (sec)	N/A	0.020	0.113	0.149	0.238	0.289	0.000	0.333	0.076

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	77	60	36	51	0	36	35
N.S.	1	1.00	1.67	1.30	0.78	1.11	0.00	0.78	0.76
time (sec)	N/A	0.024	0.096	0.233	0.196	0.283	0.000	0.375	0.088

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	98	77	46	61	0	46	45
N.S.	1	1.00	1.61	1.26	0.75	1.00	0.00	0.75	0.74
time (sec)	N/A	0.024	0.104	0.301	0.191	0.267	0.000	0.352	0.083

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	55	48	57	189	60	89
N.S.	1	1.00	0.59	0.62	0.55	0.65	2.15	0.68	1.01
time (sec)	N/A	0.042	0.120	0.356	0.192	0.306	0.646	0.304	0.763

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	40	44	37	47	136	46	43
N.S.	1	1.00	0.60	0.66	0.55	0.70	2.03	0.69	0.64
time (sec)	N/A	0.032	0.057	0.266	0.194	0.287	0.321	0.311	0.167

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	23	19	24	36	92	18	50
N.S.	1	1.00	0.50	0.41	0.52	0.78	2.00	0.39	1.09
time (sec)	N/A	0.026	0.055	0.088	0.193	0.303	0.161	0.314	0.116

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	24	23	46	18	18
N.S.	1	1.00	0.92	0.76	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.007	0.013	0.000	0.196	0.305	0.083	0.302	0.001

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	34	36	3160	36	27
N.S.	1	1.00	1.00	1.22	1.48	1.57	137.39	1.57	1.17
time (sec)	N/A	0.010	0.011	0.099	0.200	0.317	18.822	0.311	0.125

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	48	46	61	0	48	69
N.S.	1	1.00	1.00	1.41	1.35	1.79	0.00	1.41	2.03
time (sec)	N/A	0.026	0.013	0.106	0.191	0.303	0.000	0.334	0.550

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	66	65	71	0	82	125
N.S.	1	1.00	1.00	1.20	1.18	1.29	0.00	1.49	2.27
time (sec)	N/A	0.030	0.015	0.181	0.192	0.304	0.000	0.349	5.676

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	84	91	84	0	73	177
N.S.	1	1.00	1.00	1.11	1.20	1.11	0.00	0.96	2.33
time (sec)	N/A	0.038	0.015	0.256	0.185	0.292	0.000	0.356	7.172

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	48	26	36	26	44	27	25
N.S.	1	1.00	1.55	0.84	1.16	0.84	1.42	0.87	0.81
time (sec)	N/A	0.020	0.097	0.187	0.197	0.302	0.627	0.318	0.104

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	26	26	46	27	26
N.S.	1	1.00	0.87	0.84	0.84	0.84	1.48	0.87	0.84
time (sec)	N/A	0.021	0.073	0.150	0.188	0.288	0.454	0.315	0.053

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	35	26	26	26	44	26	37
N.S.	1	1.00	1.13	0.84	0.84	0.84	1.42	0.84	1.19
time (sec)	N/A	0.021	0.040	0.103	0.190	0.314	0.305	0.292	0.241

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	26	26	46	27	26
N.S.	1	1.00	0.87	0.84	0.84	0.84	1.48	0.87	0.84
time (sec)	N/A	0.022	0.036	0.089	0.194	0.290	0.208	0.314	0.076

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	24	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.60	1.33	0.87	0.87
time (sec)	N/A	0.013	0.002	0.054	0.185	0.316	0.145	0.328	0.085

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	25	25	0	29	25
N.S.	1	1.00	0.89	0.89	0.89	0.89	0.00	1.04	0.89
time (sec)	N/A	0.017	0.014	0.132	0.189	0.318	0.000	0.307	0.165

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	33	19	22	0	23	20
N.S.	1	1.00	1.00	1.57	0.90	1.05	0.00	1.10	0.95
time (sec)	N/A	0.015	0.066	0.066	0.184	0.328	0.000	0.304	0.236

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	31	34	0	42	27
N.S.	1	1.00	0.93	0.85	1.15	1.26	0.00	1.56	1.00
time (sec)	N/A	0.010	0.019	0.125	0.189	0.320	0.000	0.362	0.117

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	25	25	0	25	23
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.00	0.93	0.85
time (sec)	N/A	0.015	0.070	0.094	0.190	0.295	0.000	0.306	0.192

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	26	39	25	0	25	13
N.S.	1	1.00	1.00	1.73	2.60	1.67	0.00	1.67	0.87
time (sec)	N/A	0.018	0.004	0.093	0.183	0.306	0.000	0.335	0.110

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	0	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.00	0.81	0.81
time (sec)	N/A	0.020	0.106	0.155	0.191	0.303	0.000	0.424	0.282

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	49	25	0	25	25
N.S.	1	1.00	1.00	0.84	1.58	0.81	0.00	0.81	0.81
time (sec)	N/A	0.021	0.017	0.166	0.186	0.289	0.000	0.356	0.126

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	0	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.00	0.81	0.81
time (sec)	N/A	0.021	0.086	0.208	0.186	0.299	0.000	0.349	0.379

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	59	25	0	25	35
N.S.	1	1.00	1.00	0.84	1.90	0.81	0.00	0.81	1.13
time (sec)	N/A	0.021	0.017	0.274	0.183	0.310	0.000	0.416	0.151

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	47	46	63	88	82	45
N.S.	1	1.00	0.77	0.77	0.75	1.03	1.44	1.34	0.74
time (sec)	N/A	0.025	0.135	0.283	0.187	0.319	1.831	0.327	0.111

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	36	36	53	66	68	36
N.S.	1	1.00	0.80	0.78	0.78	1.15	1.43	1.48	0.78
time (sec)	N/A	0.023	0.067	0.210	0.189	0.337	0.933	0.324	0.113

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	26	26	41	44	26	26
N.S.	1	1.00	0.87	0.84	0.84	1.32	1.42	0.84	0.84
time (sec)	N/A	0.020	0.042	0.128	0.187	0.299	0.459	0.322	0.034

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	31	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	2.07	1.33	0.87	0.87
time (sec)	N/A	0.012	0.002	0.073	0.189	0.341	0.204	0.307	0.085

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	42	41	42	0	41	38
N.S.	1	1.00	0.78	1.05	1.02	1.05	0.00	1.02	0.95
time (sec)	N/A	0.025	0.154	0.117	0.276	0.290	0.000	0.328	0.189

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	28	29	46	0	29	24
N.S.	1	1.00	1.36	1.00	1.04	1.64	0.00	1.04	0.86
time (sec)	N/A	0.012	0.010	0.102	0.273	0.303	0.000	0.341	0.130

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	39	0	13	13
N.S.	1	1.00	1.00	1.47	0.87	2.60	0.00	0.87	0.87
time (sec)	N/A	0.018	0.005	0.127	0.193	0.305	0.000	0.353	0.144

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	77	42	26	49	0	26	25
N.S.	1	1.00	2.48	1.35	0.84	1.58	0.00	0.84	0.81
time (sec)	N/A	0.020	0.110	0.206	0.189	0.282	0.000	0.416	0.133

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	98	60	36	61	0	36	35
N.S.	1	1.00	2.13	1.30	0.78	1.33	0.00	0.78	0.76
time (sec)	N/A	0.023	0.102	0.260	0.196	0.309	0.000	0.364	0.168

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	62	66	48	66	231	74	109
N.S.	1	1.00	0.56	0.59	0.43	0.59	2.08	0.67	0.98
time (sec)	N/A	0.064	0.131	0.326	0.192	0.314	1.295	0.325	1.760

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	33	31	33	56	189	32	90
N.S.	1	1.00	0.37	0.34	0.37	0.62	2.10	0.36	1.00
time (sec)	N/A	0.051	0.063	0.216	0.186	0.329	0.660	0.329	1.325

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	40	42	37	46	136	46	43
N.S.	1	1.00	0.58	0.61	0.54	0.67	1.97	0.67	0.62
time (sec)	N/A	0.043	0.042	0.137	0.190	0.312	0.332	0.315	0.285

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	31	33	36	95	32	50
N.S.	1	1.00	0.72	0.67	0.72	0.78	2.07	0.70	1.09
time (sec)	N/A	0.014	0.023	0.000	0.200	0.324	0.166	0.312	0.002

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	38	46	48	0	48	53
N.S.	1	1.00	1.00	1.00	1.21	1.26	0.00	1.26	1.39
time (sec)	N/A	0.025	0.012	0.120	0.261	0.295	0.000	0.310	0.338

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	58	56	74	0	58	98
N.S.	1	1.00	1.08	1.18	1.14	1.51	0.00	1.18	2.00
time (sec)	N/A	0.020	0.014	0.149	0.269	0.326	0.000	0.341	3.884

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	73	76	71	74	0	63	126
N.S.	1	1.00	1.33	1.38	1.29	1.35	0.00	1.15	2.29
time (sec)	N/A	0.030	0.017	0.168	0.250	0.306	0.000	0.362	6.588

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	99	94	91	84	0	73	177
N.S.	1	1.00	1.27	1.21	1.17	1.08	0.00	0.94	2.27
time (sec)	N/A	0.049	0.019	0.250	0.238	0.314	0.000	0.338	7.168

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	120	112	111	94	0	107	229
N.S.	1	1.00	1.21	1.13	1.12	0.95	0.00	1.08	2.31
time (sec)	N/A	0.058	0.020	0.391	0.216	0.310	0.000	0.369	7.379

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	68	47	46	36	65	85	35
N.S.	1	1.00	1.48	1.02	1.00	0.78	1.41	1.85	0.76
time (sec)	N/A	0.026	0.236	0.362	0.228	0.286	2.628	0.341	0.146

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	37	36	36	68	82	36
N.S.	1	1.00	0.80	0.80	0.78	0.78	1.48	1.78	0.78
time (sec)	N/A	0.023	0.178	0.250	0.229	0.324	1.856	0.406	0.134

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	50	36	36	36	65	43	36
N.S.	1	1.00	1.09	0.78	0.78	0.78	1.41	0.93	0.78
time (sec)	N/A	0.025	0.049	0.220	0.197	0.299	1.252	0.361	0.211

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	37	36	36	68	68	36
N.S.	1	1.00	0.80	0.80	0.78	0.78	1.48	1.48	0.78
time (sec)	N/A	0.022	0.091	0.201	0.187	0.298	0.909	0.326	0.046

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	48	26	26	36	65	26	26
N.S.	1	1.00	1.55	0.84	0.84	1.16	2.10	0.84	0.84
time (sec)	N/A	0.020	0.071	0.158	0.214	0.286	0.635	0.352	0.120

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	37	37	36	36	68	54	36
N.S.	1	1.00	0.80	0.80	0.78	0.78	1.48	1.17	0.78
time (sec)	N/A	0.025	0.057	0.119	0.201	0.313	0.442	0.315	0.096

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	34	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	2.27	1.33	0.87	0.87
time (sec)	N/A	0.012	0.003	0.091	0.197	0.302	0.307	0.304	0.051

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	35	37	35	0	226	53
N.S.	1	1.00	0.88	0.88	0.92	0.88	0.00	5.65	1.32
time (sec)	N/A	0.018	0.025	0.160	0.186	0.324	0.000	0.345	0.279

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	46	32	33	0	99	31
N.S.	1	1.00	1.05	1.24	0.86	0.89	0.00	2.68	0.84
time (sec)	N/A	0.025	0.092	0.121	0.195	0.302	0.000	0.319	0.242

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	51	41	54	0	182	37
N.S.	1	1.00	0.77	1.19	0.95	1.26	0.00	4.23	0.86
time (sec)	N/A	0.025	0.031	0.178	0.193	0.346	0.000	0.336	0.152

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	35	35	0	100	35
N.S.	1	1.00	1.00	1.42	0.92	0.92	0.00	2.63	0.92
time (sec)	N/A	0.016	0.080	0.158	0.190	0.304	0.000	0.340	0.273

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	35	54	44	0	226	38
N.S.	1	1.00	0.86	0.81	1.26	1.02	0.00	5.26	0.88
time (sec)	N/A	0.015	0.039	0.182	0.194	0.312	0.000	0.347	0.140

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	35	35	0	72	35
N.S.	1	1.00	1.00	0.78	0.85	0.85	0.00	1.76	0.85
time (sec)	N/A	0.017	0.082	0.132	0.191	0.302	0.000	0.359	0.274

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	32	59	35	0	48	13
N.S.	1	1.00	1.00	2.13	3.93	2.33	0.00	3.20	0.87
time (sec)	N/A	0.021	0.006	0.143	0.193	0.294	0.000	0.420	0.144

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	35	0	116	35
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.00	2.52	0.76
time (sec)	N/A	0.025	0.089	0.200	0.191	0.291	0.000	0.342	0.328

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	36	69	35	0	93	25
N.S.	1	1.00	1.48	1.16	2.23	1.13	0.00	3.00	0.81
time (sec)	N/A	0.021	0.021	0.239	0.188	0.289	0.000	0.366	0.147

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	35	0	160	35
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.00	3.48	0.76
time (sec)	N/A	0.023	0.087	0.263	0.186	0.318	0.000	0.364	0.525

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	79	35	0	139	35
N.S.	1	1.00	1.00	0.78	1.72	0.76	0.00	3.02	0.76
time (sec)	N/A	0.026	0.020	0.381	0.204	0.270	0.000	0.395	0.184

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	35	0	204	35
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.00	4.43	0.76
time (sec)	N/A	0.023	0.091	0.489	0.198	0.337	0.000	0.359	0.771

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	89	35	0	183	45
N.S.	1	1.00	1.00	0.78	1.93	0.76	0.00	3.98	0.98
time (sec)	N/A	0.026	0.021	0.575	0.185	0.290	0.000	0.360	0.168

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	78	68	66	84	0	68	147
N.S.	1	1.00	1.18	1.03	1.00	1.27	0.00	1.03	2.23
time (sec)	N/A	0.027	0.019	0.241	0.185	0.329	0.000	0.334	7.100

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	70	45	45	0	144	50
N.S.	1	1.00	1.00	1.40	0.90	0.90	0.00	2.88	1.00
time (sec)	N/A	0.018	0.094	0.296	0.186	0.301	0.000	0.332	0.285

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	75	48	56	60	1085	145	88
N.S.	1	1.00	1.42	0.91	1.06	1.13	20.47	2.74	1.66
time (sec)	N/A	0.020	0.080	0.144	0.194	0.336	2.060	0.335	5.266

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	35	35	1086	170	66
N.S.	1	1.00	1.00	0.82	0.88	0.88	27.15	4.25	1.65
time (sec)	N/A	0.018	0.012	0.129	0.185	0.312	1.332	0.326	0.244

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	60	37	46	50	473	101	62
N.S.	1	1.00	1.58	0.97	1.21	1.32	12.45	2.66	1.63
time (sec)	N/A	0.017	0.065	0.103	0.192	0.305	0.881	0.306	1.471

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	25	25	369	25	35
N.S.	1	1.00	1.00	0.85	0.93	0.93	13.67	0.93	1.30
time (sec)	N/A	0.015	0.012	0.082	0.183	0.303	0.609	0.298	0.153

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	23	34	38	92	57	35
N.S.	1	1.00	1.83	1.00	1.48	1.65	4.00	2.48	1.52
time (sec)	N/A	0.009	0.030	0.066	0.191	0.312	0.434	0.320	0.191

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	13	17	12	26
N.S.	1	1.00	2.09	1.09	1.00	1.18	1.55	1.09	2.36
time (sec)	N/A	0.003	0.008	0.029	0.181	0.295	0.207	0.310	0.141

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	28	30	0	56	11
N.S.	1	1.00	2.82	1.09	2.55	2.73	0.00	5.09	1.00
time (sec)	N/A	0.006	0.019	0.080	0.190	0.290	0.000	0.422	0.129

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	42	30	36	52	0	55	23
N.S.	1	1.00	1.83	1.30	1.57	2.26	0.00	2.39	1.00
time (sec)	N/A	0.015	0.084	0.092	0.184	0.293	0.000	0.327	0.092

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	39	23	40	56	0	124	35
N.S.	1	1.00	1.44	0.85	1.48	2.07	0.00	4.59	1.30
time (sec)	N/A	0.014	0.016	0.160	0.184	0.301	0.000	0.316	0.088

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	57	40	50	67	0	101	33
N.S.	1	1.00	1.50	1.05	1.32	1.76	0.00	2.66	0.87
time (sec)	N/A	0.016	0.055	0.156	0.190	0.317	0.000	0.321	0.064

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	54	33	65	67	0	170	46
N.S.	1	1.00	1.38	0.85	1.67	1.72	0.00	4.36	1.18
time (sec)	N/A	0.017	0.015	0.221	0.185	0.316	0.000	0.366	0.133

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	72	50	60	77	0	145	45
N.S.	1	1.00	1.36	0.94	1.13	1.45	0.00	2.74	0.85
time (sec)	N/A	0.018	0.065	0.239	0.185	0.298	0.000	0.336	0.134

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	43	85	77	0	214	56
N.S.	1	1.00	1.21	0.75	1.49	1.35	0.00	3.75	0.98
time (sec)	N/A	0.020	0.025	0.348	0.190	0.308	0.000	0.339	0.129

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	62	42	43	82	42	43
N.S.	1	1.00	1.00	1.24	0.84	0.86	1.64	0.84	0.86
time (sec)	N/A	0.025	0.019	0.248	0.181	0.281	0.818	0.325	0.225

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	63	63	52	119	55	47
N.S.	1	1.00	0.67	1.03	1.03	0.85	1.95	0.90	0.77
time (sec)	N/A	0.030	0.198	0.145	0.272	0.287	0.647	0.315	0.333

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	52	32	33	61	32	35
N.S.	1	1.00	1.00	1.37	0.84	0.87	1.61	0.84	0.92
time (sec)	N/A	0.023	0.014	0.214	0.187	0.282	0.527	0.312	0.182

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	31	54	43	40	75	43	43
N.S.	1	1.00	0.78	1.35	1.08	1.00	1.88	1.08	1.08
time (sec)	N/A	0.023	0.180	0.099	0.268	0.330	0.427	0.327	0.320

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	20	22	39	20	23
N.S.	1	1.00	1.00	1.52	0.87	0.96	1.70	0.87	1.00
time (sec)	N/A	0.014	0.013	0.173	0.202	0.291	0.361	0.316	0.169

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	21	18	29	29	35	15
N.S.	1	1.00	1.93	1.40	1.20	1.93	1.93	2.33	1.00
time (sec)	N/A	0.006	0.012	0.051	0.276	0.283	0.330	0.426	0.150

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	13	13	20	13	13
N.S.	1	1.00	1.00	1.27	1.18	1.18	1.82	1.18	1.18
time (sec)	N/A	0.006	0.008	0.032	0.191	0.286	0.308	0.353	0.158

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	27	30	0	50	0	38	22
N.S.	1	1.00	1.17	1.30	0.00	2.17	0.00	1.65	0.96
time (sec)	N/A	0.015	0.012	0.084	0.000	0.303	0.000	0.383	0.022

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	13	31	22	33	0	16	14
N.S.	1	1.00	0.59	1.41	1.00	1.50	0.00	0.73	0.64
time (sec)	N/A	0.021	0.038	0.262	0.181	0.275	0.000	0.390	0.127

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	27	50	61	85	0	63	48
N.S.	1	1.00	0.55	1.02	1.24	1.73	0.00	1.29	0.98
time (sec)	N/A	0.034	0.011	0.165	0.190	0.297	0.000	0.362	0.072

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	46	32	43	0	32	33
N.S.	1	1.00	1.21	1.21	0.84	1.13	0.00	0.84	0.87
time (sec)	N/A	0.024	0.113	0.157	0.184	0.294	0.000	0.361	0.135

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	27	68	79	95	0	73	67
N.S.	1	1.00	0.39	0.97	1.13	1.36	0.00	1.04	0.96
time (sec)	N/A	0.030	0.012	0.266	0.192	0.319	0.000	0.336	0.165

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	63	45	71	1484	231	85
N.S.	1	1.00	1.00	1.09	0.78	1.22	25.59	3.98	1.47
time (sec)	N/A	0.030	0.019	0.204	0.186	0.320	3.460	0.371	0.460

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	103	70	66	93	719	163	129
N.S.	1	1.00	1.56	1.06	1.00	1.41	10.89	2.47	1.95
time (sec)	N/A	0.028	0.117	0.170	0.189	0.324	2.250	0.331	1.161

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	53	35	61	614	187	62
N.S.	1	1.00	0.81	1.23	0.81	1.42	14.28	4.35	1.44
time (sec)	N/A	0.025	0.046	0.141	0.187	0.322	1.325	0.355	0.208

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	86	60	56	83	241	140	77
N.S.	1	1.00	1.76	1.22	1.14	1.69	4.92	2.86	1.57
time (sec)	N/A	0.020	0.065	0.104	0.196	0.324	0.866	0.362	0.303

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	25	23	41	42	36	36
N.S.	1	1.00	1.21	0.89	0.82	1.46	1.50	1.29	1.29
time (sec)	N/A	0.012	0.060	0.083	0.194	0.298	0.389	0.318	0.173

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	75	43	46	72	58	93	48
N.S.	1	1.00	2.21	1.26	1.35	2.12	1.71	2.74	1.41
time (sec)	N/A	0.018	0.064	0.079	0.201	0.320	0.506	0.329	0.207

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	18	24	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.20	1.60	0.87	0.87
time (sec)	N/A	0.014	0.010	0.035	0.191	0.321	0.371	0.333	0.134

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	39	23	36	65	0	119	34
N.S.	1	1.00	1.44	0.85	1.33	2.41	0.00	4.41	1.26
time (sec)	N/A	0.016	0.016	0.109	0.200	0.288	0.000	0.333	0.126

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	143	52	61	96	0	140	49
N.S.	1	1.00	2.92	1.06	1.24	1.96	0.00	2.86	1.00
time (sec)	N/A	0.032	0.279	0.138	0.209	0.279	0.000	0.352	0.038

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	61	43	64	102	0	188	39
N.S.	1	1.00	1.42	1.00	1.49	2.37	0.00	4.37	0.91
time (sec)	N/A	0.026	0.046	0.212	0.200	0.296	0.000	0.369	0.122

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	205	70	73	112	0	163	60
N.S.	1	1.00	3.11	1.06	1.11	1.70	0.00	2.47	0.91
time (sec)	N/A	0.026	0.378	0.231	0.203	0.291	0.000	0.341	0.102

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	61	82	112	0	232	74
N.S.	1	1.00	1.16	1.05	1.41	1.93	0.00	4.00	1.28
time (sec)	N/A	0.027	0.020	0.338	0.204	0.355	0.000	0.357	0.179

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	90	56	68	105	56	55
N.S.	1	1.00	1.00	1.32	0.82	1.00	1.54	0.82	0.81
time (sec)	N/A	0.028	0.024	0.309	0.187	0.309	1.480	0.377	0.286

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	53	93	75	89	141	68	56
N.S.	1	1.00	0.66	1.16	0.94	1.11	1.76	0.85	0.70
time (sec)	N/A	0.032	0.288	0.311	0.285	0.290	1.154	0.502	1.463

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	61	44	56	82	41	45
N.S.	1	1.00	1.00	1.15	0.83	1.06	1.55	0.77	0.85
time (sec)	N/A	0.026	0.020	0.138	0.194	0.329	0.888	0.329	0.233

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	78	55	79	97	55	46
N.S.	1	1.00	0.75	1.37	0.96	1.39	1.70	0.96	0.81
time (sec)	N/A	0.028	0.200	0.148	0.285	0.286	0.725	0.320	0.508

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	52	35	48	63	35	32
N.S.	1	1.00	1.00	1.41	0.95	1.30	1.70	0.95	0.86
time (sec)	N/A	0.017	0.015	0.098	0.195	0.301	0.582	0.342	0.207

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	26	34	69	48	62	24
N.S.	1	1.00	1.22	0.96	1.26	2.56	1.78	2.30	0.89
time (sec)	N/A	0.012	0.009	0.095	0.279	0.307	0.507	0.387	0.207

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	54	25	38	42	25	22
N.S.	1	1.00	1.00	2.08	0.96	1.46	1.62	0.96	0.85
time (sec)	N/A	0.014	0.012	0.083	0.193	0.274	0.512	0.309	0.197

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	34	71	13	13
N.S.	1	1.00	1.00	1.47	0.87	2.27	4.73	0.87	0.87
time (sec)	N/A	0.017	0.007	0.062	0.203	0.266	0.731	0.313	0.142

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	26	24	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.73	1.60	0.87	0.87
time (sec)	N/A	0.013	0.009	0.049	0.188	0.272	0.415	0.296	0.202

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	40	50	94	0	52	32
N.S.	1	1.00	0.82	1.05	1.32	2.47	0.00	1.37	0.84
time (sec)	N/A	0.018	0.011	0.132	0.209	0.281	0.000	0.308	0.030

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	46	35	54	0	35	36
N.S.	1	1.00	1.22	1.24	0.95	1.46	0.00	0.95	0.97
time (sec)	N/A	0.024	0.116	0.145	0.186	0.308	0.000	0.326	0.175

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	31	68	73	130	0	72	61
N.S.	1	1.00	0.47	1.03	1.11	1.97	0.00	1.09	0.92
time (sec)	N/A	0.029	0.012	0.260	0.184	0.305	0.000	0.314	0.051

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	46	44	66	0	31	45
N.S.	1	1.00	0.81	0.87	0.83	1.25	0.00	0.58	0.85
time (sec)	N/A	0.025	0.040	0.234	0.181	0.284	0.000	0.315	0.218

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	31	86	91	140	0	85	79
N.S.	1	1.00	0.35	0.97	1.02	1.57	0.00	0.96	0.89
time (sec)	N/A	0.033	0.013	0.415	0.186	0.286	0.000	0.448	0.237

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	89	56	100	1664	277	92
N.S.	1	1.00	1.00	1.29	0.81	1.45	24.12	4.01	1.33
time (sec)	N/A	0.031	0.027	0.263	0.254	0.325	7.676	0.316	1.287

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	141	98	89	132	869	209	157
N.S.	1	1.00	1.58	1.10	1.00	1.48	9.76	2.35	1.76
time (sec)	N/A	0.036	0.128	0.213	0.242	0.302	4.654	0.350	0.315

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	81	49	90	733	232	74
N.S.	1	1.00	1.00	1.40	0.84	1.55	12.64	4.00	1.28
time (sec)	N/A	0.032	0.020	0.181	0.187	0.328	3.218	0.344	0.408

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	123	88	79	122	330	164	105
N.S.	1	1.00	1.76	1.26	1.13	1.74	4.71	2.34	1.50
time (sec)	N/A	0.024	0.079	0.143	0.187	0.292	1.713	0.313	0.350

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	33	38	70	61	164	52
N.S.	1	1.00	1.26	0.79	0.90	1.67	1.45	3.90	1.24
time (sec)	N/A	0.015	0.017	0.116	0.204	0.364	0.574	0.357	0.184

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	69	71	112	92	139	78
N.S.	1	1.00	2.05	1.25	1.29	2.04	1.67	2.53	1.42
time (sec)	N/A	0.031	0.072	0.108	0.183	0.292	0.988	0.335	0.242

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	25	39	44	25	25
N.S.	1	1.00	1.00	1.47	1.67	2.60	2.93	1.67	1.67
time (sec)	N/A	0.020	0.005	0.085	0.198	0.429	0.550	0.327	0.141

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	113	51	65	111	58	98	48
N.S.	1	1.00	2.05	0.93	1.18	2.02	1.05	1.78	0.87
time (sec)	N/A	0.036	0.084	0.094	0.189	0.505	0.929	0.323	0.216

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	27	24	13	23
N.S.	1	1.00	1.00	0.93	0.87	1.80	1.60	0.87	1.53
time (sec)	N/A	0.014	0.008	0.059	0.181	0.292	0.480	0.309	0.170

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	54	33	51	105	0	165	79
N.S.	1	1.00	1.35	0.82	1.28	2.62	0.00	4.12	1.98
time (sec)	N/A	0.020	0.016	0.145	0.181	0.371	0.000	0.311	0.196

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	129	70	79	132	0	163	66
N.S.	1	1.00	1.84	1.00	1.13	1.89	0.00	2.33	0.94
time (sec)	N/A	0.031	2.884	0.211	0.183	0.291	0.000	0.302	0.057

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	61	74	138	0	232	82
N.S.	1	1.00	0.93	1.05	1.28	2.38	0.00	4.00	1.41
time (sec)	N/A	0.026	0.223	0.314	0.192	0.307	0.000	0.505	0.108

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	268	88	91	148	0	209	78
N.S.	1	1.00	3.01	0.99	1.02	1.66	0.00	2.35	0.88
time (sec)	N/A	0.031	0.454	0.329	0.186	0.348	0.000	0.318	0.147

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	91	79	92	148	0	278	64
N.S.	1	1.00	1.32	1.14	1.33	2.14	0.00	4.03	0.93
time (sec)	N/A	0.030	0.048	0.469	0.180	0.334	0.000	0.317	0.141

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	18	14	33	29	14	19
N.S.	1	1.00	1.59	1.06	0.82	1.94	1.71	0.82	1.12
time (sec)	N/A	0.018	0.025	0.082	0.182	0.296	0.020	0.293	0.092

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	13
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.76
time (sec)	N/A	0.018	0.007	0.074	0.184	0.304	0.042	0.303	0.152

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	24	37	0	18
N.S.	1	1.00	1.00	0.86	0.82	1.09	1.68	0.00	0.82
time (sec)	N/A	0.021	0.023	0.086	0.192	0.314	2.885	0.000	0.149

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	37	21	18
N.S.	1	1.00	1.00	0.86	0.82	0.95	1.68	0.95	0.82
time (sec)	N/A	0.017	0.013	0.027	0.188	0.300	0.254	0.280	0.152

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	36	18	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.80	0.90	0.90
time (sec)	N/A	0.017	0.011	0.033	0.190	0.334	0.426	0.306	0.200

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	26	34	0	37
N.S.	1	1.00	1.00	0.95	0.90	1.30	1.70	0.00	1.85
time (sec)	N/A	0.019	0.017	0.029	0.183	0.310	0.771	0.000	0.271

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	36	0	53
N.S.	1	1.00	1.00	0.86	0.82	1.18	1.64	0.00	2.41
time (sec)	N/A	0.019	0.020	0.030	0.198	0.306	3.456	0.000	0.581

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	36	0	65
N.S.	1	1.00	1.00	0.86	0.82	1.18	1.64	0.00	2.95
time (sec)	N/A	0.019	0.027	0.027	0.192	0.292	29.620	0.000	6.663

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	0	0	65
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.00	0.00	2.95
time (sec)	N/A	0.016	0.041	0.032	0.187	0.315	0.000	0.000	3.909

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	60	249	0	119	0	0	0
N.S.	1	1.00	0.48	1.98	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.070	0.099	4.692	0.000	0.133	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	60	236	0	107	0	0	0
N.S.	1	1.00	0.48	1.87	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.072	0.095	1.938	0.000	0.133	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	223	0	106	0	0	0
N.S.	1	1.00	0.58	2.28	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.057	0.037	1.552	0.000	0.120	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	208	0	90	0	0	0
N.S.	1	1.00	0.58	2.12	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.050	0.046	0.660	0.000	0.124	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	194	0	87	0	0	0
N.S.	1	1.00	0.84	2.81	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.040	0.061	0.503	0.000	0.108	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	188	0	78	0	0	0
N.S.	1	1.00	0.84	2.72	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.042	0.071	0.362	0.000	0.091	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	198	0	105	0	0	0
N.S.	1	1.00	0.88	2.91	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.046	0.064	0.584	0.000	0.093	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	242	0	103	0	0	0
N.S.	1	1.00	0.83	3.36	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.042	0.058	0.592	0.000	0.099	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	59	365	0	120	0	0	0
N.S.	1	1.00	0.59	3.65	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.055	0.052	1.027	0.000	0.102	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	59	396	0	114	0	0	0
N.S.	1	1.00	0.59	3.96	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.060	0.047	1.371	0.000	0.100	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	37	36	34	73	53	0
N.S.	1	1.00	1.27	0.82	0.80	0.76	1.62	1.18	0.00
time (sec)	N/A	0.028	0.208	0.066	0.193	0.316	0.862	0.332	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	57	37	36	28	71	46	0
N.S.	1	1.00	1.33	0.86	0.84	0.65	1.65	1.07	0.00
time (sec)	N/A	0.029	0.126	0.072	0.185	0.279	0.781	0.341	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	35	36	70	0	0
N.S.	1	1.00	1.07	0.84	0.81	0.84	1.63	0.00	0.00
time (sec)	N/A	0.032	0.053	0.051	0.187	0.277	0.773	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	35	34	38	71	0	66
N.S.	1	1.00	1.12	0.81	0.79	0.88	1.65	0.00	1.53
time (sec)	N/A	0.033	0.069	0.055	0.189	0.287	3.543	0.000	0.810

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	70	37	37	38	71	0	93
N.S.	1	1.00	1.63	0.86	0.86	0.88	1.65	0.00	2.16
time (sec)	N/A	0.032	0.187	0.052	0.190	0.264	30.187	0.000	3.460

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	37	37	38	0	0	93
N.S.	1	1.00	1.56	0.82	0.82	0.84	0.00	0.00	2.07
time (sec)	N/A	0.034	0.202	0.048	0.184	0.269	0.000	0.000	3.662

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	94	37	37	38	0	0	279
N.S.	1	1.00	2.09	0.82	0.82	0.84	0.00	0.00	6.20
time (sec)	N/A	0.036	0.374	0.052	0.200	0.297	0.000	0.000	5.091

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	57	275	0	133	0	0	0
N.S.	1	1.00	0.37	1.76	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.107	0.101	13.672	0.000	0.132	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	57	262	0	121	0	0	0
N.S.	1	1.00	0.37	1.68	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.096	0.064	11.613	0.000	0.113	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	65	249	0	120	0	0	0
N.S.	1	1.00	0.51	1.95	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.093	0.054	11.091	0.000	0.121	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	65	255	0	102	0	0	0
N.S.	1	1.00	0.51	1.99	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.084	0.073	1.274	0.000	0.111	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	58	221	0	100	0	0	0
N.S.	1	1.00	0.59	2.23	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.066	0.045	0.952	0.000	0.100	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	58	208	0	88	0	0	0
N.S.	1	1.00	0.59	2.10	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.072	0.051	0.387	0.000	0.096	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	213	0	115	0	0	0
N.S.	1	1.00	0.60	2.13	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.074	0.046	0.480	0.000	0.104	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	286	0	113	0	0	0
N.S.	1	1.00	0.59	2.80	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.076	0.050	0.469	0.000	0.105	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	65	366	0	120	0	0	0
N.S.	1	1.00	0.64	3.59	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.078	0.051	0.552	0.000	0.130	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	65	398	0	114	0	0	0
N.S.	1	1.00	0.64	3.90	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.075	0.047	0.534	0.000	0.101	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	111	37	36	44	0	0	35
N.S.	1	1.00	2.13	0.71	0.69	0.85	0.00	0.00	0.67
time (sec)	N/A	0.024	0.186	0.092	0.203	0.305	0.000	0.000	0.374

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	83	331	98	313	0	0	0
N.S.	1	1.00	0.83	3.31	0.98	3.13	0.00	0.00	0.00
time (sec)	N/A	0.068	0.380	0.404	0.286	0.389	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	80	292	98	299	0	0	0
N.S.	1	1.00	0.81	2.95	0.99	3.02	0.00	0.00	0.00
time (sec)	N/A	0.048	0.268	0.368	0.275	0.402	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	68	253	83	281	0	0	0
N.S.	1	1.00	0.87	3.24	1.06	3.60	0.00	0.00	0.00
time (sec)	N/A	0.042	0.220	0.362	0.267	0.416	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	212	83	259	0	0	0
N.S.	1	1.00	0.84	2.75	1.08	3.36	0.00	0.00	0.00
time (sec)	N/A	0.041	0.160	0.108	0.271	0.371	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	182	67	238	0	48	0
N.S.	1	1.00	0.88	3.14	1.16	4.10	0.00	0.83	0.00
time (sec)	N/A	0.034	0.142	0.095	0.279	0.335	0.000	0.339	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	181	68	246	0	52	0
N.S.	1	1.00	0.85	3.07	1.15	4.17	0.00	0.88	0.00
time (sec)	N/A	0.035	0.131	0.101	0.270	0.344	0.000	0.348	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	441	79	309	0	0	0
N.S.	1	1.00	0.85	5.65	1.01	3.96	0.00	0.00	0.00
time (sec)	N/A	0.044	0.168	0.102	0.271	0.315	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	649	80	318	0	0	0
N.S.	1	1.00	0.86	8.01	0.99	3.93	0.00	0.00	0.00
time (sec)	N/A	0.042	0.209	0.129	0.288	0.334	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	81	862	100	342	0	0	0
N.S.	1	1.00	0.81	8.62	1.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.049	0.287	0.135	0.291	0.352	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	1057	102	342	0	0	0
N.S.	1	1.00	0.84	10.26	0.99	3.32	0.00	0.00	0.00
time (sec)	N/A	0.048	0.335	0.161	0.287	0.343	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	89	242	0	122	0	0	0
N.S.	1	1.00	0.72	1.95	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.071	0.723	3.504	0.000	0.115	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	74	229	0	121	0	0	0
N.S.	1	1.00	0.77	2.39	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.052	0.485	2.852	0.000	0.112	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	73	216	0	109	0	0	0
N.S.	1	1.00	0.76	2.25	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.056	0.384	2.441	0.000	0.104	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	58	203	0	105	0	0	0
N.S.	1	1.00	0.88	3.08	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.039	0.307	2.115	0.000	0.102	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	190	0	91	0	0	0
N.S.	1	1.00	0.85	2.88	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.039	0.237	0.405	0.000	0.109	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	203	0	102	0	0	0
N.S.	1	1.00	0.86	3.12	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.037	0.228	0.429	0.000	0.113	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	188	0	93	0	0	0
N.S.	1	1.00	0.73	2.94	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.036	0.230	0.394	0.000	0.097	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	209	0	131	0	0	0
N.S.	1	1.00	0.69	2.22	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.050	0.305	0.500	0.000	0.101	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	190	0	129	0	0	0
N.S.	1	1.00	0.63	1.94	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.061	0.309	0.701	0.000	0.097	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	408	0	145	0	0	0
N.S.	1	1.00	0.65	3.24	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.069	0.342	1.064	0.000	0.100	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	407	133	419	0	0	0
N.S.	1	1.00	1.01	3.01	0.99	3.10	0.00	0.00	0.00
time (sec)	N/A	0.058	1.868	6.099	0.276	0.465	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	371	118	405	0	0	0
N.S.	1	1.00	0.69	3.28	1.04	3.58	0.00	0.00	0.00
time (sec)	N/A	0.051	0.769	6.062	0.279	0.419	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	310	118	393	0	0	0
N.S.	1	1.00	1.04	2.74	1.04	3.48	0.00	0.00	0.00
time (sec)	N/A	0.053	1.039	6.025	0.279	0.419	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	286	103	380	0	0	0
N.S.	1	1.00	0.71	3.14	1.13	4.18	0.00	0.00	0.00
time (sec)	N/A	0.045	0.391	5.921	0.280	0.383	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	280	103	347	0	0	0
N.S.	1	1.00	0.84	3.08	1.13	3.81	0.00	0.00	0.00
time (sec)	N/A	0.044	0.283	0.085	0.270	0.334	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	62	275	102	340	0	95	0
N.S.	1	1.00	0.67	2.96	1.10	3.66	0.00	1.02	0.00
time (sec)	N/A	0.041	0.338	0.085	0.275	0.356	0.000	0.333	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	69	283	103	334	0	91	0
N.S.	1	1.00	0.74	3.04	1.11	3.59	0.00	0.98	0.00
time (sec)	N/A	0.040	0.327	0.091	0.278	0.360	0.000	0.377	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	91	689	117	406	0	0	0
N.S.	1	1.00	0.79	5.99	1.02	3.53	0.00	0.00	0.00
time (sec)	N/A	0.051	0.350	0.115	0.284	0.341	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	92	885	117	418	0	0	0
N.S.	1	1.00	0.80	7.70	1.02	3.63	0.00	0.00	0.00
time (sec)	N/A	0.055	0.448	0.134	0.275	0.360	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	102	1140	134	438	0	0	0
N.S.	1	1.00	0.74	8.32	0.98	3.20	0.00	0.00	0.00
time (sec)	N/A	0.057	0.534	0.181	0.286	0.365	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	37	21	18
N.S.	1	1.00	1.00	0.86	0.82	0.95	1.68	0.95	0.82
time (sec)	N/A	0.014	0.014	0.031	0.202	0.297	1.566	0.324	0.094

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	14	170	13	25
N.S.	1	1.00	0.86	0.67	0.62	0.67	8.10	0.62	1.19
time (sec)	N/A	0.015	0.012	0.071	0.196	0.328	3.658	0.312	0.217

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	24	13	25
N.S.	1	1.00	0.86	0.67	0.62	0.95	1.14	0.62	1.19
time (sec)	N/A	0.015	0.011	0.047	0.205	0.310	3.275	0.318	0.191

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	24	13	25
N.S.	1	1.00	0.86	0.67	0.62	0.95	1.14	0.62	1.19
time (sec)	N/A	0.015	0.010	5.122	0.204	0.309	32.682	0.326	0.171

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	323	13	25
N.S.	1	1.00	0.84	0.74	0.68	0.63	17.00	0.68	1.32
time (sec)	N/A	0.015	0.008	0.033	0.196	0.288	4.339	0.318	0.188

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	70	423	0	0	0	0	0
N.S.	1	1.00	0.53	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.081	0.906	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	409	0	0	0	0	0
N.S.	1	1.00	0.74	4.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.063	0.342	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	67	393	0	0	0	0	0
N.S.	1	1.00	1.26	7.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.043	0.258	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	70	357	0	172	0	0	0
N.S.	1	1.00	0.75	3.84	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.067	0.076	0.500	0.000	0.135	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	412	0	193	0	0	0
N.S.	1	1.00	0.52	3.07	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.102	0.097	0.309	0.000	0.131	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	70	437	0	1033	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	0.180	0.084	2.829	0.000	0.475	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	67	362	0	933	0	0	0
N.S.	1	1.00	0.24	1.29	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.112	0.042	0.394	0.000	0.457	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	35	0	42	0	0	50
N.S.	1	1.00	1.00	0.95	0.00	1.14	0.00	0.00	1.35
time (sec)	N/A	0.031	0.060	0.226	0.000	0.309	0.000	0.000	0.767

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	54	0	54	0	0	95
N.S.	1	1.00	0.76	0.72	0.00	0.72	0.00	0.00	1.27
time (sec)	N/A	0.065	0.168	0.229	0.000	0.329	0.000	0.000	1.916

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	67	65	0	64	0	0	216
N.S.	1	1.00	0.60	0.58	0.00	0.57	0.00	0.00	1.93
time (sec)	N/A	0.102	0.169	0.340	0.000	0.379	0.000	0.000	6.247

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	131	131	71	1744	0	0	0	0	0
N.S.	1	1.00	0.54	13.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.089	1.474	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	67	211	0	0	0	0	0
N.S.	1	1.00	0.72	2.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.058	1.343	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	213	0	108	0	0	0
N.S.	1	1.00	0.68	2.17	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.073	0.113	0.193	0.000	0.113	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	70	227	0	121	0	0	0
N.S.	1	1.00	0.53	1.71	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.112	0.101	0.237	0.000	0.110	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	67	433	0	1015	0	0	0
N.S.	1	1.00	0.21	1.35	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.169	0.049	0.197	0.000	0.453	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	313	313	67	687	0	1092	0	0	0
N.S.	1	1.00	0.21	2.19	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.180	0.112	0.250	0.000	0.520	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	50	0	0	64
N.S.	1	1.00	1.08	1.03	0.00	1.35	0.00	0.00	1.73
time (sec)	N/A	0.039	0.077	0.202	0.000	0.328	0.000	0.000	1.332

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	59	0	61	0	0	207
N.S.	1	1.00	0.54	0.56	0.00	0.58	0.00	0.00	1.95
time (sec)	N/A	0.113	0.210	0.155	0.000	0.376	0.000	0.000	6.178

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	67	68	0	73	0	0	193
N.S.	1	1.00	0.48	0.48	0.00	0.52	0.00	0.00	1.37
time (sec)	N/A	0.146	0.217	0.310	0.000	0.446	0.000	0.000	6.656

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	72	439	0	0	0	0	0
N.S.	1	1.00	0.43	2.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.122	1.700	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	70	426	0	0	0	0	0
N.S.	1	1.00	0.53	3.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.123	1.421	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	67	410	0	0	0	0	0
N.S.	1	1.00	0.71	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.065	0.263	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	67	398	0	0	0	0	0
N.S.	1	1.00	0.71	4.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.084	0.217	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	70	416	0	212	0	0	0
N.S.	1	1.00	0.53	3.13	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.117	0.124	0.227	0.000	0.140	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	72	442	0	224	0	0	0
N.S.	1	1.00	0.43	2.63	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.151	0.119	0.238	0.000	0.119	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	67	447	0	1028	0	0	0
N.S.	1	1.00	0.21	1.40	0.00	3.21	0.00	0.00	0.00
time (sec)	N/A	0.169	0.081	0.421	0.000	0.527	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	315	315	67	492	0	1164	0	0	0
N.S.	1	1.00	0.21	1.56	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.176	0.084	0.260	0.000	0.498	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	40	0	60	0	0	89
N.S.	1	1.00	1.08	1.08	0.00	1.62	0.00	0.00	2.41
time (sec)	N/A	0.034	0.101	0.193	0.000	0.365	0.000	0.000	1.769

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	57	61	0	74	0	0	176
N.S.	1	1.00	0.54	0.58	0.00	0.70	0.00	0.00	1.66
time (sec)	N/A	0.104	0.220	0.145	0.000	0.429	0.000	0.000	6.541

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	67	70	0	87	0	0	207
N.S.	1	1.00	0.48	0.50	0.00	0.62	0.00	0.00	1.47
time (sec)	N/A	0.150	0.318	0.309	0.000	0.594	0.000	0.000	6.508

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	57	892	0	766	0	0	44
N.S.	1	1.00	0.25	3.95	0.00	3.39	0.00	0.00	0.19
time (sec)	N/A	0.091	0.037	2.293	0.000	0.476	0.000	0.000	1.949

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	11	0	16	0	0	53
N.S.	1	1.00	1.00	0.69	0.00	1.00	0.00	0.00	3.31
time (sec)	N/A	0.013	0.014	0.408	0.000	0.311	0.000	0.000	0.652

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	38	195	0	305	0	0	25
N.S.	1	1.00	0.31	1.60	0.00	2.50	0.00	0.00	0.20
time (sec)	N/A	0.051	0.012	2.773	0.000	0.422	0.000	0.000	0.460

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	38	388	0	315	0	0	25
N.S.	1	1.00	0.27	2.71	0.00	2.20	0.00	0.00	0.17
time (sec)	N/A	0.067	0.010	0.292	0.000	0.377	0.000	0.000	0.552

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	132	70	1727	0	0	0	0	0
N.S.	1	1.00	0.53	13.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.083	0.511	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	68	203	0	0	0	0	0
N.S.	1	1.00	0.74	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	0.079	0.259	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	65	113	0	62	0	0	0
N.S.	1	1.00	1.23	2.13	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.065	0.043	0.220	0.000	0.110	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	65	207	0	108	0	0	0
N.S.	1	1.00	0.67	2.13	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.134	0.071	0.235	0.000	0.109	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	226	0	122	0	0	0
N.S.	1	1.00	0.52	1.69	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.189	0.092	0.276	0.000	0.114	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	65	362	0	911	0	0	0
N.S.	1	1.00	0.23	1.29	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.226	0.042	0.885	0.000	0.436	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	36	35	0	39	0	0	31
N.S.	1	1.00	1.03	1.00	0.00	1.11	0.00	0.00	0.89
time (sec)	N/A	0.069	0.044	0.214	0.000	0.310	0.000	0.000	0.605

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	51	0	51	0	0	77
N.S.	1	1.00	0.69	0.68	0.00	0.68	0.00	0.00	1.03
time (sec)	N/A	0.078	0.117	0.174	0.000	0.338	0.000	0.000	1.332

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	67	65	0	61	0	0	123
N.S.	1	1.00	0.60	0.58	0.00	0.54	0.00	0.00	1.10
time (sec)	N/A	0.121	0.171	0.188	0.000	0.339	0.000	0.000	3.676

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	55	343	0	651	0	0	44
N.S.	1	1.00	0.32	1.97	0.00	3.74	0.00	0.00	0.25
time (sec)	N/A	0.064	0.021	0.317	0.000	0.406	0.000	0.000	1.396

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	55	650	0	766	0	0	44
N.S.	1	1.00	0.28	3.27	0.00	3.85	0.00	0.00	0.22
time (sec)	N/A	0.082	0.025	0.342	0.000	0.426	0.000	0.000	1.505

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	57	737	0	803	0	0	44
N.S.	1	1.00	0.28	3.67	0.00	4.00	0.00	0.00	0.22
time (sec)	N/A	0.079	0.025	4.133	0.000	0.398	0.000	0.000	1.785

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	57	799	0	901	0	0	44
N.S.	1	1.00	0.25	3.54	0.00	3.99	0.00	0.00	0.19
time (sec)	N/A	0.100	0.031	0.373	0.000	0.406	0.000	0.000	1.786

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	144	0	0	44
N.S.	1	1.00	0.45	0.00	0.00	1.12	0.00	0.00	0.34
time (sec)	N/A	0.107	0.030	0.000	0.000	0.304	0.000	0.000	1.074

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	57	0	0	390	0	0	44
N.S.	1	1.00	0.25	0.00	0.00	1.74	0.00	0.00	0.20
time (sec)	N/A	0.209	0.031	0.000	0.000	0.340	0.000	0.000	0.826

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	249	57	0	0	444	0	0	44
N.S.	1	1.00	0.23	0.00	0.00	1.78	0.00	0.00	0.18
time (sec)	N/A	0.229	0.039	0.000	0.000	0.312	0.000	0.000	1.533

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	197	0	0	44
N.S.	1	1.00	0.37	0.00	0.00	1.27	0.00	0.00	0.28
time (sec)	N/A	0.109	0.038	0.000	0.000	0.323	0.000	0.000	1.013

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	195	0	0	44
N.S.	1	1.00	0.37	0.00	0.00	1.26	0.00	0.00	0.28
time (sec)	N/A	0.067	0.037	0.000	0.000	0.302	0.000	0.000	1.558

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	152	0	0	44
N.S.	1	1.00	0.45	0.00	0.00	1.19	0.00	0.00	0.34
time (sec)	N/A	0.049	0.021	0.000	0.000	0.312	0.000	0.000	1.371

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	225	55	0	0	388	0	0	44
N.S.	1	1.00	0.24	0.00	0.00	1.72	0.00	0.00	0.20
time (sec)	N/A	0.183	0.021	0.000	0.000	0.324	0.000	0.000	0.948

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	250	250	55	0	0	446	0	0	44
N.S.	1	1.00	0.22	0.00	0.00	1.78	0.00	0.00	0.18
time (sec)	N/A	0.204	0.024	0.000	0.000	0.345	0.000	0.000	1.596

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	189	0	0	44
N.S.	1	1.00	0.37	0.00	0.00	1.22	0.00	0.00	0.28
time (sec)	N/A	0.068	0.025	0.000	0.000	0.317	0.000	0.000	1.030

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	219	0	0	44
N.S.	1	1.00	0.37	0.00	0.00	1.41	0.00	0.00	0.28
time (sec)	N/A	0.073	0.026	0.000	0.000	0.348	0.000	0.000	1.853

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	18	0	0	10
N.S.	1	1.00	1.00	0.00	0.00	1.12	0.00	0.00	0.62
time (sec)	N/A	0.013	0.009	0.000	0.000	0.324	0.000	0.000	0.585

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	10	0	0	94
N.S.	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	5.88
time (sec)	N/A	0.013	0.012	0.000	0.000	0.274	0.000	0.000	0.634

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0	71
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.024	0.075	0.000	0.000	0.000	0.000	0.000	2.251

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	85	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	87	77	70	2040	248	132
N.S.	1	1.00	0.74	1.18	1.04	0.95	27.57	3.35	1.78
time (sec)	N/A	0.042	0.227	0.859	0.209	0.312	4.409	0.334	1.467

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	50	53	46	525	118	62
N.S.	1	1.00	0.96	1.00	1.06	0.92	10.50	2.36	1.24
time (sec)	N/A	0.031	0.064	0.560	0.206	0.273	1.261	0.329	0.683

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	24	56	24	25
N.S.	1	1.00	1.04	1.04	1.00	1.00	2.33	1.00	1.04
time (sec)	N/A	0.014	0.007	1.286	0.202	0.285	0.348	0.324	0.348

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.039	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.010	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	78	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	83	87	78	84	2451	249	132
N.S.	1	1.00	1.09	1.14	1.03	1.11	32.25	3.28	1.74
time (sec)	N/A	0.043	0.411	2.082	0.304	0.288	4.419	0.324	1.399

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	52	52	50	688	117	65
N.S.	1	1.00	1.00	1.04	1.04	1.00	13.76	2.34	1.30
time (sec)	N/A	0.030	0.236	0.629	0.211	0.304	1.172	0.309	0.544

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	26	25	25	60	25	26
N.S.	1	1.00	1.04	1.04	1.00	1.00	2.40	1.00	1.04
time (sec)	N/A	0.014	0.008	0.895	0.187	0.285	0.343	0.299	0.190

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	52	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	154	0	0	0	0	0	0
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.898	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	244	0	0	0	0	0	0
N.S.	1	1.00	4.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	1.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.277	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.324	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	158	0	0	0	0	0	0
N.S.	1	1.00	2.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.363	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	79	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.445	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	445	63	56	0	100	0
N.S.	1	1.00	0.68	5.24	0.74	0.66	0.00	1.18	0.00
time (sec)	N/A	0.035	0.580	1.011	0.220	0.285	0.000	0.303	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	435	50	46	0	77	0
N.S.	1	1.00	0.76	6.90	0.79	0.73	0.00	1.22	0.00
time (sec)	N/A	0.033	0.347	0.209	0.215	0.284	0.000	0.302	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	425	35	34	0	53	0
N.S.	1	1.00	0.88	10.37	0.85	0.83	0.00	1.29	0.00
time (sec)	N/A	0.029	0.311	0.260	0.220	0.308	0.000	0.308	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	23	0	22	23
N.S.	1	1.00	1.00	0.94	1.28	1.28	0.00	1.22	1.28
time (sec)	N/A	0.021	0.121	0.083	0.215	0.312	0.000	0.304	0.231

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	73	151	72	247	0	61	0
N.S.	1	1.00	1.26	2.60	1.24	4.26	0.00	1.05	0.00
time (sec)	N/A	0.031	0.305	0.194	0.295	0.318	0.000	0.309	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	95	461	106	354	0	98	0
N.S.	1	1.00	1.02	4.96	1.14	3.81	0.00	1.05	0.00
time (sec)	N/A	0.048	0.576	0.305	0.292	0.323	0.000	0.343	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	509	138	438	0	127	0
N.S.	1	1.00	0.87	4.14	1.12	3.56	0.00	1.03	0.00
time (sec)	N/A	0.051	0.788	0.234	0.328	0.347	0.000	0.467	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	179	0	106	0	0	0
N.S.	1	1.00	0.59	1.46	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.095	0.344	4.207	0.000	0.101	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	61	162	0	94	0	0	0
N.S.	1	1.00	0.64	1.71	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.059	0.266	0.859	0.000	0.100	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	147	0	83	0	0	0
N.S.	1	1.00	0.76	2.19	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.043	0.211	0.485	0.000	0.100	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	77	0	57	0	0	35
N.S.	1	1.00	1.00	2.03	0.00	1.50	0.00	0.00	0.92
time (sec)	N/A	0.016	0.039	0.340	0.000	0.091	0.000	0.000	0.176

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	47	139	0	98	0	0	0
N.S.	1	1.00	0.76	2.24	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.042	0.277	0.408	0.000	0.102	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	186	0	148	0	0	0
N.S.	1	1.00	0.66	1.96	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.061	0.335	0.585	0.000	0.091	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	205	0	187	0	0	0
N.S.	1	1.00	0.59	1.67	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.095	0.595	0.903	0.000	0.125	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	52	256	72	54	0	98	0
N.S.	1	1.00	0.63	3.08	0.87	0.65	0.00	1.18	0.00
time (sec)	N/A	0.045	0.393	0.803	0.203	0.272	0.000	0.303	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	835	55	43	0	75	0
N.S.	1	1.00	0.67	13.25	0.87	0.68	0.00	1.19	0.00
time (sec)	N/A	0.038	0.238	0.237	0.196	0.289	0.000	0.290	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	825	37	31	0	46	0
N.S.	1	1.00	0.73	20.12	0.90	0.76	0.00	1.12	0.00
time (sec)	N/A	0.035	0.217	0.232	0.214	0.288	0.000	0.425	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	18	0	25	18
N.S.	1	1.00	1.00	0.94	1.28	1.00	0.00	1.39	1.00
time (sec)	N/A	0.024	0.099	0.047	0.209	0.273	0.000	0.295	0.226

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	85	220	87	278	0	75	0
N.S.	1	1.00	1.10	2.86	1.13	3.61	0.00	0.97	0.00
time (sec)	N/A	0.037	0.318	0.230	0.288	0.347	0.000	0.305	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	601	123	388	0	127	0
N.S.	1	1.00	0.86	5.32	1.09	3.43	0.00	1.12	0.00
time (sec)	N/A	0.054	0.672	0.235	0.299	0.348	0.000	0.293	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	70	203	0	108	0	0	0
N.S.	1	1.00	0.55	1.59	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.101	0.326	2.596	0.000	0.106	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	60	194	0	98	0	0	0
N.S.	1	1.00	0.61	1.98	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.270	1.425	0.000	0.108	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	175	0	85	0	0	0
N.S.	1	1.00	0.73	2.65	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.044	0.204	0.678	0.000	0.105	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	392	0	84	0	0	0
N.S.	1	1.00	0.73	5.94	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.027	0.055	0.509	0.000	0.139	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	57	265	0	113	0	0	0
N.S.	1	1.00	0.63	2.94	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.055	0.301	0.637	0.000	0.102	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	77	276	0	167	0	0	0
N.S.	1	1.00	0.62	2.23	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.090	0.371	0.934	0.000	0.126	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	52	446	66	70	0	100	0
N.S.	1	1.00	0.61	5.25	0.78	0.82	0.00	1.18	0.00
time (sec)	N/A	0.046	0.646	0.870	0.198	0.289	0.000	0.310	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	436	52	57	0	74	0
N.S.	1	1.00	0.67	6.92	0.83	0.90	0.00	1.17	0.00
time (sec)	N/A	0.036	0.465	0.257	0.211	0.291	0.000	0.383	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	319	36	42	0	49	50
N.S.	1	1.00	0.78	7.78	0.88	1.02	0.00	1.20	1.22
time (sec)	N/A	0.030	0.312	65.197	0.204	0.272	0.000	0.303	0.495

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	39
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.95
time (sec)	N/A	0.023	0.097	2.855	0.219	0.305	0.000	0.341	0.382

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	212	87	328	0	86	0
N.S.	1	1.00	1.12	2.72	1.12	4.21	0.00	1.10	0.00
time (sec)	N/A	0.036	0.263	1.555	0.282	0.325	0.000	0.294	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	109	502	123	448	0	121	0
N.S.	1	1.00	0.96	4.44	1.09	3.96	0.00	1.07	0.00
time (sec)	N/A	0.053	1.464	31.803	0.300	0.340	0.000	0.310	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	119	556	155	542	0	150	0
N.S.	1	1.00	0.83	3.89	1.08	3.79	0.00	1.05	0.00
time (sec)	N/A	0.061	1.021	246.762	0.282	0.362	0.000	0.336	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	74	174	0	129	0	0	0
N.S.	1	1.00	0.57	1.34	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.093	0.425	1396.795	0.000	0.107	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	64	155	0	114	0	0	0
N.S.	1	1.00	0.64	1.55	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.064	0.278	201.960	0.000	0.101	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	142	0	101	0	0	0
N.S.	1	1.00	0.74	2.03	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.043	0.242	15.908	0.000	0.090	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	51	144	0	101	0	0	0
N.S.	1	1.00	0.73	2.06	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.025	0.073	0.701	0.000	0.102	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	159	0	131	0	0	0
N.S.	1	1.00	0.68	1.62	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.065	0.332	7.194	0.000	0.103	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	79	204	0	193	0	0	0
N.S.	1	1.00	0.64	1.66	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.097	0.439	95.065	0.000	0.108	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	55	63	61	0	108	0
N.S.	1	1.00	0.60	0.63	0.72	0.70	0.00	1.24	0.00
time (sec)	N/A	0.038	0.347	0.252	0.203	0.318	0.000	0.303	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	45	50	51	0	85	0
N.S.	1	1.00	0.65	0.69	0.77	0.78	0.00	1.31	0.00
time (sec)	N/A	0.032	0.258	0.185	0.202	0.293	0.000	0.314	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	35	37	41	0	62	0
N.S.	1	1.00	0.74	0.81	0.86	0.95	0.00	1.44	0.00
time (sec)	N/A	0.030	0.215	0.168	0.214	0.303	0.000	0.303	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.40
time (sec)	N/A	0.020	0.103	0.047	0.185	0.325	0.000	0.304	0.286

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	73	142	73	253	0	56	0
N.S.	1	1.00	1.24	2.41	1.24	4.29	0.00	0.95	0.00
time (sec)	N/A	0.028	0.183	0.183	0.292	0.370	0.000	0.298	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	281	105	361	0	104	0
N.S.	1	1.00	1.00	3.02	1.13	3.88	0.00	1.12	0.00
time (sec)	N/A	0.043	0.527	0.210	0.288	0.339	0.000	0.312	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	107	345	138	450	0	135	0
N.S.	1	1.00	0.87	2.80	1.12	3.66	0.00	1.10	0.00
time (sec)	N/A	0.053	0.627	0.218	0.313	0.344	0.000	0.306	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	73	483	0	117	0	0	0
N.S.	1	1.00	0.59	3.93	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.090	0.494	3.903	0.000	0.120	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	63	451	0	107	0	0	0
N.S.	1	1.00	0.66	4.75	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.066	0.390	1.533	0.000	0.097	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	60	417	0	94	0	0	0
N.S.	1	1.00	0.90	6.22	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.037	0.242	0.738	0.000	0.133	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	299	0	66	0	0	0
N.S.	1	1.00	1.00	7.87	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.014	0.042	0.421	0.000	0.098	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	48	253	0	109	0	0	0
N.S.	1	1.00	0.76	4.02	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.040	0.273	0.286	0.000	0.089	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	74	305	0	158	0	0	0
N.S.	1	1.00	0.78	3.21	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.062	0.271	0.465	0.000	0.095	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	333	0	199	0	0	0
N.S.	1	1.00	0.70	2.71	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.099	0.378	0.755	0.000	0.104	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	52	60	63	61	0	110	0
N.S.	1	1.00	0.60	0.69	0.72	0.70	0.00	1.26	0.00
time (sec)	N/A	0.044	0.528	0.220	0.194	0.319	0.000	0.336	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	50	50	51	0	87	0
N.S.	1	1.00	0.65	0.77	0.77	0.78	0.00	1.34	0.00
time (sec)	N/A	0.039	0.329	0.175	0.207	0.306	0.000	0.345	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	40	37	41	0	64	0
N.S.	1	1.00	0.74	0.93	0.86	0.95	0.00	1.49	0.00
time (sec)	N/A	0.034	0.268	0.175	0.212	0.325	0.000	0.321	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	35	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.75	1.40
time (sec)	N/A	0.023	0.102	0.048	0.239	0.289	0.000	0.286	0.308

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	36	197	89	314	0	73	0
N.S.	1	1.00	0.46	2.53	1.14	4.03	0.00	0.94	0.00
time (sec)	N/A	0.035	0.313	0.185	0.344	0.433	0.000	0.289	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	98	281	106	364	0	93	0
N.S.	1	1.00	1.05	3.02	1.14	3.91	0.00	1.00	0.00
time (sec)	N/A	0.051	0.443	0.192	0.302	0.371	0.000	0.306	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	109	342	137	454	0	128	0
N.S.	1	1.00	0.89	2.78	1.11	3.69	0.00	1.04	0.00
time (sec)	N/A	0.061	0.578	0.195	0.293	0.352	0.000	0.302	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	81	176	0	109	0	0	0
N.S.	1	1.00	0.64	1.40	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.088	0.309	1.158	0.000	0.111	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	71	160	0	99	0	0	0
N.S.	1	1.00	0.72	1.63	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.059	0.225	0.648	0.000	0.104	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	144	0	87	0	0	0
N.S.	1	1.00	0.82	2.00	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.026	0.065	0.365	0.000	0.095	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	143	0	101	0	0	0
N.S.	1	1.00	0.85	2.10	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.044	0.263	0.323	0.000	0.098	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	62	181	0	148	0	0	0
N.S.	1	1.00	0.61	1.77	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.067	0.332	0.325	0.000	0.107	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	74	210	0	197	0	0	0
N.S.	1	1.00	0.56	1.59	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.095	0.512	0.517	0.000	0.106	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	62	60	63	61	0	110	0
N.S.	1	1.00	0.71	0.69	0.72	0.70	0.00	1.26	0.00
time (sec)	N/A	0.039	0.545	0.248	0.192	0.316	0.000	0.314	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	50	50	51	0	87	0
N.S.	1	1.00	0.80	0.77	0.77	0.78	0.00	1.34	0.00
time (sec)	N/A	0.035	0.335	0.183	0.179	0.324	0.000	0.318	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	40	37	41	0	64	0
N.S.	1	1.00	0.98	0.93	0.86	0.95	0.00	1.49	0.00
time (sec)	N/A	0.032	0.292	0.167	0.191	0.286	0.000	0.306	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	35	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.75	1.40
time (sec)	N/A	0.022	0.126	0.045	0.179	0.291	0.000	0.304	0.311

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	90	318	90	319	0	81	0
N.S.	1	1.00	1.11	3.93	1.11	3.94	0.00	1.00	0.00
time (sec)	N/A	0.034	0.292	0.185	0.275	0.410	0.000	0.309	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	98	290	106	370	0	101	0
N.S.	1	1.00	1.05	3.12	1.14	3.98	0.00	1.09	0.00
time (sec)	N/A	0.045	0.679	0.192	0.270	0.358	0.000	0.299	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	110	348	136	458	0	131	0
N.S.	1	1.00	0.89	2.83	1.11	3.72	0.00	1.07	0.00
time (sec)	N/A	0.055	1.216	0.191	0.264	0.388	0.000	0.307	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	83	486	0	117	0	0	0
N.S.	1	1.00	0.66	3.86	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.084	0.569	2.641	0.000	0.129	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	454	0	107	0	0	0
N.S.	1	1.00	0.67	4.63	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.057	0.447	1.690	0.000	0.111	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	420	0	95	0	0	0
N.S.	1	1.00	0.83	5.83	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.026	0.082	0.842	0.000	0.110	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	268	0	109	0	0	0
N.S.	1	1.00	0.75	3.94	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.044	0.349	0.580	0.000	0.102	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	79	308	0	162	0	0	0
N.S.	1	1.00	0.77	3.02	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.074	0.357	0.456	0.000	0.110	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	87	336	0	203	0	0	0
N.S.	1	1.00	0.66	2.55	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.094	0.520	0.711	0.000	0.112	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	169	537	0	1145	0	0	0
N.S.	1	1.00	0.38	1.20	0.00	2.55	0.00	0.00	0.00
time (sec)	N/A	0.288	2.097	3.492	0.000	0.514	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	157	447	0	1129	0	0	0
N.S.	1	1.00	0.38	1.08	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.230	1.322	4.457	0.000	0.500	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	122	362	0	1041	0	0	0
N.S.	1	1.00	0.32	0.96	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.166	1.021	3.991	0.000	0.494	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	37	35	0	44	0	0	36
N.S.	1	1.00	1.12	1.06	0.00	1.33	0.00	0.00	1.09
time (sec)	N/A	0.032	0.401	0.655	0.000	0.287	0.000	0.000	0.607

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	52	53	0	73	0	0	83
N.S.	1	1.00	0.73	0.75	0.00	1.03	0.00	0.00	1.17
time (sec)	N/A	0.066	0.560	0.708	0.000	0.298	0.000	0.000	1.563

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	65	65	0	96	0	0	169
N.S.	1	1.00	0.61	0.61	0.00	0.91	0.00	0.00	1.59
time (sec)	N/A	0.099	0.766	0.768	0.000	0.326	0.000	0.000	5.746

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	128	90	1739	0	0	0	0	0
N.S.	1	1.00	0.70	13.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	15.837	7.714	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	211	0	0	0	0	0
N.S.	1	1.00	0.73	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	11.002	1.882	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	113	0	59	0	0	0
N.S.	1	1.00	1.25	2.13	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.109	0.548	0.866	0.000	0.100	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	207	0	125	0	0	0
N.S.	1	1.00	0.79	2.18	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.185	0.704	0.843	0.000	0.124	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	111	229	0	173	0	0	0
N.S.	1	1.00	0.85	1.76	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.246	1.296	0.934	0.000	0.120	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	398	0	0	0	0	0
N.S.	1	1.00	0.75	3.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.671	0.984	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	74	385	0	0	0	0	0
N.S.	1	1.00	0.87	4.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.394	0.935	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	371	0	0	0	0	0
N.S.	1	1.00	1.18	7.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	1.148	0.836	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	63	356	0	171	0	0	0
N.S.	1	1.00	0.78	4.40	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.087	0.381	0.761	0.000	0.117	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	82	441	0	239	0	0	0
N.S.	1	1.00	0.71	3.83	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.115	0.539	0.888	0.000	0.133	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	145	704	0	895	0	0	0
N.S.	1	1.00	0.40	1.94	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.178	1.332	1.359	0.000	0.419	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	113	351	0	843	0	0	0
N.S.	1	1.00	0.34	1.07	0.00	2.57	0.00	0.00	0.00
time (sec)	N/A	0.135	0.808	4.809	0.000	0.432	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	0	48	0	0	57
N.S.	1	1.00	1.00	1.00	0.00	1.60	0.00	0.00	1.90
time (sec)	N/A	0.034	0.242	0.587	0.000	0.302	0.000	0.000	0.976

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	43	0	72	0	0	103
N.S.	1	1.00	0.69	0.70	0.00	1.18	0.00	0.00	1.69
time (sec)	N/A	0.053	0.326	0.631	0.000	0.312	0.000	0.000	2.415

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	52	53	0	95	0	0	163
N.S.	1	1.00	0.57	0.58	0.00	1.04	0.00	0.00	1.79
time (sec)	N/A	0.081	0.496	0.707	0.000	0.349	0.000	0.000	6.075

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	63	0	116	0	0	192
N.S.	1	1.00	0.51	0.52	0.00	0.96	0.00	0.00	1.59
time (sec)	N/A	0.105	0.705	0.802	0.000	0.380	0.000	0.000	6.147

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	490	176	624	0	1201	0	0	0
N.S.	1	1.00	0.36	1.27	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	0.339	1.850	5.786	0.000	0.591	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	453	453	165	534	0	1188	0	0	0
N.S.	1	1.00	0.36	1.18	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.289	1.389	5.013	0.000	0.568	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	151	439	0	1127	0	0	0
N.S.	1	1.00	0.36	1.05	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	0.226	1.135	4.751	0.000	0.525	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	144	455	0	1224	0	0	0
N.S.	1	1.00	0.35	1.11	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.244	1.249	4.513	0.000	0.508	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	45	40	0	68	0	0	84
N.S.	1	1.00	1.29	1.14	0.00	1.94	0.00	0.00	2.40
time (sec)	N/A	0.040	0.555	0.566	0.000	0.315	0.000	0.000	1.641

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	103	243	0	0	0	0	0
N.S.	1	1.00	0.60	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	1.043	2.283	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	135	87	1746	0	0	0	0	0
N.S.	1	1.00	0.64	12.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.481	1.967	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	205	0	0	0	0	0
N.S.	1	1.00	0.89	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.663	1.741	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	78	208	0	131	0	0	0
N.S.	1	1.00	0.78	2.08	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.103	0.665	0.751	0.000	0.114	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	119	231	0	178	0	0	0
N.S.	1	1.00	0.87	1.69	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.138	1.158	0.819	0.000	0.117	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	131	253	0	230	0	0	0
N.S.	1	1.00	0.75	1.45	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.182	1.523	0.914	0.000	0.111	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	5.941	0.000	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	5.680	0.000	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	106	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	5.460	0.000	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	89	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	18.322	0.000	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	116	0	0	0	0	0	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	7.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	285	0	0	0	0	0	0
N.S.	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	1.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.350	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	289	0	0	0	0	0	0
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	89	85	85	0	0	134
N.S.	1	1.00	1.00	1.11	1.06	1.06	0.00	0.00	1.68
time (sec)	N/A	0.048	0.611	2.259	0.191	0.298	0.000	0.000	1.396

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	47	54	59	53	0	0	67
N.S.	1	1.00	0.90	1.04	1.13	1.02	0.00	0.00	1.29
time (sec)	N/A	0.035	0.267	0.925	0.205	0.303	0.000	0.000	0.714

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	28	28	28	0	0	27
N.S.	1	1.00	0.88	1.12	1.12	1.12	0.00	0.00	1.08
time (sec)	N/A	0.021	0.068	0.693	0.190	0.302	0.000	0.000	0.208

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	50	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	201	0	0	0	0	0	0
N.S.	1	1.00	4.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	2.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.300	0.000	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	57	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	104	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	31.911	0.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	10.377	0.000	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	72	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	10.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	10.500	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	67	252	0	95	0	0	0
N.S.	1	1.00	0.67	2.52	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.051	0.268	1.019	0.000	0.119	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	248	0	93	0	0	0
N.S.	1	1.00	0.83	3.31	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.034	0.173	0.890	0.000	0.108	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	233	0	82	0	0	0
N.S.	1	1.00	0.76	3.24	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.036	0.180	0.875	0.000	0.101	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	314	0	59	0	0	0
N.S.	1	1.00	0.98	7.14	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.021	0.050	0.947	0.000	0.100	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	120	0	55	0	0	63
N.S.	1	1.00	0.98	2.79	0.00	1.28	0.00	0.00	1.47
time (sec)	N/A	0.014	0.043	0.753	0.000	0.092	0.000	0.000	0.382

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	57	409	0	80	0	0	0
N.S.	1	1.00	0.84	6.01	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.028	0.111	0.918	0.000	0.091	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	227	0	96	0	0	0
N.S.	1	1.00	0.74	3.07	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.027	0.151	0.851	0.000	0.122	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	68	430	0	128	0	0	0
N.S.	1	1.00	0.68	4.30	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.041	0.167	0.931	0.000	0.107	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	68	186	0	99	0	0	0
N.S.	1	1.00	0.66	1.81	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.053	0.226	1.260	0.000	0.105	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	249	0	98	0	0	0
N.S.	1	1.00	0.81	3.23	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.041	0.221	1.007	0.000	0.106	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	56	159	0	85	0	0	0
N.S.	1	1.00	0.75	2.12	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.035	0.110	0.892	0.000	0.103	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	316	0	61	0	0	0
N.S.	1	1.00	0.98	6.87	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.025	0.011	0.918	0.000	0.100	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	121	0	57	0	0	0
N.S.	1	1.00	0.98	2.75	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.019	0.006	0.782	0.000	0.106	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	413	0	83	0	0	0
N.S.	1	1.00	0.76	5.82	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.026	0.007	0.839	0.000	0.105	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	227	0	99	0	0	0
N.S.	1	1.00	0.81	3.15	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.031	0.142	0.937	0.000	0.091	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	68	431	0	138	0	0	0
N.S.	1	1.00	0.66	4.18	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.044	0.266	0.980	0.000	0.102	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	70	246	0	98	0	0	0
N.S.	1	1.00	0.69	2.41	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.050	0.179	1.148	0.000	0.113	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	442	0	96	0	0	0
N.S.	1	1.00	0.79	6.14	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.035	0.136	1.003	0.000	0.119	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	233	0	85	0	0	0
N.S.	1	1.00	0.86	3.15	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.034	0.066	0.945	0.000	0.091	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	297	0	62	0	0	0
N.S.	1	1.00	0.98	6.91	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.017	0.014	0.979	0.000	0.093	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	125	0	58	0	0	0
N.S.	1	1.00	0.98	2.72	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.019	0.006	0.930	0.000	0.096	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	415	0	83	0	0	0
N.S.	1	1.00	0.74	5.93	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.030	0.172	0.956	0.000	0.090	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	250	0	99	0	0	0
N.S.	1	1.00	0.78	3.25	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.029	0.113	0.957	0.000	0.097	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	281	0	98	0	0	0
N.S.	1	1.00	0.68	2.73	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.052	0.121	1.068	0.000	0.103	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	445	0	96	0	0	0
N.S.	1	1.00	0.81	6.01	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.035	0.073	0.998	0.000	0.101	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	262	0	85	0	0	0
N.S.	1	1.00	0.82	3.40	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.034	0.008	1.056	0.000	0.099	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	303	0	62	0	0	0
N.S.	1	1.00	0.98	6.59	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.017	0.012	1.166	0.000	0.093	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	131	0	58	0	0	0
N.S.	1	1.00	0.98	2.85	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.017	0.007	0.944	0.000	0.097	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	421	0	83	0	0	0
N.S.	1	1.00	0.75	5.77	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.030	0.145	0.922	0.000	0.107	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	254	0	99	0	0	0
N.S.	1	1.00	0.78	3.30	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.025	0.143	0.960	0.000	0.100	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	453	0	134	0	0	0
N.S.	1	1.00	0.70	4.31	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.044	0.212	1.014	0.000	0.100	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	102	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	8.100	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [290] had the largest ratio of [.6153999999999999947]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	2	1	1.00	8	0.125
8	A	5	2	1.00	8	0.250
9	A	3	2	1.00	8	0.250
10	A	2	2	1.00	8	0.250
11	A	2	2	1.00	8	0.250
12	A	1	1	1.00	8	0.125
13	A	1	1	1.00	8	0.125
14	A	2	2	1.00	8	0.250
15	A	2	2	1.00	8	0.250
16	A	3	2	1.00	8	0.250
17	A	3	2	1.00	10	0.200
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	10	0.200
20	A	1	1	1.00	10	0.100
21	A	1	1	1.00	10	0.100
22	A	2	2	1.00	10	0.200
23	A	2	2	1.00	10	0.200
24	A	3	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	12	0.250
26	A	3	3	1.00	12	0.250
27	A	3	3	1.00	12	0.250
28	A	2	2	1.00	12	0.167
29	A	2	2	1.00	12	0.167
30	A	3	3	1.00	12	0.250
31	A	3	3	1.00	12	0.250
32	A	4	3	1.00	12	0.250
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	C	1	1	0.11	12	0.083
36	C	1	1	0.23	12	0.083
37	C	1	1	0.21	12	0.083
38	A	1	1	1.00	12	0.083
39	A	1	1	1.00	8	0.125
40	A	1	1	1.00	10	0.100
41	A	2	2	1.00	21	0.095
42	A	2	2	1.00	15	0.133
43	A	2	2	1.00	15	0.133
44	A	2	2	1.00	13	0.154
45	A	1	1	1.00	6	0.167
46	A	2	2	1.00	13	0.154
47	A	2	2	1.00	15	0.133
48	A	2	2	1.00	15	0.133
49	A	3	2	1.00	17	0.118
50	A	3	2	1.00	17	0.118
51	A	3	2	1.00	17	0.118
52	A	2	2	1.00	15	0.133
53	A	2	2	1.00	8	0.250
54	A	2	2	1.00	17	0.118
55	A	3	2	1.00	17	0.118
56	A	3	2	1.00	17	0.118
57	A	3	2	1.00	17	0.118
58	A	5	3	1.00	17	0.176
59	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
60	A	3	3	1.00	17	0.176
61	A	2	2	1.00	8	0.250
62	A	3	3	1.00	13	0.231
63	A	2	2	1.00	15	0.133
64	A	3	3	1.00	17	0.176
65	A	4	3	1.00	17	0.176
66	A	3	2	1.00	17	0.118
67	A	3	2	1.00	17	0.118
68	A	3	2	1.00	17	0.118
69	A	3	2	1.00	17	0.118
70	A	2	2	1.00	15	0.133
71	A	3	2	1.00	15	0.133
72	A	3	2	1.00	15	0.133
73	A	2	2	1.00	8	0.250
74	A	2	1	1.00	15	0.067
75	A	2	2	1.00	17	0.118
76	A	3	2	1.00	17	0.118
77	A	3	2	1.00	17	0.118
78	A	3	2	1.00	17	0.118
79	A	3	2	1.00	17	0.118
80	A	3	2	1.00	17	0.118
81	A	3	2	1.00	17	0.118
82	A	3	2	1.00	17	0.118
83	A	2	2	1.00	15	0.133
84	A	4	4	1.00	17	0.235
85	A	3	2	1.00	8	0.250
86	A	2	2	1.00	17	0.118
87	A	3	2	1.00	17	0.118
88	A	3	2	1.00	17	0.118
89	A	6	3	1.00	17	0.176
90	A	5	3	1.00	17	0.176
91	A	4	3	1.00	17	0.176
92	A	3	2	1.00	8	0.250
93	A	4	3	1.00	15	0.200
94	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	15	0.133
96	A	4	3	1.00	17	0.176
97	A	5	3	1.00	17	0.176
98	A	4	3	1.00	17	0.176
99	A	3	2	1.00	17	0.118
100	A	4	3	1.00	17	0.176
101	A	3	2	1.00	17	0.118
102	A	3	2	1.00	17	0.118
103	A	3	2	1.00	17	0.118
104	A	2	2	1.00	15	0.133
105	A	4	3	1.00	15	0.200
106	A	3	2	1.00	17	0.118
107	A	4	3	1.00	17	0.176
108	A	3	2	1.00	15	0.133
109	A	3	2	1.00	8	0.250
110	A	3	2	1.00	15	0.133
111	A	2	2	1.00	17	0.118
112	A	3	2	1.00	17	0.118
113	A	3	2	1.00	17	0.118
114	A	3	2	1.00	17	0.118
115	A	4	3	1.00	17	0.176
116	A	3	2	1.00	17	0.118
117	A	4	3	1.00	17	0.176
118	A	5	4	1.00	17	0.235
119	A	3	2	1.00	15	0.133
120	A	4	3	1.00	15	0.200
121	A	4	3	1.00	15	0.200
122	A	4	3	1.00	15	0.200
123	A	3	2	1.00	15	0.133
124	A	3	3	1.00	13	0.231
125	A	1	1	1.00	6	0.167
126	A	2	2	1.00	13	0.154
127	A	3	3	1.00	15	0.200
128	A	3	2	1.00	15	0.133
129	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	3	1.00	15	0.200
131	A	4	3	1.00	15	0.200
132	A	4	3	1.00	15	0.200
133	A	3	2	1.00	17	0.118
134	A	5	4	1.00	17	0.235
135	A	3	2	1.00	17	0.118
136	A	4	4	1.00	17	0.235
137	A	3	2	1.00	15	0.133
138	A	2	2	1.00	8	0.250
139	A	2	2	1.00	13	0.154
140	A	3	3	1.00	15	0.200
141	A	3	2	1.00	17	0.118
142	A	4	4	1.00	17	0.235
143	A	3	2	1.00	17	0.118
144	A	5	4	1.00	17	0.235
145	A	4	3	1.00	17	0.176
146	A	5	4	1.00	17	0.235
147	A	4	3	1.00	17	0.176
148	A	4	4	1.00	15	0.267
149	A	2	2	1.00	8	0.250
150	A	2	2	1.00	15	0.133
151	A	2	2	1.00	15	0.133
152	A	3	2	1.00	15	0.133
153	A	4	4	1.00	17	0.235
154	A	4	3	1.00	17	0.176
155	A	5	4	1.00	17	0.235
156	A	4	3	1.00	17	0.176
157	A	3	2	1.00	17	0.118
158	A	6	4	1.00	17	0.235
159	A	3	2	1.00	17	0.118
160	A	5	4	1.00	17	0.235
161	A	3	2	1.00	15	0.133
162	A	3	2	1.00	8	0.250
163	A	2	1	1.00	15	0.067
164	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	2	1.00	15	0.133
166	A	4	3	1.00	15	0.200
167	A	3	2	1.00	17	0.118
168	A	5	4	1.00	17	0.235
169	A	3	2	1.00	17	0.118
170	A	6	4	1.00	17	0.235
171	A	4	3	1.00	17	0.176
172	A	6	4	1.00	17	0.235
173	A	4	3	1.00	17	0.176
174	A	5	4	1.00	15	0.267
175	A	3	2	1.00	8	0.250
176	A	3	2	1.00	15	0.133
177	A	2	2	1.00	17	0.118
178	A	3	3	1.00	17	0.176
179	A	2	2	1.00	15	0.133
180	A	4	3	1.00	15	0.200
181	A	5	4	1.00	17	0.235
182	A	4	3	1.00	17	0.176
183	A	6	4	1.00	17	0.235
184	A	4	3	1.00	17	0.176
185	A	3	2	1.00	9	0.222
186	A	3	2	1.00	9	0.222
187	A	2	2	1.00	19	0.105
188	A	2	2	1.00	19	0.105
189	A	2	2	1.00	19	0.105
190	A	2	2	1.00	19	0.105
191	A	2	2	1.00	19	0.105
192	A	2	2	1.00	19	0.105
193	A	2	2	1.00	19	0.105
194	A	5	4	1.00	21	0.190
195	A	5	4	1.00	21	0.190
196	A	4	4	1.00	21	0.190
197	A	4	4	1.00	21	0.190
198	A	3	3	1.00	21	0.143
199	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	3	3	1.00	21	0.143
201	A	3	3	1.00	21	0.143
202	A	4	4	1.00	21	0.190
203	A	4	4	1.00	21	0.190
204	A	3	2	1.00	21	0.095
205	A	3	2	1.00	21	0.095
206	A	3	2	1.00	21	0.095
207	A	3	2	1.00	21	0.095
208	A	3	2	1.00	21	0.095
209	A	3	2	1.00	21	0.095
210	A	3	2	1.00	21	0.095
211	A	6	4	1.00	21	0.190
212	A	6	4	1.00	21	0.190
213	A	5	4	1.00	21	0.190
214	A	5	4	1.00	21	0.190
215	A	4	3	1.00	21	0.143
216	A	4	3	1.00	21	0.143
217	A	4	4	1.00	21	0.190
218	A	4	4	1.00	21	0.190
219	A	4	3	1.00	21	0.143
220	A	4	3	1.00	21	0.143
221	A	3	2	1.00	19	0.105
222	A	7	6	1.00	19	0.316
223	A	7	6	1.00	19	0.316
224	A	6	6	1.00	19	0.316
225	A	6	6	1.00	19	0.316
226	A	5	5	1.00	19	0.263
227	A	5	5	1.00	19	0.263
228	A	6	6	1.00	19	0.316
229	A	6	6	1.00	19	0.316
230	A	7	6	1.00	19	0.316
231	A	7	6	1.00	19	0.316
232	A	5	4	1.00	21	0.190
233	A	4	4	1.00	21	0.190
234	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	3	3	1.00	21	0.143
236	A	3	3	1.00	21	0.143
237	A	3	3	1.00	21	0.143
238	A	3	3	1.00	21	0.143
239	A	4	4	1.00	21	0.190
240	A	4	4	1.00	21	0.190
241	A	5	4	1.00	21	0.190
242	A	8	7	1.00	21	0.333
243	A	7	7	1.00	21	0.333
244	A	7	7	1.00	21	0.333
245	A	6	6	1.00	21	0.286
246	A	6	6	1.00	21	0.286
247	A	6	6	1.00	21	0.286
248	A	6	6	1.00	21	0.286
249	A	7	7	1.00	21	0.333
250	A	7	7	1.00	21	0.333
251	A	8	7	1.00	21	0.333
252	A	2	2	1.00	19	0.105
253	A	3	2	1.00	11	0.182
254	A	3	2	1.00	11	0.182
255	A	3	2	1.00	11	0.182
256	A	3	2	1.00	11	0.182
257	A	4	3	1.00	25	0.120
258	A	3	3	1.00	25	0.120
259	A	2	2	1.00	25	0.080
260	A	3	3	1.00	25	0.120
261	A	4	3	1.00	25	0.120
262	A	11	8	1.00	25	0.320
263	A	10	7	1.00	25	0.280
264	A	1	1	1.00	25	0.040
265	A	2	2	1.00	25	0.080
266	A	3	2	1.00	25	0.080
267	A	4	4	1.00	25	0.160
268	A	3	3	1.00	25	0.120
269	A	3	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	4	1.00	25	0.160
271	A	11	8	1.00	25	0.320
272	A	11	8	1.00	25	0.320
273	A	1	1	1.00	25	0.040
274	A	3	3	1.00	25	0.120
275	A	4	3	1.00	25	0.120
276	A	5	4	1.00	25	0.160
277	A	4	4	1.00	25	0.160
278	A	3	3	1.00	25	0.120
279	A	3	3	1.00	25	0.120
280	A	4	4	1.00	25	0.160
281	A	5	4	1.00	25	0.160
282	A	11	8	1.00	25	0.320
283	A	11	8	1.00	25	0.320
284	A	1	1	1.00	25	0.040
285	A	3	3	1.00	25	0.120
286	A	4	3	1.00	25	0.120
287	A	12	8	1.00	21	0.381
288	A	1	1	1.00	13	0.077
289	A	10	7	1.00	13	0.538
290	A	11	8	1.00	13	0.615
291	A	4	3	1.00	25	0.120
292	A	3	3	1.00	25	0.120
293	A	2	2	1.00	25	0.080
294	A	3	3	1.00	25	0.120
295	A	4	3	1.00	25	0.120
296	A	10	7	1.00	25	0.280
297	A	1	1	1.00	25	0.040
298	A	2	2	1.00	25	0.080
299	A	3	2	1.00	25	0.080
300	A	10	7	1.00	21	0.333
301	A	11	8	1.00	21	0.381
302	A	11	8	1.00	21	0.381
303	A	12	8	1.00	21	0.381
304	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	1	1	1.00	21	0.048
306	A	1	1	1.00	12	0.083
307	A	1	1	1.00	21	0.048
308	A	1	1	1.00	21	0.048
309	A	1	1	1.00	21	0.048
310	A	1	1	1.00	21	0.048
311	A	1	1	1.00	12	0.083
312	A	1	1	1.00	21	0.048
313	A	1	1	1.00	21	0.048
314	A	1	1	1.00	21	0.048
315	A	1	1	1.00	21	0.048
316	A	1	1	1.00	12	0.083
317	A	1	1	1.00	21	0.048
318	A	1	1	1.00	21	0.048
319	A	1	1	1.00	21	0.048
320	A	1	1	1.00	21	0.048
321	A	1	1	1.00	12	0.083
322	A	1	1	1.00	21	0.048
323	A	1	1	1.00	21	0.048
324	A	8	8	1.00	21	0.381
325	A	11	7	1.00	21	0.333
326	A	12	8	1.00	21	0.381
327	A	9	9	1.00	21	0.429
328	A	9	9	1.00	21	0.429
329	A	8	8	1.00	21	0.381
330	A	11	7	1.00	21	0.333
331	A	12	8	1.00	21	0.381
332	A	9	9	1.00	21	0.429
333	A	9	9	1.00	21	0.429
334	A	1	1	1.00	13	0.077
335	A	1	1	1.00	13	0.077
336	A	1	1	1.00	17	0.059
337	A	1	1	1.00	19	0.053
338	A	1	1	1.00	19	0.053
339	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	3	2	1.00	19	0.105
341	A	3	2	1.00	19	0.105
342	A	2	2	1.00	17	0.118
343	A	2	2	1.00	17	0.118
344	A	2	2	1.00	19	0.105
345	A	1	1	1.00	19	0.053
346	A	1	1	1.00	19	0.053
347	A	1	1	1.00	10	0.100
348	A	1	1	1.00	19	0.053
349	A	1	1	1.00	19	0.053
350	A	1	1	1.00	23	0.043
351	A	1	1	1.00	23	0.043
352	A	1	1	1.00	23	0.043
353	A	1	1	1.00	23	0.043
354	A	1	1	1.00	23	0.043
355	A	3	2	1.00	19	0.105
356	A	3	2	1.00	19	0.105
357	A	2	2	1.00	17	0.118
358	A	2	2	1.00	17	0.118
359	A	2	2	1.00	19	0.105
360	A	2	2	1.00	19	0.105
361	A	1	1	1.00	19	0.053
362	A	1	1	1.00	19	0.053
363	A	1	1	1.00	10	0.100
364	A	1	1	1.00	19	0.053
365	A	1	1	1.00	19	0.053
366	A	1	1	1.00	23	0.043
367	A	1	1	1.00	23	0.043
368	A	1	1	1.00	23	0.043
369	A	1	1	1.00	23	0.043
370	A	1	1	1.00	23	0.043
371	A	3	2	1.00	21	0.095
372	A	3	2	1.00	21	0.095
373	A	3	2	1.00	21	0.095
374	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	5	1.00	19	0.263
376	A	6	6	1.00	21	0.286
377	A	7	6	1.00	21	0.286
378	A	5	3	1.00	21	0.143
379	A	4	3	1.00	21	0.143
380	A	3	3	1.00	21	0.143
381	A	2	2	1.00	12	0.167
382	A	3	3	1.00	21	0.143
383	A	4	3	1.00	21	0.143
384	A	5	3	1.00	21	0.143
385	A	3	2	1.00	21	0.095
386	A	3	2	1.00	21	0.095
387	A	3	2	1.00	21	0.095
388	A	2	2	1.00	19	0.105
389	A	6	6	1.00	19	0.316
390	A	7	7	1.00	21	0.333
391	A	5	4	1.00	21	0.190
392	A	4	4	1.00	21	0.190
393	A	3	3	1.00	21	0.143
394	A	3	3	1.00	12	0.250
395	A	4	4	1.00	21	0.190
396	A	5	4	1.00	21	0.190
397	A	3	2	1.00	21	0.095
398	A	3	2	1.00	21	0.095
399	A	3	2	1.00	21	0.095
400	A	2	2	1.00	19	0.105
401	A	6	6	1.00	19	0.316
402	A	7	7	1.00	21	0.333
403	A	8	7	1.00	21	0.333
404	A	5	4	1.00	21	0.190
405	A	4	4	1.00	21	0.190
406	A	3	3	1.00	21	0.143
407	A	3	3	1.00	12	0.250
408	A	4	4	1.00	21	0.190
409	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	3	2	1.00	21	0.095
411	A	3	2	1.00	21	0.095
412	A	3	2	1.00	21	0.095
413	A	2	2	1.00	19	0.105
414	A	5	5	1.00	19	0.263
415	A	6	6	1.00	21	0.286
416	A	7	6	1.00	21	0.286
417	A	5	3	1.00	21	0.143
418	A	4	3	1.00	21	0.143
419	A	3	3	1.00	21	0.143
420	A	2	2	1.00	12	0.167
421	A	3	3	1.00	21	0.143
422	A	4	3	1.00	21	0.143
423	A	5	3	1.00	21	0.143
424	A	3	2	1.00	21	0.095
425	A	3	2	1.00	21	0.095
426	A	3	2	1.00	21	0.095
427	A	2	2	1.00	19	0.105
428	A	6	6	1.00	19	0.316
429	A	6	6	1.00	21	0.286
430	A	7	7	1.00	21	0.333
431	A	5	4	1.00	21	0.190
432	A	4	4	1.00	21	0.190
433	A	3	3	1.00	12	0.250
434	A	3	3	1.00	21	0.143
435	A	4	4	1.00	21	0.190
436	A	5	4	1.00	21	0.190
437	A	3	2	1.00	21	0.095
438	A	3	2	1.00	21	0.095
439	A	3	2	1.00	21	0.095
440	A	2	2	1.00	19	0.105
441	A	6	6	1.00	19	0.316
442	A	6	6	1.00	21	0.286
443	A	7	7	1.00	21	0.333
444	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	4	4	1.00	21	0.190
446	A	3	3	1.00	12	0.250
447	A	3	3	1.00	21	0.143
448	A	4	4	1.00	21	0.190
449	A	5	4	1.00	21	0.190
450	A	13	9	1.00	25	0.360
451	A	12	9	1.00	25	0.360
452	A	11	8	1.00	25	0.320
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	25	0.080
455	A	3	2	1.00	25	0.080
456	A	5	4	1.00	25	0.160
457	A	4	4	1.00	25	0.160
458	A	3	3	1.00	25	0.120
459	A	4	4	1.00	25	0.160
460	A	5	4	1.00	25	0.160
461	A	5	4	1.00	23	0.174
462	A	4	4	1.00	23	0.174
463	A	3	3	1.00	23	0.130
464	A	4	4	1.00	23	0.174
465	A	5	4	1.00	23	0.174
466	A	12	9	1.00	23	0.391
467	A	11	8	1.00	23	0.348
468	A	1	1	1.00	23	0.043
469	A	2	2	1.00	23	0.087
470	A	3	2	1.00	23	0.087
471	A	4	2	1.00	23	0.087
472	A	14	10	1.00	25	0.400
473	A	13	10	1.00	25	0.400
474	A	12	9	1.00	25	0.360
475	A	12	9	1.00	25	0.360
476	A	1	1	1.00	25	0.040
477	A	6	5	1.00	25	0.200
478	A	5	5	1.00	25	0.200
479	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
480	A	4	4	1.00	25	0.160
481	A	5	5	1.00	25	0.200
482	A	6	5	1.00	25	0.200
483	A	2	2	1.00	23	0.087
484	A	2	2	1.00	23	0.087
485	A	2	2	1.00	23	0.087
486	A	2	2	1.00	23	0.087
487	A	2	2	1.00	23	0.087
488	A	2	2	1.00	17	0.118
489	A	2	2	1.00	19	0.105
490	A	2	2	1.00	19	0.105
491	A	2	2	1.00	21	0.095
492	A	3	2	1.00	19	0.105
493	A	3	2	1.00	19	0.105
494	A	2	2	1.00	17	0.118
495	A	2	2	1.00	17	0.118
496	A	2	2	1.00	19	0.105
497	A	2	2	1.00	19	0.105
498	A	2	2	1.00	19	0.105
499	A	2	2	1.00	19	0.105
500	A	2	2	1.00	10	0.200
501	A	2	2	1.00	19	0.105
502	A	2	2	1.00	19	0.105
503	A	2	2	1.00	23	0.087
504	A	2	2	1.00	23	0.087
505	A	2	2	1.00	23	0.087
506	A	2	2	1.00	23	0.087
507	A	5	4	1.00	21	0.190
508	A	4	4	1.00	21	0.190
509	A	4	4	1.00	21	0.190
510	A	3	3	1.00	19	0.158
511	A	2	2	1.00	12	0.167
512	A	4	4	1.00	19	0.210
513	A	4	4	1.00	21	0.190
514	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	5	4	1.00	21	0.190
516	A	4	4	1.00	21	0.190
517	A	4	4	1.00	21	0.190
518	A	3	3	1.00	21	0.143
519	A	3	3	1.00	19	0.158
520	A	3	3	1.00	12	0.250
521	A	4	4	1.00	19	0.210
522	A	5	4	1.00	21	0.190
523	A	5	4	1.00	21	0.190
524	A	4	4	1.00	21	0.190
525	A	4	4	1.00	19	0.210
526	A	2	2	1.00	12	0.167
527	A	3	3	1.00	19	0.158
528	A	4	4	1.00	21	0.190
529	A	4	4	1.00	21	0.190
530	A	5	4	1.00	21	0.190
531	A	4	4	1.00	19	0.210
532	A	3	3	1.00	12	0.250
533	A	3	3	1.00	19	0.158
534	A	3	3	1.00	21	0.143
535	A	4	4	1.00	21	0.190
536	A	4	4	1.00	21	0.190
537	A	5	4	1.00	21	0.190
538	A	2	2	1.00	21	0.095

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sin(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2718}

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\cos(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a)\cos(bx)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

```
[In] Integrate[Sin[a + b*x],x]
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\cos(bx+a)}{b}$	12
default	$-\frac{\cos(bx+a)}{b}$	12
risch	$-\frac{\cos(bx+a)}{b}$	12
parallelrisch	$\frac{-\cos(bx+a)-1}{b}$	15
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	32
meijerg	$\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	34

```
[In] int(sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -cos(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a),x, algorithm="fricas")
[Out] -cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \sin(a+bx)\,dx = \begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a),x)
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
[Out] -cos(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a),x, algorithm="giac")
[Out] -cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+b\,x)}{b}$$

```
[In] int(sin(a + b*x),x)
[Out] -cos(a + b*x)/b
```

3.2 $\int \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)\sin(a+bx)}{2b} + \frac{\int 1\,dx}{2} \\
&= \frac{x}{2} - \frac{\cos(a+bx)\sin(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$
\int \sin^2(a+bx)\,dx = -\frac{-2(a+bx)+\sin(2(a+bx))}{4b}
$$

```
[In] Integrate[Sin[a + b*x]^2,x]
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx-\sin(2bx+2a)}{4b}$	22
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

```
[In] int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/4/b*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = \frac{bx - \cos(bx+a)\sin(bx+a)}{2\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**2,x)
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \sin(2\,bx + 2\,a)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{1}{2}\,x - \frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

```
[In] int(sin(a + b*x)^2,x)
```

```
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

3.3 $\int \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sin^3(a+bx)\,dx = -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\int \sin^3(a+bx)\,dx = \frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int (1-x^2)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sin^3(a+bx)\,dx=-\frac{3\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}$$

```
[In] Integrate[Sin[a + b*x]^3,x]
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$	22
default	$-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$	22
parallelrisch	$\frac{-8-9\cos(bx+a)+\cos(3bx+3a)}{12b}$	25
risch	$-\frac{3\cos(bx+a)}{4b}+\frac{\cos(3bx+3a)}{12b}$	27
norman	$\frac{-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	39

```
[In] int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx=\frac{\cos(bx+a)^3-3\cos(bx+a)}{3b}$$

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**3,x)
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3}{3b} - \frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sin^3(a+bx)\,dx = -\frac{3\,\cos\left(a+b\,x\right)-{\cos\left(a+b\,x\right)}^3}{3\,b}$$

```
[In] int(sin(a + b*x)^3,x)
```

```
[Out] -(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

3.4 $\int \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sin^4(a+bx)\,dx = \frac{3x}{8} - \frac{3\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

```
[Out] 3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sin^4(a+bx)\,dx = -\frac{\sin^3(a+bx)\cos(a+bx)}{4b} - \frac{3\sin(a+bx)\cos(a+bx)}{8b} + \frac{3x}{8}$$

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

*n]

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3}{4}\int \sin^2(a+bx)\,dx\\ &= -\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx=\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

```
[In] Integrate[Sin[a + b*x]^4,x]
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sin(4bx+4a)-8\sin(2bx+2a)}{32b}$
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}-\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{11\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{11\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

```
[In] int(sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/32*(12*b*x+sin(4*b*x+4*a)-8*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^4(a+bx)\,dx = \frac{3\,bx + \left(2\,\cos\left(bx+a\right)^3 - 5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^4(a+bx)}{8} + \frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{3x\cos^4(a+bx)}{8} - \frac{5\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**4,x)
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(4\,bx+4\,a\right) - 8\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sin^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{\sin\left(4\,bx+4\,a\right)}{32\,b} - \frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \sin^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{5\tan(a+b\,x)^3}{8}+\frac{3\tan(a+b\,x)}{8}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(sin(a + b*x)^4,x)
[Out] (3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
+ tan(a + b*x)^4 + 1))
```

3.5 $\int \sin^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \sin^5(a+bx)\,dx=-\frac{\cos(a+bx)}{b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -cos(b*x+a)/b+2/3*cos(b*x+a)^3/b-1/5*cos(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\int \sin^5(a+bx)\,dx=-\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]^5,x]
```

```
[Out] -(Cos[a + b*x]/b) + (2*Cos[a + b*x]^3)/(3*b) - Cos[a + b*x]^5/(5*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,\cos(a+bx)\right)}{b}\\&=-\frac{\cos(a+bx)}{b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \sin^5(a+bx)\,dx = -\frac{5\cos(a+bx)}{8b} + \frac{5\cos(3(a+bx))}{48b} - \frac{\cos(5(a+bx))}{80b}$$

```
[In] Integrate[Sin[a + b*x]^5,x]
[Out] (-5*Cos[a + b*x])/(8*b) + (5*Cos[3*(a + b*x)])/(48*b) - Cos[5*(a + b*x)]/(8
0*b)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{5b}$	32
default	$-\frac{\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{5b}$	32
parallelrisch	$\frac{-128-150\cos(bx+a)+25\cos(3bx+3a)-3\cos(5bx+5a)}{240b}$	38
risch	$-\frac{5\cos(bx+a)}{8b}-\frac{\cos(5bx+5a)}{80b}+\frac{5\cos(3bx+3a)}{48b}$	41
norman	$\frac{-\frac{16}{15b}-\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{32\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$	55

```
[In] int(sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/5/b*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \sin^5(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^5 - 10\,\cos\left(bx+a\right)^3 + 15\,\cos\left(bx+a\right)}{15\,b}$$

```
[In] integrate(sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos(a+bx)}{b} - \frac{4\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{8\cos^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sin^5(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)/b - 4*sin(a + b*x)**2*cos(a + b*x)
**3/(3*b) - 8*cos(a + b*x)**5/(15*b), Ne(b, 0)), (x*sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \sin^5(a+bx)\,dx = -\frac{3\cos(bx+a)^5 - 10\cos(bx+a)^3 + 15\cos(bx+a)}{15\,b}$$

```
[In] integrate(sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \sin^5(a+bx)\,dx = -\frac{\cos(bx+a)^5}{5\,b} + \frac{2\cos(bx+a)^3}{3\,b} - \frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(b*x + a)^5/b + 2/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \sin^5(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^5}{5} - \frac{2\cos(a+b\,x)^3}{3} + \cos\left(a+b\,x\right)}{b}$$

```
[In] int(sin(a + b*x)^5,x)
[Out] -(cos(a + b*x) - (2*cos(a + b*x)^3)/3 + cos(a + b*x)^5/5)/b
```

3.6 $\int \sin^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sin^6(a+bx)\,dx = \frac{5x}{16} - \frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b}$$

```
[Out] 5/16*x-5/16*cos(b*x+a)*sin(b*x+a)/b-5/24*cos(b*x+a)*sin(b*x+a)^3/b-1/6*cos(
b*x+a)*sin(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sin^6(a+bx)\,dx = -\frac{\sin^5(a+bx)\cos(a+bx)}{6b} - \frac{5\sin^3(a+bx)\cos(a+bx)}{24b} - \frac{5\sin(a+bx)\cos(a+bx)}{16b} + \frac{5x}{16}$$

```
[In] Int[Sin[a + b*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) - (5*Cos[a + b*x]*Sin[a + b
*x]^3)/(24*b) - (Cos[a + b*x]*Sin[a + b*x]^5)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)\sin^5(a+bx)}{6b} + \frac{5}{6}\int \sin^4(a+bx)\,dx \\
&= -\frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b} + \frac{5}{8}\int \sin^2(a+bx)\,dx \\
&= -\frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b} \\
&\quad + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} - \frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \sin^6(a+bx)\,dx = \frac{60a+60bx-45\sin(2(a+bx))+9\sin(4(a+bx))-\sin(6(a+bx))}{192b}$$

```
[In] Integrate[Sin[a + b*x]^6,x]
[Out] (60*a + 60*b*x - 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] - Sin[6*(a + b*x)
])/(192*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{60bx-\sin(6bx+6a)+9\sin(4bx+4a)-45\sin(2bx+2a)}{192b}$
risch	$\frac{5x}{16}-\frac{\sin(6bx+6a)}{192b}+\frac{3\sin(4bx+4a)}{64b}-\frac{15\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\left(\sin^5(bx+a)+\frac{5\left(\sin^3(bx+a)\right)}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
default	$\frac{-\frac{\left(\sin^5(bx+a)+\frac{5\left(\sin^3(bx+a)\right)}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
norman	$\frac{\frac{5x}{16}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{85\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{33\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{33\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{85\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{5\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+15}{\left(1+\tan^2\left(\right.\right.}$

```
[In] int(sin(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] 1/192*(60*b*x-sin(6*b*x+6*a)+9*sin(4*b*x+4*a)-45*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \sin^6(a+bx)\,dx = \frac{15\,bx - \left(8\,\cos\left(bx+a\right)^5 - 26\,\cos\left(bx+a\right)^3 + 33\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

```
[In] integrate(sin(b*x+a)^6,x, algorithm="fricas")
[Out] 1/48*(15*b*x - (8*cos(b*x + a)^5 - 26*cos(b*x + a)^3 + 33*cos(b*x + a))*sin
(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \sin^6(a+bx)\,dx = \begin{cases} \frac{5x\sin^6(a+bx)}{16}+\frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{5x\cos^6(a+bx)}{16}-\frac{11\sin^5(a+bx)\cos(a+bx)}{16b}-\frac{5\sin^3(}{} \\ x\sin^6(a) \end{cases}$$

```
[In] integrate(sin(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/16
 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 - 11*si
n(a + b*x)**5*cos(a + b*x)/(16*b) - 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*b)
 - 5*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \sin^6(a+bx)\,dx = \frac{4\,\sin\left(2\,bx+2\,a\right)^3+60\,bx+60\,a+9\,\sin\left(4\,bx+4\,a\right)-48\,\sin\left(2\,bx+2\,a\right)}{192\,b}$$

```
[In] integrate(sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 60*b*x + 60*a + 9*sin(4*b*x + 4*a) - 48*sin(2
*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \sin^6(a+bx)\,dx = \frac{5}{16}\,x - \frac{\sin\left(6\,bx+6\,a\right)}{192\,b} + \frac{3\,\sin\left(4\,bx+4\,a\right)}{64\,b} - \frac{15\,\sin\left(2\,bx+2\,a\right)}{64\,b}$$

```
[In] integrate(sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b - 15/64*sin(2*b
*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sin^6(a+bx)\,dx = \frac{5\,x}{16} - \frac{\frac{15\,\sin(2\,a+2\,b\,x)}{64} - \frac{3\,\sin(4\,a+4\,b\,x)}{64} + \frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

```
[In] int(sin(a + b*x)^6,x)
```

```
[Out] (5*x)/16 - ((15*sin(2*a + 2*b*x))/64 - (3*sin(4*a + 4*b*x))/64 + sin(6*a +
6*b*x)/192)/b
```

3.7 $\int \sin^7(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \sin^7(a+bx)\,dx = -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}$$

```
[Out] -cos(b*x+a)/b+cos(b*x+a)^3/b-3/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\int \sin^7(a+bx)\,dx = \frac{\cos^7(a+bx)}{7b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{b}-\frac{\cos(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]^7,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/b - (3*Cos[a + b*x]^5)/(5*b) + Cos[a + b
*x]^7/(7*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \sin^7(a+bx)\,dx = -\frac{35\cos(a+bx)}{64b} + \frac{7\cos(3(a+bx))}{64b} - \frac{7\cos(5(a+bx))}{320b} + \frac{\cos(7(a+bx))}{448b}$$

[In] Integrate[Sin[a + b*x]^7,x]

[Out] (-35*Cos[a + b*x])/(64*b) + (7*Cos[3*(a + b*x)])/(64*b) - (7*Cos[5*(a + b*x
)])/(320*b) + Cos[7*(a + b*x)]/(448*b)

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\left(\frac{16}{5}+\sin^6(bx+a)+\frac{6\left(\sin^4(bx+a)\right)}{5}+\frac{8\left(\sin^2(bx+a)\right)}{5}\right)\cos(bx+a)}{7b}$	42
default	$-\frac{\left(\frac{16}{5}+\sin^6(bx+a)+\frac{6\left(\sin^4(bx+a)\right)}{5}+\frac{8\left(\sin^2(bx+a)\right)}{5}\right)\cos(bx+a)}{7b}$	42
parallelrisch	$\frac{-1024+245\cos(3bx+3a)-49\cos(5bx+5a)-1225\cos(bx+a)+5\cos(7bx+7a)}{2240b}$	49
risch	$-\frac{35\cos(bx+a)}{64b}+\frac{\cos(7bx+7a)}{448b}-\frac{7\cos(5bx+5a)}{320b}+\frac{7\cos(3bx+3a)}{64b}$	55
norman	$\frac{-\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{32}{35b}-\frac{32\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{96\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^7}$	71

[In] int(sin(b*x+a)^7,x,method=_RETURNVERBOSE)

[Out] -1/7/b*(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x+a)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin^7(a+bx)\,dx = \frac{5\cos(bx+a)^7 - 21\cos(bx+a)^5 + 35\cos(bx+a)^3 - 35\cos(bx+a)}{35\,b}$$

[In] integrate(sin(b*x+a)^7,x, algorithm="fricas")

[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \sin^7(a+bx)\,dx = \begin{cases} -\frac{\sin^6(a+bx)\cos(a+bx)}{b} - \frac{2\sin^4(a+bx)\cos^3(a+bx)}{b} - \frac{8\sin^2(a+bx)\cos^5(a+bx)}{5b} - \frac{16\cos^7(a+bx)}{35b} & \text{for } b \neq 0 \\ x\sin^7(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x)**6*cos(a + b*x)/b - 2*sin(a + b*x)**4*cos(a + b*x)
**3/b - 8*sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 16*cos(a + b*x)**7/(35*b)
, Ne(b, 0)), (x*sin(a)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin^7(a+bx)\,dx = \frac{5\cos(bx+a)^7 - 21\cos(bx+a)^5 + 35\cos(bx+a)^3 - 35\cos(bx+a)}{35\,b}$$

```
[In] integrate(sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sin^7(a+bx)\,dx = \frac{\cos(bx+a)^7}{7\,b} - \frac{3\cos(bx+a)^5}{5\,b} + \frac{\cos(bx+a)^3}{b} - \frac{\cos(bx+a)}{b}$$

```
[In] integrate(sin(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/7*cos(b*x + a)^7/b - 3/5*cos(b*x + a)^5/b + cos(b*x + a)^3/b - cos(b*x +
a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sin^7(a+bx)\,dx = \frac{\cos(a+b\,x)\,\left(5\cos(a+b\,x)^6-21\cos(a+b\,x)^4+35\cos(a+b\,x)^2-35\right)}{35\,b}$$

```
[In] int(sin(a + b*x)^7,x)
[Out] (cos(a + b*x)*(35*cos(a + b*x)^2 - 21*cos(a + b*x)^4 + 5*cos(a + b*x)^6 - 3
5))/(35*b)
```

3.8 $\int \sin^8(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 88

$$\int \sin^8(a+bx)\,dx = \frac{35x}{128} - \frac{35\cos(a+bx)\sin(a+bx)}{128b} - \frac{35\cos(a+bx)\sin^3(a+bx)}{192b} - \frac{7\cos(a+bx)\sin^5(a+bx)}{48b} - \frac{\cos(a+bx)\sin^7(a+bx)}{8b}$$

```
[Out] 35/128*x-35/128*cos(b*x+a)*sin(b*x+a)/b-35/192*cos(b*x+a)*sin(b*x+a)^3/b-7/
48*cos(b*x+a)*sin(b*x+a)^5/b-1/8*cos(b*x+a)*sin(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\int \sin^8(a+bx)\,dx = -\frac{\sin^7(a+bx)\cos(a+bx)}{8b} - \frac{7\sin^5(a+bx)\cos(a+bx)}{48b} - \frac{35\sin^3(a+bx)\cos(a+bx)}{192b} - \frac{35\sin(a+bx)\cos(a+bx)}{128b} + \frac{35x}{128}$$

```
[In] Int[Sin[a + b*x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) - (35*Cos[a + b*x]*Sin[
a + b*x]^3)/(192*b) - (7*Cos[a + b*x]*Sin[a + b*x]^5)/(48*b) - (Cos[a + b*x
]*Sin[a + b*x]^7)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{7}{8}\int \sin^6(a+bx)\,dx\\
&= -\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{35}{48}\int \sin^4(a+bx)\,dx\\
&= -\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}-\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}\\
&\quad -\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{35}{64}\int \sin^2(a+bx)\,dx\\
&= -\frac{35\cos(a+bx)\sin(a+bx)}{128b}-\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}\\
&\quad -\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\frac{\cos(a+bx)\sin^7(a+bx)}{8b}+\frac{35\int 1\,dx}{128}\\
&= \frac{35x}{128}-\frac{35\cos(a+bx)\sin(a+bx)}{128b}-\frac{35\cos(a+bx)\sin^3(a+bx)}{192b}\\
&\quad -\frac{7\cos(a+bx)\sin^5(a+bx)}{48b}-\frac{\cos(a+bx)\sin^7(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \sin^8(a+bx)\,dx = \frac{840a+840bx-672\sin(2(a+bx))+168\sin(4(a+bx))-32\sin(6(a+bx))+3\sin(8(a+bx))}{3072b}$$

```
[In] Integrate[Sin[a + b*x]^8,x]
```

```
[Out] (840*a + 840*b*x - 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] - 32*Sin[6*(
a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{840bx+3\sin(8bx+8a)-32\sin(6bx+6a)+168\sin(4bx+4a)-672\sin(2bx+2a)}{3072b}$
derivativedivides	$\frac{-\frac{\left(\sin^{7}(bx+a)+\frac{7\left(\sin^{5}(bx+a)\right)}{6}+\frac{35\left(\sin^{3}(bx+a)\right)}{24}+\frac{35\sin(bx+a)}{16}\right)\cos(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
default	$\frac{-\frac{\left(\sin^{7}(bx+a)+\frac{7\left(\sin^{5}(bx+a)\right)}{6}+\frac{35\left(\sin^{3}(bx+a)\right)}{24}+\frac{35\sin(bx+a)}{16}\right)\cos(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
risch	$\frac{35x}{128}+\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(6bx+6a)}{96b}+\frac{7\sin(4bx+4a)}{128b}-\frac{7\sin(2bx+2a)}{32b}$
norman	$\frac{35x}{128}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{805\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{2681\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{5053\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{5053\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{2681(\tan^{11}(\ldots}{192b}$

```
[In] int(sin(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072*(840*b*x+3*sin(8*b*x+8*a)-32*sin(6*b*x+6*a)+168*sin(4*b*x+4*a)-672*s
in(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sin^{8}(a+bx)\,dx = \frac{105\,bx+\left(48\,\cos(bx+a)^{7}-200\,\cos(bx+a)^{5}+326\,\cos(bx+a)^{3}-279\,\cos(bx+a)\right)\sin(bx+a)}{384\,b}$$

```
[In] integrate(sin(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(105*b*x + (48*cos(b*x + a)^7 - 200*cos(b*x + a)^5 + 326*cos(b*x + a)
^3 - 279*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.

Time = 0.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

$$\int \sin^8(a+bx)\,dx$$
$$= \begin{cases} \frac{35x\sin^8(a+bx)}{128} + \frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{35x\cos^8(a+bx)}{128} - \frac{93\,\mathrm{s}}{} \\ x\sin^8(a) \end{cases}$$

```
[In] integrate(sin(b*x+a)**8,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2/
32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(a
+ b*x)**6/32 + 35*x*cos(a + b*x)**8/128 - 93*sin(a + b*x)**7*cos(a + b*x)/(
128*b) - 511*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) - 385*sin(a + b*x)**3*
cos(a + b*x)**5/(384*b) - 35*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)
), (x*sin(a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \sin^8(a+bx)\,dx$$
$$= \frac{128\,\sin(2\,bx+2\,a)^3 + 840\,bx + 840\,a + 3\,\sin(8\,bx+8\,a) + 168\,\sin(4\,bx+4\,a) - 768\,\sin(2\,bx+2\,a)}{3072\,b}$$

```
[In] integrate(sin(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*b*x + 2*a)^3 + 840*b*x + 840*a + 3*sin(8*b*x + 8*a) + 168
*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \sin^8(a+bx)\,dx = \frac{35}{128}\,x + \frac{\sin(8\,bx+8\,a)}{1024\,b} - \frac{\sin(6\,bx+6\,a)}{96\,b} + \frac{7\,\sin(4\,bx+4\,a)}{128\,b} - \frac{7\,\sin(2\,bx+2\,a)}{32\,b}$$

```
[In] integrate(sin(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/96*sin(6*b*x + 6*a)/b + 7/128*sin(
4*b*x + 4*a)/b - 7/32*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \sin^8(a+bx)\,dx$$
$$=\frac{35\,x}{128}-\frac{\frac{93\tan(a+b\,x)^7}{128}+\frac{511\tan(a+b\,x)^5}{384}+\frac{385\tan(a+b\,x)^3}{384}+\frac{35\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8+4\tan\left(a+b\,x\right)^6+6\tan\left(a+b\,x\right)^4+4\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(sin(a + b*x)^8,x)
```

```
[Out] (35*x)/128 - ((35*tan(a + b*x))/128 + (385*tan(a + b*x)^3)/384 + (511*tan(a
 + b*x)^5)/384 + (93*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a +
b*x)^4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

3.9 $\int \sin^{\frac{7}{2}}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int \sin^{\frac{7}{2}}(bx)\,dx=-\frac{10\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{21b}-\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b}-\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}$$

```
[Out] -10/21*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*
Pi+1/2*b*x),2^(1/2))/b-2/7*cos(b*x)*sin(b*x)^(5/2)/b-10/21*cos(b*x)*sin(b*x
)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2720}

$$\int \sin^{\frac{7}{2}}(bx)\,dx=-\frac{10\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}-\frac{10\sqrt{\sin(bx)}\cos(bx)}{21b}$$

```
[In] Int[Sin[b*x]^(7/2),x]
```

```
[Out] (-10*EllipticF[Pi/4 - (b*x)/2, 2])/(21*b) - (10*Cos[b*x]*Sqrt[Sin[b*x]])/(2
1*b) - (2*Cos[b*x]*Sin[b*x]^(5/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b} + \frac{5}{7}\int \sin^{\frac{3}{2}}(bx)\,dx \\ &= -\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b} - \frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sin(bx)}}\,dx \\ &= -\frac{10\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{21b} - \frac{10\cos(bx)\sqrt{\sin(bx)}}{21b} - \frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \frac{-20\,\text{EllipticF}\left(\frac{1}{4}(\pi-2bx),2\right) + (-23\cos(bx)+3\cos(3bx))\sqrt{\sin(bx)}}{42b}$$

```
[In] Integrate[Sin[b*x]^(7/2),x]
[Out] (-20*EllipticF[(Pi - 2*b*x)/4, 2] + (-23*Cos[b*x] + 3*Cos[3*b*x])*Sqrt[Sin[
b*x]])/(42*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\frac{2\left(\cos^4(bx)\right)\sin(bx)}{7}+\frac{5\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{21}-\frac{16\left(\cos^2(bx)\right)\sin(bx)}{21}}{\cos(bx)\sqrt{\sin(bx)}\,b}$	84

```
[In] int(sin(b*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] (2/7*cos(b*x)^4*sin(b*x)+5/21*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-si
n(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-16/21*cos(b*x)^2*si
n(b*x))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \sin^{\frac{7}{2}}(bx)\,dx$$
$$= \frac{5\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx))+5\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx)-i\sin}{21\,b}$$

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x)) +
 5*sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x)) + 2*(3*
cos(b*x)^3 - 8*cos(b*x))*sqrt(sin(b*x)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \int \sin(bx)^{\frac{7}{2}}\,dx$$

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(7/2), x)
```

Giac [F]

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \int \sin(bx)^{\frac{7}{2}}\,dx$$

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \sin^{\frac{7}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\;\frac{3}{2};\;\cos(b\,x)^2\right)}{b\left(\sin(b\,x)^2\right)^{9/4}}$$

```
[In] int(sin(b*x)^(7/2),x)
```

```
[Out] -(cos(b*x)*sin(b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(9/4))
```

3.10 $\int \sin^{\frac{5}{2}}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}$$

```
[Out] -6/5*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/5*cos(b*x)*sin(b*x)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2719}

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

```
[In] Int[Sin[b*x]^(5/2),x]
```

```
[Out] (-6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x]*Sin[b*x]^(3/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b} + \frac{3}{5}\int\sqrt{\sin(bx)}\,dx \\ &= -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b} - \frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{2\left(3E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\cos(bx)\sin^{\frac{3}{2}}(bx)\right)}{5b}$$

```
[In] Integrate[Sin[b*x]^(5/2),x]
[Out] (-2*(3*EllipticE[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sin[b*x]^(3/2)))/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.88

method	result
default	$\frac{\frac{2\left(\sin^4(bx)\right)}{5}-\frac{2\left(\sin^2(bx)\right)}{5}-\frac{6\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,E\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)}\right.}{5}}{\cos(bx)\sqrt{\sin(bx)}\,b}$

```
[In] int(sin(b*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] (2/5*sin(b*x)^4-2/5*sin(b*x)^2-6/5*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)
*(-sin(b*x))^(1/2)*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))+3/5*(sin(b*x)+
1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/
2),1/2*2^(1/2)))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \sin^{\frac{5}{2}}(bx)\,dx =$$
$$-\frac{2\cos(bx)\sin(bx)^{\frac{3}{2}} - 3i\sqrt{2}\sqrt{-i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx))) + 3}{5b}$$

[In] integrate(sin(b*x)^(5/2),x, algorithm="fricas")

[Out] -1/5*(2*cos(b*x)*sin(b*x)^(3/2) - 3*I*sqrt(2)*sqrt(-I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x))) + 3*I*sqrt(2)*sqrt(I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x))))/b

Sympy [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin^{\frac{5}{2}}(bx)\,dx$$

[In] integrate(sin(b*x)**(5/2),x)

[Out] Integral(sin(b*x)**(5/2), x)

Maxima [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin(bx)^{\frac{5}{2}}\,dx$$

[In] integrate(sin(b*x)^(5/2),x, algorithm="maxima")

[Out] integrate(sin(b*x)^(5/2), x)

Giac [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin(bx)^{\frac{5}{2}}\,dx$$

[In] integrate(sin(b*x)^(5/2),x, algorithm="giac")

[Out] integrate(sin(b*x)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\left(\sin(b\,x)^2\right)^{7/4}}$$

```
[In] int(sin(b*x)^(5/2),x)
[Out] -(cos(b*x)*sin(b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(7/4))
```

3.11 $\int \sin^{\frac{3}{2}}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sin^{\frac{3}{2}}(bx)\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}$$

```
[Out] -2/3*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/3*cos(b*x)*sin(b*x)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2720}

$$\int \sin^{\frac{3}{2}}(bx)\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

```
[In] Int[Sin[b*x]^(3/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b} + \frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$
$$= -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4} - \frac{bx}{2}, 2\right)}{3b} - \frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sin^{\frac{3}{2}}(bx)\,dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi - 2bx), 2\right) + \cos(bx)\sqrt{\sin(bx)}\right)}{3b}$$

```
[In] Integrate[Sin[b*x]^(3/2),x]
[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sqrt[Sin[b*x]]))/(3*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)+1}, \frac{\sqrt{2}}{2}\right)}{3} - \frac{2\left(\cos^2(bx)\right)\sin(bx)}{3}}{\cos(bx)\sqrt{\sin(bx)}\,b}$	72

```
[In] int(sin(b*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] (1/3*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((
sin(b*x)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x)^2*sin(b*x))/cos(b*x)/sin(b*x)^(
1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \frac{\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx)+i\,\sin(bx))+\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx)-i\,\sin(bx$$

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x)) + sq
rt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x)) - 2*cos(b*x)
*sqrt(sin(b*x)))/b
```

Sympy [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin^{\frac{3}{2}}(bx)\,dx$$

```
[In] integrate(sin(b*x)**(3/2),x)
[Out] Integral(sin(b*x)**(3/2), x)
```

Maxima [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin(bx)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x)^(3/2), x)
```

Giac [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin(bx)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sin^{\frac{3}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\left(\sin(b\,x)^2\right)^{5/4}}$$

```
[In] int(sin(b*x)^(3/2),x)
```

```
[Out] -(cos(b*x)*sin(b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(5/4))
```

3.12 $\int \sqrt{\sin(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

```
[Out] -2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+1
/2*b*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2719}

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

```
[In] Int[Sqrt[Sin[b*x]],x]
[Out] (-2*EllipticE[Pi/4 - (b*x)/2, 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

```
[In] Integrate[Sqrt[Sin[b*x]],x]
```

```
[Out] (-2*EllipticE[(Pi/2 - b*x)/2, 2])/b
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

method	result
default	$-\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\left(2E\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx)\sqrt{\sin(bx)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(e^{2ibx}-1\right)e^{-ibx}}}{b}+\frac{i\left(\frac{2i\left(i-ie^{2ibx}\right)}{\sqrt{e^{ibx}\left(i-ie^{2ibx}\right)}}-\frac{\sqrt{e^{ibx}+1}\,\sqrt{-2\,e^{ibx}+2}\,\sqrt{-e^{ibx}}\,\left(-2E\left(\sqrt{e^{ibx}+1},\frac{\sqrt{2}}{2}\right)+F\left(\sqrt{e^{ibx}+1},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{-ie^{3ibx}+ie^{ibx}}}\right)\sqrt{2}\,\sqrt{-}}{b\left(e^{2ibx}-1\right)}$

```
[In] int(sin(b*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*(2*EllipticE((s
in(b*x)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2)))/co
s(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.89

$$\int \sqrt{\sin(bx)}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{-i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)))-i\sqrt{2}\sqrt{i}\text{weierstrassZeta}(4}{b}$$

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(-I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x
) + I*sin(b*x))) - I*sqrt(2)*sqrt(I)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(b*x) - I*sin(b*x))))/b
```

Sympy [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

```
[In] integrate(sin(b*x)**(1/2),x)
[Out] Integral(sqrt(sin(b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(b*x)), x)
```

Giac [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(b*x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2\,\mathrm{E}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

```
[In] int(sin(b*x)^(1/2),x)
[Out] -(2*ellipticE(pi/4 - (b*x)/2, 2))/b
```

3.13 $\int \frac{1}{\sqrt{\sin(bx)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{b}$$

```
[Out] -2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi+1
/2*b*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2720}

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{b}$$

```
[In] Int[1/Sqrt[Sin[b*x]],x]
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right),2\right)}{b}$$

```
[In] Integrate[1/Sqrt[Sin[b*x]],x]
[Out] (-2*EllipticF[(Pi/2 - b*x)/2, 2])/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

method	result	size
default	$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{\cos(bx)\sqrt{\sin(bx)}\,b}$	57

```
[In] int(1/sin(b*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b
*x)+1)^(1/2),1/2*2^(1/2))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \frac{\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx)+i\,\sin(bx))+\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx)-i\,\sin(bx))}{b}$$

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="fricas")
[Out] (sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x)) + sqrt(2
)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x)))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

```
[In] integrate(1/sin(b*x)**(1/2),x)
[Out] Integral(1/sqrt(sin(b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sin(b*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sin(b*x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

```
[In] int(1/sin(b*x)^(1/2),x)
[Out] -(2*ellipticF(pi/4 - (b*x)/2, 2))/b
```

3.14 $\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b} - \frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

```
[Out] 2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+1/
2*b*x),2^(1/2))/b-2*cos(b*x)/b/sin(b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2719}

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b} - \frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

```
[In] Int[Sin[b*x]^(-3/2),x]
```

```
[Out] (2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(bx)}{b\sqrt{\sin(bx)}} - \int \sqrt{\sin(bx)}\,dx \\ &= \frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|\,2\right)}{b} - \frac{2\cos(bx)}{b\sqrt{\sin(bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{2\left(E\left(\frac{1}{4}(\pi-2bx)\middle|\,2\right)-\frac{\cos(bx)}{\sqrt{\sin(bx)}}\right)}{b}$$

```
[In] Integrate[Sin[b*x]^(-3/2),x]
[Out] (2*(EllipticE[(Pi - 2*b*x)/4, 2] - Cos[b*x]/Sqrt[Sin[b*x]]))/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.97

method	result
default	$\frac{2\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,E\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-2(c}{\cos(bx)\sqrt{\sin(bx)}\,b}$

```
[In] int(1/sin(b*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] (2*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticE((si
n(b*x)+1)^(1/2),1/2*2^(1/2))-(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin
(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x)^2)/cos(b*
x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{-i\sqrt{2}\sqrt{-i}\sin(bx)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)))+i\sqrt{2}\sqrt{i}\sin(bx)\,\mathrm{w}}{b\sin(bx)}$$

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(-I)*sin(b*x)*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(b*x) + I*sin(b*x))) + I*sqrt(2)*sqrt(I)*sin(b*x)*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x))) - 2*cos(b*x)*sqrt(sin(
b*x)))/(b*sin(b*x))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$$

```
[In] integrate(1/sin(b*x)**(3/2),x)
```

```
[Out] Integral(sin(b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin{(bx)}^{\frac{3}{2}}}\,dx$$

[In] integrate(1/sin(b*x)^(3/2),x, algorithm="giac")

[Out] integrate(sin(b*x)^(-3/2), x)

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = -\frac{\cos{(b\,x)}\,\left(\sin{(b\,x)}^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos{(b\,x)}^2\right)}{b\,\sqrt{\sin{(b\,x)}}}$$

[In] int(1/sin(b*x)^(3/2),x)

[Out] -(cos(b*x)*(sin(b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(b*x)^2))/(b*si
n(b*x)^(1/2))

3.15 $\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b} - \frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

```
[Out] -2/3*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/3*cos(b*x)/b/sin(b*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2720}

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b} - \frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

```
[In] Int[Sin[b*x]^(-5/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x])/(3*b*Sin[b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}+\frac{1}{3}\int\frac{1}{\sqrt{\sin(bx)}}\,dx\\ &= -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int\frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx=-\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi-2bx),2\right)+\frac{\cos(bx)}{\sin^{\frac{3}{2}}(bx)}\right)}{3b}$$

```
[In] Integrate[Sin[b*x]^(-5/2),x]
[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]/Sin[b*x]^(3/2)))/(3*b)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,F\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\sin(bx)-2\left(\cos^2(bx)\right)}{3\sin(bx)^{\frac{3}{2}}\cos(bx)b}$	72

```
[In] int(1/sin(b*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/sin(b*x)^(3/2)*((sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1
/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))*sin(b*x)-2*cos(b*x)^2)/cos(b*
x)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$$
$$= \frac{\sqrt{-i}\left(\sqrt{2}\cos(bx)^2 - \sqrt{2}\right)\text{weierstrassPInverse}(4, 0, \cos(bx) + i\,\sin(bx)) + \sqrt{i}\left(\sqrt{2}\cos(bx)^2 - \sqrt{2}\right)\text{weierstra}}{3\left(b\cos(bx)^2 - b\right)}$$

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(-I)*(sqrt(2)*cos(b*x)^2 - sqrt(2))*weierstrassPInverse(4, 0, cos(
b*x) + I*sin(b*x)) + sqrt(I)*(sqrt(2)*cos(b*x)^2 - sqrt(2))*weierstrassPInv
erse(4, 0, cos(b*x) - I*sin(b*x)) + 2*cos(b*x)*sqrt(sin(b*x)))/(b*cos(b*x)^
2 - b)
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$$

```
[In] integrate(1/sin(b*x)**(5/2),x)
```

```
[Out] Integral(sin(b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\sin(b\,x)^{3/2}}$$

```
[In] int(1/sin(b*x)^(5/2),x)
[Out] -(cos(b*x)*(sin(b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(b*x)^2))/(b*si
n(b*x)^(3/2))
```

3.16 $\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

```
[Out] 6/5*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+
1/2*b*x),2^(1/2))/b-2/5*cos(b*x)/b/sin(b*x)^(5/2)-6/5*cos(b*x)/b/sin(b*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2719}

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

```
[In] Int[Sin[b*x]^(-7/2),x]
```

```
[Out] (6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x])/(5*b*Sin[b*x]^(5/2))
- (6*Cos[b*x])/(5*b*Sqrt[Sin[b*x]])
```

<u>Rule 2716</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} + \frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx \\
&= -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} - \frac{6\cos(bx)}{5b\sqrt{\sin(bx)}} - \frac{3}{5}\int \sqrt{\sin(bx)}\,dx \\
&= \frac{6E\left(\frac{\pi}{4} - \frac{bx}{2}\middle|\,2\right)}{5b} - \frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} - \frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \frac{-7\cos(bx) + 3\cos(3bx) + 12E\left(\frac{1}{4}(\pi - 2bx)\middle|\,2\right)\sin^{\frac{5}{2}}(bx)}{10b\sin^{\frac{5}{2}}(bx)}$$

```
[In] Integrate[Sin[b*x]^(-7/2),x]
[Out] (-7*Cos[b*x] + 3*Cos[3*b*x] + 12*EllipticE[(Pi - 2*b*x)/4, 2]*Sin[b*x]^(5/2
))/(10*b*Sin[b*x]^(5/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20

method	result
default	$\frac{6\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,(\sin^2(bx))\,E\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,(\sin^2(bx))\,F\left(\right.}{5\sin(bx)^{\frac{5}{2}}\cos(bx)b}$

```
[In] int(1/sin(b*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/5/sin(b*x)^(5/2)*(6*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^
(1/2)*sin(b*x)^2*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))-3*(sin(b*x)+1)^(
1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*sin(b*x)^2*EllipticF((sin(b*x)
+1)^(1/2),1/2*2^(1/2))+6*sin(b*x)^4-4*sin(b*x)^2-2)/cos(b*x)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = -\frac{3\sqrt{-i}\left(i\sqrt{2}\cos(bx)^2 - i\sqrt{2}\right)\sin(bx)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx)+i\,\sin(bx)$$

[In] integrate(1/sin(b*x)^(7/2),x, algorithm="fricas")

[Out] -1/5*(3*sqrt(-I)*(I*sqrt(2)*cos(b*x)^2 - I*sqrt(2))*sin(b*x)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x))) + 3*sqrt(I)*(-I*sqrt(2)*cos(b*x)^2 + I*sqrt(2))*sin(b*x)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x))) + 2*(3*cos(b*x)^3 - 4*cos(b*x))*sqrt(sin(b*x)))/((b*cos(b*x)^2 - b)*sin(b*x))

Sympy [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$$

[In] integrate(1/sin(b*x)**(7/2),x)

[Out] Integral(sin(b*x)**(-7/2), x)

Maxima [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{7}{2}}}\,dx$$

[In] integrate(1/sin(b*x)^(7/2),x, algorithm="maxima")

[Out] integrate(sin(b*x)^(-7/2), x)

Giac [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin\left(bx\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(-7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = -\frac{\cos\left(b\,x\right)\,\left(\sin\left(b\,x\right)^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \cos\left(b\,x\right)^2\right)}{b\sin\left(b\,x\right)^{5/2}}$$

```
[In] int(1/sin(b*x)^(7/2),x)
```

```
[Out] -(cos(b*x)*(sin(b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(b*x)^2))/(b*si
n(b*x)^(5/2))
```

3.17 $\int \sin^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{21b} - \frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b} - \frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}$$

```
[Out] -10/21*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/7*cos(b*x+a)*sin(b*x+a)^(5/2)/b-
10/21*cos(b*x+a)*sin(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2720}

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{21b} - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b} - \frac{10\sqrt{\sin(a+bx)}\cos(a+bx)}{21b}$$

```
[In] Int[Sin[a + b*x]^(7/2),x]
```

```
[Out] (10*EllipticF[(a - Pi/2 + b*x)/2, 2])/(21*b) - (10*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(21*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\int\sin^{\frac{3}{2}}(a+bx)\,dx\\ &= -\frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b}-\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{21}\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{21b}-\frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b}-\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int\sin^{\frac{7}{2}}(a+bx)\,dx = \frac{-20\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+(-23\cos(a+bx)+3\cos(3(a+bx)))\sqrt{\sin(a+bx)}}{42b}$$

```
[In] Integrate[Sin[a + b*x]^(7/2),x]
[Out] (-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3*(a +
 b*x)])*Sqrt[Sin[a + b*x]])/(42*b)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\frac{2\left(\cos^{4}(bx+a)\right)\sin(bx+a)}{7}+\frac{5\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{21}-\frac{16\left(\cos^{2}(bx+a)\right)\sin(bx+a)}{21}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	104

```
[In] int(sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/7*cos(b*x+a)^4*sin(b*x+a)+5/21*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1
/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-16/21*c
os(b*x+a)^2*sin(b*x+a))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{5\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+5\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx}{21\,b}$$

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x
 + a)) + 5*sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b
*x + a)) + 2*(3*cos(b*x + a)^3 - 8*cos(b*x + a))*sqrt(sin(b*x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{7}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = -\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{9/4}}$$

```
[In] int(sin(a + b*x)^(7/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(9/4))
```

3.18 $\int \sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx=\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}$$

```
[Out] -6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
E(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)*sin(b*x+a)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2715, 2719}

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx=\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

```
[In] Int[Sin[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(
3/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b} + \frac{3}{5}\int\sqrt{\sin(a+bx)}\,dx\\ &= \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{5b} - \frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = -\frac{6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)+\sqrt{\sin(a+bx)}\sin(2(a+bx))}{5b}$$

```
[In] Integrate[Sin[a + b*x]^(5/2),x]
[Out] -1/5*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*Sin[2*(a +
 b*x)])/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.02

method	result
default	$\frac{\frac{2\left(\sin^4(bx+a)\right)}{5}-\frac{2\left(\sin^2(bx+a)\right)}{5}-\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}}{}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

```
[In] int(sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] (2/5*sin(b*x+a)^4-2/5*sin(b*x+a)^2-6/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+3/
5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elliptic
F((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx =$$

$$-\frac{2\cos(bx+a)\sin(bx+a)^{\frac{3}{2}} - 3i\sqrt{2}\sqrt{-i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\text{s}}{5\,b}$$

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*cos(b*x + a)*sin(b*x + a)^(3/2) - 3*I*sqrt(2)*sqrt(-I)*weierstrassZ
eta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*s
qrt(2)*sqrt(I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a)
 - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin^{\frac{5}{2}}(a+bx)\,dx$$

```
[In] integrate(sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = -\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\,\left(\sin(a+b\,x)^2\right)^{7/4}}$$

```
[In] int(sin(a + b*x)^(5/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(7/4))
```

3.19 $\int \sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b}-\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
F(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/3*cos(b*x+a)*sin(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2715, 2720}

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

```
[In] Int[Sin[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b
*x]])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}+\frac{1}{3}\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b}-\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+\cos(a+bx)\sqrt{\sin(a+bx)}\right)}{3b}$$

```
[In] Integrate[Sin[a + b*x]^(3/2),x]
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
)/(3*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\left(\cos^2(bx+a)\right)\sin(bx+a)}{3}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	88

```
[In] int(sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] (1/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b*x
+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx+}{3b}$$

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x +
a)) + sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x +
a)) - 2*cos(b*x + a)*sqrt(sin(b*x + a)))/b
```

Sympy [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] integrate(sin(b*x+a)**(3/2),x)
[Out] Integral(sin(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = -\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{5/4}}$$

```
[In] int(sin(a + b*x)^(3/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(5/4))
```

3.20 $\int \sqrt{\sin(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{b}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2719}

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{b}$$

```
[In] Int[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sqrt{\sin(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right)\middle|\,2\right)}{b}$$

```
[In] Integrate[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticE[(-a + Pi/2 - b*x)/2, 2])/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.33

method	result
default	$-\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\left(2E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}{b}+\frac{i\left(\frac{2i\left(i-ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(i-ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\,\left(-2E\left(\sqrt{e^{i(bx+a)}+1},\ldots\right.\right.}{\sqrt{-ie^{3i(bx+a)}+ie^{i(bx+a)}}}\ldots\right)}{b\left(e^{2i(b\ldots}\right.}$

```
[In] int(sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*(2*Ellipt
icE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^
(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \sqrt{\sin(a+bx)}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{-i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\sqrt{i}\text{weierstra}\ldots}{b}$$

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(-I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x
 + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(I)*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin{(a+bx)}}\,dx$$

```
[In] integrate(sin(b*x+a)**(1/2),x)
[Out] Integral(sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin{(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin{(bx+a)}}\,dx$$

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\middle|2\right)}{b}$$

```
[In] int(sin(a + b*x)^(1/2),x)
[Out] (2*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/b
```

3.21 $\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{b}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2720}

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b}$$

```
[In] Int[1/Sqrt[Sin[a + b*x]],x]
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right),2\right)}{b}$$

```
[In] Integrate[1/Sqrt[Sin[a + b*x]],x]
[Out] (-2*EllipticF[(-a + Pi/2 - b*x)/2, 2])/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

method	result	size
default	$\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	69

```
[In] int(1/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF(
(sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{-i}\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a))+\sqrt{2}\sqrt{i}\text{weierstrassPInverse}(4,0,\cos(bx+a)-i\,\sin(bx+a))}{b}$$

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="fricas")
[Out] (sqrt(2)*sqrt(-I)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))
+ sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))
/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(a+bx)}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx+a)}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx+a)}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b}$$

```
[In] int(1/sin(a + b*x)^(1/2),x)
[Out] -(2*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/b
```

3.22 $\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

```
[Out] 2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(c
os(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2*cos(b*x+a)/b/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin[a +
b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}} - \int \sqrt{\sin(a+bx)}\,dx$$
$$= -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right) - \frac{\cos(a+bx)}{\sqrt{\sin(a+bx)}}\right)}{b}$$

```
[In] Integrate[Sin[a + b*x]^(-3/2),x]
[Out] (2*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - Cos[a + b*x]/Sqrt[Sin[a + b*x]]))
/b
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.07

method	result
default	$\frac{2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\right.}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

```
[In] int(1/sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] (2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipti
cE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)
^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*co
s(b*x+a)^2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{-i}\sin(bx+a)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a)))+i\sqrt{2}}{}$$

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(-I)*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(I)*sin(b*x + a)*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))
) - 2*cos(b*x + a)*sqrt(sin(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\,\sqrt{\sin(a+b\,x)}}$$

```
[In] int(1/sin(a + b*x)^(3/2),x)
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(1/2))
```

3.23 $\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
F(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/3*cos(b*x+a)/b/sin(b*x+a)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2720}

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[In] Int[Sin[a + b*x]^(-5/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin[a +
b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2bx),2\right) - \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

```
[In] Integrate[Sin[a + b*x]^(-5/2),x]
[Out] (2*(EllipticF[(2*a - Pi + 2*b*x)/4, 2] - Cos[a + b*x]/Sin[a + b*x]^(3/2)))/
(3*b)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sin(bx+a)-2\left(\cos^2(bx+a)\right)}{3\sin(bx+a)^{\frac{3}{2}}\cos(bx+a)b}$	88

```
[In] int(1/sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/sin(b*x+a)^(3/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*
x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*sin(b*x+a)-2*cos(b*
x+a)^2)/cos(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.45

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{-i}\left(\sqrt{2}\cos(bx+a)^2-\sqrt{2}\right)\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+\sqrt{i}\left(\sqrt{2}\cos(bx+a)^2\right.}{3\left(b\cos(bx+a)^2\right.}$$

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(-I)*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*weierstrassPInverse(4, 0,
cos(b*x + a) + I*sin(b*x + a)) + sqrt(I)*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))
*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*cos(b*x + a)*
sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\sin(a+b\,x)^{3/2}}$$

```
[In] int(1/sin(a + b*x)^(5/2),x)
```

```
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(3/2))
```

3.24 $\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)} dx = -\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

```
[Out] 6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)/b/sin(b*x+a)^(5/2)-6/5
*cos(b*x+a)/b/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)} dx = -\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^(-7/2),x]
```

```
[Out] (-6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x])/(5*b*Sin[a +
 b*x]^(5/2)) - (6*Cos[a + b*x])/(5*b*Sqrt[Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}} - \frac{3}{5}\int \sqrt{\sin(a+bx)}\,dx \\
&= -\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{5b} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$
\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\left(3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right) - \frac{\cos(a+bx)(1+3\sin^2(a+bx))}{\sin^{\frac{5}{2}}(a+bx)}\right)}{5b}
$$

```
[In] Integrate[Sin[a + b*x]^(-7/2),x]
[Out] (2*(3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - (Cos[a + b*x]*(1 + 3*Sin[a + b*
x]^2))/Sin[a + b*x]^(5/2)))/(5*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.29

method	result
default	$\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,(\sin^2(bx+a))\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-}}{5\sin(bx+a)^{\frac{5}{2}}\cos(bx+a)b}$

```
[In] int(1/sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/5/sin(b*x+a)^(5/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(
b*x+a))^(1/2)*sin(b*x+a)^2*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(s
in(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*sin(b*x+a)^2
*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x+a)^4-4*sin(b*x+a)^2-
2)/cos(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{3\sqrt{-i}\left(i\sqrt{2}\cos\left(bx+a\right)^2 - i\sqrt{2}\right)\sin\left(bx+a\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)}{}$$

[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="fricas")

[Out] -1/5*(3*sqrt(-I)*(I*sqrt(2)*cos(b*x + a)^2 - I*sqrt(2))*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*sqrt(I)*(-I*sqrt(2)*cos(b*x + a)^2 + I*sqrt(2))*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*(3*cos(b*x + a)^3 - 4*cos(b*x + a))*sqrt(sin(b*x + a)))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))

Sympy [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$

[In] integrate(1/sin(b*x+a)**(7/2),x)

[Out] Integral(sin(a + b*x)**(-7/2), x)

Maxima [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="maxima")

[Out] integrate(sin(b*x + a)^(-7/2), x)

Giac [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin{(bx+a)}^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\cos{(a+b\,x)}\,\left(\sin{(a+b\,x)}^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \cos{(a+b\,x)}^2\right)}{b\sin{(a+b\,x)}^{5/2}}$$

```
[In] int(1/sin(a + b*x)^(7/2),x)
```

```
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(5/2))
```

3.25 $\int (c\sin(a+bx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{10c^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{21b\sqrt{c\sin(a+bx)}} - \frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

```
[Out] -2/7*c*cos(b*x+a)*(c*sin(b*x+a))^(5/2)/b-10/21*c^4*(sin(1/2*a+1/4*Pi+1/2*b*
x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2
^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^(1/2)-10/21*c^3*cos(b*x+a)*(c*sin
(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{10c^4\sqrt{\sin(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{21b\sqrt{c\sin(a+bx)}} - \frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

```
[In] Int[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (10*c^4*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(21*b*Sqrt[c*S
in[a + b*x]]) - (10*c^3*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(21*b) - (2*c*Co
s[a + b*x]*(c*Sin[a + b*x])^(5/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}+\frac{1}{7}\left(5c^2\right)\int (c\sin(a+bx))^{3/2}\,dx\\
&= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}\\
&\quad-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}+\frac{1}{21}\left(5c^4\right)\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx\\
&= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}\\
&\quad+\frac{\left(5c^4\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{21\sqrt{c\sin(a+bx)}}\\
&= \frac{10c^4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{21b\sqrt{c\sin(a+bx)}}\\
&\quad-\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{c^3\left(-20\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+(-23\cos(a+bx)+3\cos(3(a+bx)))\sqrt{\sin(a+bx)}\right)}{42b\sqrt{\sin(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (c^3*(-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3
*(a + b*x)])*Sqrt[Sin[a + b*x]])*Sqrt[c*Sin[a + b*x]])/(42*b*Sqrt[Sin[a + b
*x]])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

method	result
default	$-\frac{c^4\left(-6\left(\sin^5(bx+a)\right)+5\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)F\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(bx+a)\right)+10\sin(bx+a)\right)}{21\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

```
[In] int((c*sin(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*c^4*(-6*sin(b*x+a)^5+5*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*s
in(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-4*sin(b*x+a)^3
+10*sin(b*x+a))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{5\sqrt{2}\sqrt{-i\,c}c^3\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a))+5\sqrt{2}\sqrt{i\,c}c^3\text{weierstrassPI}}{}$$

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*sqrt(2)*sqrt(-I*c)*c^3*weierstrassPInverse(4, 0, cos(b*x + a) + I*s
in(b*x + a)) + 5*sqrt(2)*sqrt(I*c)*c^3*weierstrassPInverse(4, 0, cos(b*x +
a) - I*sin(b*x + a)) + 2*(3*c^3*cos(b*x + a)^3 - 8*c^3*cos(b*x + a))*sqrt(c
*sin(b*x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\sin(bx+a))^{\frac{7}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(7/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\sin(bx+a))^{\frac{7}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\,\sin(a+b\,x))^{7/2}\,dx$$

```
[In] int((c*sin(a + b*x))^(7/2),x)
[Out] int((c*sin(a + b*x))^(7/2), x)
```

3.26 $\int (c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin(a+bx))^{5/2}\,dx = \frac{6c^2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

```
[Out] -2/5*c*cos(b*x+a)*(c*sin(b*x+a))^(3/2)/b-6/5*c^2*(sin(1/2*a+1/4*Pi+1/2*b*x)
^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(
1/2))*(c*sin(b*x+a))^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2719}

$$\int (c\sin(a+bx))^{5/2}\,dx = \frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (6*c^2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*Sqrt[Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(3/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b} + \frac{1}{5}(3c^2)\int\sqrt{c\sin(a+bx)}\,dx \\
&= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b} + \frac{\left(3c^2\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{5\sqrt{\sin(a+bx)}} \\
&= \frac{6c^2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int(c\sin(a+bx))^{5/2}\,dx = -\frac{(c\sin(a+bx))^{5/2}\left(6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)+\sqrt{\sin(a+bx)}\sin(2(a+bx))\right)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2),x]
[Out] -1/5*((c*Sin[a + b*x])^(5/2)*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[
Sin[a + b*x]]*Sin[2*(a + b*x)]))/(b*Sin[a + b*x]^(5/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{c^3\left(6\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)E\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+\right.\right.}{5\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

[In] int((c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/5*c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^4+2*sin(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35

$$\int (c\sin(a+bx))^{5/2}\,dx = -\frac{2\sqrt{c\sin(bx+a)}c^2\cos(bx+a)\sin(bx+a) - 3i\sqrt{2}\sqrt{-i\,c}c^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,c}{}$$

[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="fricas")

[Out] -1/5*(2*sqrt(c*sin(b*x + a))*c^2*cos(b*x + a)*sin(b*x + a) - 3*I*sqrt(2)*sqrt(-I*c)*c^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*sqrt(I*c)*c^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b

Sympy [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(a+bx))^{\frac{5}{2}}\,dx$$

[In] integrate((c*sin(b*x+a))**(5/2),x)

[Out] Integral((c*sin(a + b*x))**(5/2), x)

Maxima [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\,\sin(a+b\,x))^{5/2}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^(5/2), x)
```

3.27 $\int (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin(a+bx))^{3/2}\,dx=\frac{2c^2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

```
[Out] -2/3*c^2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elli
pticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^
(1/2)-2/3*c*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\int (c\sin(a+bx))^{3/2}\,dx=\frac{2c^2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*c^2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b} + \frac{1}{3}c^2\int\frac{1}{\sqrt{c\sin(a+bx)}}\,dx \\
&= -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b} + \frac{\left(c^2\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{c\sin(a+bx)}} \\
&= \frac{2c^2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3b\sqrt{c\sin(a+bx)}} - \frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int(c\sin(a+bx))^{3/2}\,dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+\cos(a+bx)\sqrt{\sin(a+bx)}\right)(c\sin(a+bx))^{3/2}}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2),x]
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
*(c*Sin[a + b*x])^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{c^{2}\left(\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)F\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-2\left(\sin^{3}(bx+a)\right)+2\sin(bx+a)\right)}{3\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$	97

[In] int((c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3*c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int(c\sin(a+bx))^{3/2}\,dx=\frac{\sqrt{2}\sqrt{-i\,c}c\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a))+\sqrt{2}\sqrt{i\,c}c\text{weierstrassPInvers}}{3\,b}$$

[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*sqrt(-I*c)*c*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) + sqrt(2)*sqrt(I*c)*c*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)) - 2*sqrt(c*sin(b*x + a))*c*cos(b*x + a))/b

Sympy [F]

$$\int(c\sin(a+bx))^{3/2}\,dx=\int(c\sin(a+bx))^{\frac{3}{2}}\,dx$$

[In] integrate((c*sin(b*x+a))**(3/2),x)

[Out] Integral((c*sin(a + b*x))**(3/2), x)

Maxima [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\,\sin(a+b\,x))^{3/2}\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^(3/2), x)
```

3.28 $\int \sqrt{c \sin(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \sqrt{c \sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2719}

$$\int \sqrt{c \sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a + b
*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{c\sin(a+bx)}\int\sqrt{\sin(a+bx)}\,dx}{\sqrt{\sin(a+bx)}} \\ &= \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int\sqrt{c\sin(a+bx)}\,dx = -\frac{2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]],x]
[Out] (-2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a
 + b*x]])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

method	result
default	$-\frac{c\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)\left(2E\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-ic\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}{b}+\frac{i\left(\frac{2i\left(-ic\,e^{2i(bx+a)}+ic\right)}{c\sqrt{e^{i(bx+a)}\left(-ic\,e^{2i(bx+a)}+ic\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\,\left(-2E\left(\sqrt{e^{i(bx+\cdots}}\right.\right.\cdots}{\sqrt{-ic\,e^{3i(bx+a)}+ic\,e^{i(bx-\cdots}}}\cdots\right.}{b\left(e\cdots\right.}$

```
[In] int((c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -c*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*(2*Ellipti
cE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2
^(1/2)))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \sqrt{c\sin(a+bx)}\,dx = \frac{i\sqrt{2}\sqrt{-i\,c}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos{(bx+a)}+i\,\sin{(bx+a)}))-i\sqrt{2}\sqrt{i\,c}\mathrm{weierstr}}{b}$$

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(-I*c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b
*x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(I*c)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(a+bx)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sqrt{c\sin(a+bx)}\,dx = \frac{2\,\sqrt{c\,\sin\left(a+b\,x\right)}\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{\sin\left(a+b\,x\right)}}$$

```
[In] int((c*sin(a + b*x))^(1/2),x)
[Out] (2*(c*sin(a + b*x))^(1/2)*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/(b*sin(a + b*
x)^(1/2))
```

3.29 $\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2721, 2720}

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b\sqrt{c\sin(a+bx)}}$$

```
[In] Int[1/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a + b
*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{\sqrt{c\sin(a+bx)}}$$
$$= \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]],x]
[Out] (-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a
 + b*x]])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)F\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$	74

```
[In] int(1/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-
sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{2}\sqrt{-i\,c}\text{weierstrassPInverse}(4,0,\cos{(bx+a)}+i\,\sin{(bx+a)})+\sqrt{2}\sqrt{i\,c}\text{weierstrassPInverse}(4,0,\cos{(bx}}{bc}$$

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] (sqrt(2)*sqrt(-I*c)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)
) + sqrt(2)*sqrt(I*c)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x +
a)))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(a+bx)}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(1/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(bx+a)}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(c*sin(b*x + a)), x)

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{2\sqrt{\sin\left(a+b\,x\right)}\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{c\,\sin\left(a+b\,x\right)}}$$

[In] int(1/(c*sin(a + b*x))^(1/2),x)

[Out] -(2*sin(a + b*x)^(1/2)*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/(b*(c*sin(a + b*x))^(1/2))

3.30 $\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}$$

```
[Out] -2*cos(b*x+a)/b/c/(c*sin(b*x+a))^(1/2)+2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2
)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c
*sin(b*x+a))^(1/2)/b/c^2/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}} - \frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*x
)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{\int\sqrt{c\sin(a+bx)}\,dx}{c^2}\\ &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{\sqrt{c\sin(a+bx)}\int\sqrt{\sin(a+bx)}\,dx}{c^2\sqrt{\sin(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2\left(\cos(a+bx)-E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}\right)}{bc\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-3/2),x]
[Out] (-2*(Cos[a + b*x] - EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])
)/(b*c*Sqrt[c*Sin[a + b*x]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

method	result
default	$\frac{2\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)E\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}(bx+a)\right)F}{c\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

```
[In] int(1/(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/c*(2*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{-i\,c}\sin(bx+a)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)$$

[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")

[Out] (-I*sqrt(2)*sqrt(-I*c)*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(I*c)*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) - 2*sqrt(c*sin(b*x + a))*cos(b*x + a))/(b*c^2*sin(b*x + a))

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c*sin(b*x+a))**(3/2),x)

[Out] Integral((c*sin(a + b*x))**(-3/2), x)

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((c*sin(b*x + a))^(-3/2), x)

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\,\sin(a+b\,x))^{3/2}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(3/2),x)
[Out] int(1/(c*sin(a + b*x))^(3/2), x)
```

3.31 $\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3bc^2\sqrt{c\sin(a+bx)}}$$

```
[Out] -2/3*cos(b*x+a)/b/c/(c*sin(b*x+a))^(3/2)-2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^
(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2)
)*sin(b*x+a)^(1/2)/b/c^2/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2720}

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \frac{2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(-5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*c*(c*Sin[a + b*x])^(3/2)) + (2*EllipticF[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*c^2*Sqrt[c*Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{\int\frac{1}{\sqrt{c\sin(a+bx)}}\,dx}{3c^2}\\ &= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3c^2\sqrt{c\sin(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3bc^2\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int\frac{1}{(c\sin(a+bx))^{5/2}}\,dx=-\frac{2\left(\cos(a+bx)+\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sin^{\frac{3}{2}}(a+bx)\right)}{3bc(c\sin(a+bx))^{3/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-5/2),x]
[Out] (-2*(Cos[a + b*x] + EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2))
)/(3*b*c*(c*Sin[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sin^{\frac{5}{2}}(bx+a)\right)F\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(bx+a)\right)+2\sin(bx+a)}{3c^2\sin(bx+a)^2\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$	105

```
[In] int(1/(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

[Out] -1/3/c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(5/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/sin(b*x+a)^2/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \frac{\left(\sqrt{2}\cos\left(bx+a\right)^2-\sqrt{2}\right)\sqrt{-i\,c}\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)$$

[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="fricas")

[Out] 1/3*((sqrt(2)*cos(b*x + a)^2 - sqrt(2))*sqrt(-I*c)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) + (sqrt(2)*cos(b*x + a)^2 - sqrt(2))*sqrt(I*c)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*sqrt(c*sin(b*x + a))*cos(b*x + a))/(b*c^3*cos(b*x + a)^2 - b*c^3)

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin\left(a+bx\right))^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(c*sin(b*x+a))**(5/2),x)

[Out] Integral((c*sin(a + b*x))**(-5/2), x)

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((c*sin(b*x + a))^(-5/2), x)

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\sin{(a+b\,x)})^{5/2}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(5/2),x)
[Out] int(1/(c*sin(a + b*x))^(5/2), x)
```

3.32 $\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}$$

```
[Out] -2/5*cos(b*x+a)/b/c/(c*sin(b*x+a))^(5/2)-6/5*cos(b*x+a)/b/c^3/(c*sin(b*x+a)
)^(1/2)+6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*E
llipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/c^4/sin(
b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*c*(c*Sin[a + b*x])^(5/2)) - (6*Cos[a + b*x])/(5*b*c^
3*Sqrt[c*Sin[a + b*x]]) - (6*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a
+ b*x]])/(5*b*c^4*Sqrt[Sin[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} + \frac{3\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx}{5c^2} \\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{3\int \sqrt{c\sin(a+bx)}\,dx}{5c^4} \\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{\left(3\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx}{5c^4\sqrt{\sin(a+bx)}} \\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{2\left(\cot(a+bx) - 3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx) + \frac{3}{2}\sin(2(a+bx))\right)}{5bc^2(c\sin(a+bx))^{3/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*(Cot[a + b*x] - 3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2
) + (3*Sin[2*(a + b*x)])/2))/(5*b*c^2*(c*Sin[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.60

method	result
default	$\frac{6\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sin^{\frac{7}{2}}(bx+a)\right)E\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sin^{\frac{7}{2}}(bx+a)\right)F\left(\right.}{5c^3\sin(bx+a)^3\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

```
[In] int(1/(c*sin(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*El
lipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(b
*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2)
)+6*sin(b*x+a)^5-4*sin(b*x+a)^3-2*sin(b*x+a))/sin(b*x+a)^3/cos(b*x+a)/(c*si
n(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{3\left(i\sqrt{2}\cos\left(bx+a\right)^2-i\sqrt{2}\right)\sqrt{-i\,c}\sin\left(bx+a\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)\cdots}{}$$

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(I*sqrt(2)*cos(b*x + a)^2 - I*sqrt(2))*sqrt(-I*c)*sin(b*x + a)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)))
 + 3*(-I*sqrt(2)*cos(b*x + a)^2 + I*sqrt(2))*sqrt(I*c)*sin(b*x + a)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) +
2*(3*cos(b*x + a)^3 - 4*cos(b*x + a))*sqrt(c*sin(b*x + a)))/((b*c^4*cos(b*x
 + a)^2 - b*c^4)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+bx\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))**(7/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{{\left(c\,\sin\left(a+b\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(7/2),x)
[Out] int(1/(c*sin(a + b*x))^(7/2), x)
```

3.33 $\int (c\sin(a+bx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/3}}{7bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] 3/7*cos(b*x+a)*hypergeom([1/2, 7/6],[13/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(7/
3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(4/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(7/3))/(7*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/3}}{7bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\sin(a+bx))^{4/3}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{4/3}\tan(a+bx)}{7b}$$

```
[In] Integrate[(c*Sin[a + b*x])^(4/3),x]
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(4/3)*Tan[a + b*x])/(7*b)
```

Maple [F]

$$\int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

```
[In] int((c*sin(b*x+a))^(4/3),x)
[Out] int((c*sin(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(1/3)*c*sin(b*x + a), x)
```

Sympy [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(a+bx))^{\frac{4}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(4/3),x)
[Out] Integral((c*sin(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\,\sin(a+b\,x))^{4/3}\,dx$$

```
[In] int((c*sin(a + b*x))^(4/3),x)
[Out] int((c*sin(a + b*x))^(4/3), x)
```

3.34 $\int (c\sin(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\sin(a+bx))^{2/3}\,dx=\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/3}}{5bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] 3/5*cos(b*x+a)*hypergeom([1/2, 5/6],[11/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(5/
3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int (c\sin(a+bx))^{2/3}\,dx=\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(5/3))/(5*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/3}}{5bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\sin(a+bx))^{2/3}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{2/3}\tan(a+bx)}{5b}$$

```
[In] Integrate[(c*Sin[a + b*x])^(2/3),x]
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(2/3)*Tan[a + b*x])/(5*b)
```

Maple [F]

$$\int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

```
[In] int((c*sin(b*x+a))^(2/3),x)
[Out] int((c*sin(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(2/3), x)
```

Sympy [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin{(a+bx)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(2/3),x)
[Out] Integral((c*sin(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin{(bx+a)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin{(bx+a)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\,\sin{(a+b\,x)})^{2/3}\,dx$$

```
[In] int((c*sin(a + b*x))^(2/3),x)
[Out] int((c*sin(a + b*x))^(2/3), x)
```

3.35 $\int \sqrt[3]{c \sin(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 517

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = -\frac{3\sqrt{\frac{3}{2}\left(3-i\sqrt{3}\right)}\sqrt[3]{c}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3+i\sqrt{3}}}\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{i+\sqrt{3}}{3i+\sqrt{3}}+\frac{2(c\cdots}{(\cdots}}}{b} + \frac{3\left(1-i\sqrt{3}\right)\sqrt{3-i\sqrt{3}}\sqrt[3]{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3-i\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2\cdots}}{c^{2/3}}}\cdots}{2\sqrt{2}b}$$

```
[Out] 3/4*c^(1/3)*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3-I*3
^(1/2))^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2))*sec(b*x+a)*(1-I*3^(1/2))
*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin
(b*x+a))^(2/3)/c^(2/3)/(3+I*3^(1/2)))^(1/2)*(3-I*3^(1/2))^(1/2)*(2*(c*sin(b
*x+a))^(2/3)/c^(2/3)/(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b*2^(1/
2)-3/2*c^(1/3)*EllipticE(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3+
I*3^(1/2))^(1/2),((3*I-3^(1/2))/(3*I+3^(1/2)))^(1/2))*sec(b*x+a)*(1-(c*sin(
b*x+a))^(2/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin(b*x+a))^(2
/3)/c^(2/3)/(3+I*3^(1/2)))^(1/2)*(18-6*I*3^(1/2))^(1/2)*(2*(c*sin(b*x+a))^(
2/3)/c^(2/3)/(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \frac{3\cos(a+bx)(c\sin(a+bx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)}{4bc\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(4/3))/(4*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)(c\sin(a+bx))^{4/3}}{4bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)\sqrt[3]{c\sin(a+bx)}\tan(a+bx)}{4b}$$

```
[In] Integrate[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c
*Sin[a + b*x])^(1/3)*Tan[a + b*x])/(4*b)
```

Maple [F]

$$\int (c\sin (bx+a))^{\frac{1}{3}}\, dx$$

```
[In] int((c*sin(b*x+a))^(1/3),x)
[Out] int((c*sin(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c\sin(a+bx)}\, dx = \int (c\sin (bx+a))^{\frac{1}{3}}\, dx$$

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{c\sin(a+bx)}\, dx = \int \sqrt[3]{c\sin (a+bx)}\, dx$$

```
[In] integrate((c*sin(b*x+a))**(1/3),x)
[Out] Integral((c*sin(a + b*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{c\sin(a+bx)}\, dx = \int (c\sin (bx+a))^{\frac{1}{3}}\, dx$$

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\sin(bx+a))^{\frac{1}{3}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\,\sin(a+b\,x))^{1/3}\,dx$$

```
[In] int((c*sin(a + b*x))^(1/3),x)
[Out] int((c*sin(a + b*x))^(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 252

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = -\frac{3\sqrt{3-i\sqrt{3}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3-i\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{i+\sqrt{3}}{3i+\sqrt{3}}+\cdots}}{\sqrt{2}b\sqrt[3]{c}}$$

```
[Out] -3/2*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3-I*3^(1/2))
^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2))*sec(b*x+a)*(1-(c*sin(b*x+a))^(2
/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin(b*x+a))^(2/3)/c^(2/3
)/(3+I*3^(1/2)))^(1/2)*(3-I*3^(1/2))^(1/2)*(2*(c*sin(b*x+a))^(2/3)/c^(2/3)/
(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b/c^(1/3)*2^(1/2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec), antiderivative size = 58, normalized size of antiderivative = 0.23, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \frac{3\cos(a+bx)(c\sin(a+bx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(a+bx)\right)}{2bc\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(2/3))/(2*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(a+bx)\right)(c\sin(a+bx))^{2/3}}{2bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(a+bx)\right)\tan(a+bx)}{2b\sqrt[3]{c\sin(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*Ta
n[a + b*x])/(2*b*(c*Sin[a + b*x])^(1/3))
```

Maple [F]

$$\int \frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

```
[In] int(1/(c*sin(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(2/3)/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))**(1/3),x)
[Out] Integral((c*sin(a + b*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{\left(c\,\sin\left(a+b\,x\right)\right)^{1/3}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(1/3),x)
[Out] int(1/(c*sin(a + b*x))^(1/3), x)
```

3.37 $\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 271

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)(c\sin(a+bx))^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(a+bx)\sqrt[3]{c\sin(}}{2bc^{5/3}\sqrt{-\frac{(c\sin(a+bx))^{2/3}(c^{2/3}-}{\left(c^{2/3}-\left(1+\sqrt{3}\right)(c\sin}}}$$

```
[Out] 1/2*3^(3/4)*((c^(2/3)-(c*sin(b*x+a))^(2/3)*(1-3^(1/2)))^2/(c^(2/3)-(c*sin(b
*x+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1-3^(1/2)
))*(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))*EllipticF((1-(c^(2/3)-(c*sin(
b*x+a))^(2/3)*(1-3^(1/2)))^2/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(b*x+a)*(c*sin(b*x+a))^(1/3)*(c^(2/3)-(c*
sin(b*x+a))^(2/3))*(c^(4/3)*(1+(c*sin(b*x+a))^(2/3)/c^(2/3)+(c*sin(b*x+a))^
(4/3)/c^(4/3))/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)/b/c^(5/3
)/(-(c*sin(b*x+a))^(2/3)*(c^(2/3)-(c*sin(b*x+a))^(2/3))/(c^(2/3)-(c*sin(b*x
+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3\cos(a+bx)\sqrt[3]{c\sin(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(-2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(1/3))/(b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2(a+bx)\right)\sqrt[3]{c\sin(a+bx)}}{bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2(a+bx)\right)\tan(a+bx)}{b(c\sin(a+bx))^{2/3}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-2/3),x]
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*Ta
n[a + b*x])/(b*(c*Sin[a + b*x])^(2/3))
```

Maple [F]

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}dx$$

```
[In] int(1/(c*sin(b*x+a))^(2/3),x)
[Out] int(1/(c*sin(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(1/3)/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin\left(a+bx\right))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))**(2/3),x)
[Out] Integral((c*sin(a + b*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin\left(bx+a\right))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{\left(c\,\sin\left(a+b\,x\right)\right)^{2/3}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(2/3),x)
[Out] int(1/(c*sin(a + b*x))^(2/3), x)
```

3.38 $\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

```
[Out] -3*cos(b*x+a)*hypergeom([-1/6, 1/2],[5/6],sin(b*x+a)^2)/b/c/(c*sin(b*x+a))^
(1/3)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(-4/3),x]
[Out] (-3*Cos[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2])/(b*c*Sq
rt[Cos[a + b*x]^2]*(c*Sin[a + b*x])^(1/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)\tan(a+bx)}{b(c\sin(a+bx))^{4/3}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*(c*Sin[a + b*x])^(4/3))
```

Maple [F]

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}dx$$

```
[In] int(1/(c*sin(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(c*sin(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2 - c^2), x)
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(a+bx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\,\sin{(a+b\,x)})^{4/3}}\,dx$$

```
[In] int(1/(c*sin(a + b*x))^(4/3),x)
[Out] int(1/(c*sin(a + b*x))^(4/3), x)
```

3.39 $\int \sin^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \sin^n(a+bx)\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sin^{1+n}(a+bx)}{b(1+n)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*sin(b*x+a)^
(1+n)/b/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2722}

$$\int \sin^n(a+bx)\,dx = \frac{\cos(a+bx)\sin^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
Sin[a + b*x]^(1 + n))/(b*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sin^{1+n}(a+bx)}{b(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \sin^n(a+bx)\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^{1+n}(a+bx)}{b(1+n)}$$

```
[In] Integrate[Sin[a + b*x]^n,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*Sin[a + b*x]^(1 + n))/(b*(1 + n))
```

Maple [F]

$$\int \left(\sin^n(bx+a)\right)dx$$

```
[In] int(sin(b*x+a)^n,x)
[Out] int(sin(b*x+a)^n,x)
```

Fricas [F]

$$\int \sin^n(a+bx)\,dx = \int \sin(bx+a)^n\,dx$$

```
[In] integrate(sin(b*x+a)^n,x, algorithm="fricas")
[Out] integral(sin(b*x + a)^n, x)
```

Sympy [F]

$$\int \sin^n(a+bx)\,dx = \int \sin^n(a+bx)\,dx$$

```
[In] integrate(sin(b*x+a)**n,x)
[Out] Integral(sin(a + b*x)**n, x)
```

Maxima [F]

$$\int \sin^n(a+bx)\,dx = \int \sin(bx+a)^n\,dx$$

```
[In] integrate(sin(b*x+a)^n,x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^n, x)
```

Giac [F]

$$\int \sin^n(a+bx)\,dx = \int \sin(bx+a)^n\,dx$$

```
[In] integrate(sin(b*x+a)^n,x, algorithm="giac")
[Out] integrate(sin(b*x + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sin^n(a+bx)\,dx = -\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{n}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{\frac{n}{2}+\frac{1}{2}}}$$

```
[In] int(sin(a + b*x)^n,x)
[Out] -(cos(a + b*x)*sin(a + b*x)^(n + 1)*hypergeom([1/2, 1/2 - n/2], 3/2, cos(a
+ b*x)^2))/(b*(sin(a + b*x)^2)^(n/2 + 1/2))
```

3.40 $\int (c \sin(a+bx))^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (c\sin(a+bx))^n\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+n}}{bc(1+n)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*(c*sin(b*x+
a))^(1+n)/b/c/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2722}

$$\int (c\sin(a+bx))^n\,dx = \frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + n))/(b*c*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+n}}{bc(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int (c\sin(a+bx))^n\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^n\tan(a+bx)}{b(1+n)}$$

```
[In] Integrate[(c*Sin[a + b*x])^n,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^n*Tan[a + b*x])/(b*(1 + n))
```

Maple [F]

$$\int (c\sin(bx+a))^n\,dx$$

```
[In] int((c*sin(b*x+a))^n,x)
[Out] int((c*sin(b*x+a))^n,x)
```

Fricas [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin(bx+a))^n\,dx$$

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^n, x)
```

Sympy [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin{(a+bx)})^n\;dx$$

```
[In] integrate((c*sin(b*x+a))**n,x)
[Out] Integral((c*sin(a + b*x))**n, x)
```

Maxima [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin{(bx+a)})^n\;dx$$

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^n, x)
```

Giac [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin{(bx+a)})^n\;dx$$

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^n\,dx = \int {\left(c\,\sin\left(a+b\,x\right)\right)}^n\,dx$$

```
[In] int((c*sin(a + b*x))^n,x)
[Out] int((c*sin(a + b*x))^n, x)
```

3.41 $\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^{1+m}(b\sin(e+fx)}{af(1+m+n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],sin(f*x+e)^2)
*(a*sin(f*x+e))^(1+m)*(b*sin(f*x+e))^n/a/f/(1+m+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(\right.}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sin[e +
f*x]^2]*(a*Sin[e + f*x])^(1 + m)*(b*Sin[e + f*x])^n)/(a*f*(1 + m + n)*Sqrt[
Cos[e + f*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \left((a\sin(e+fx))^{-n}(b\sin(e+fx))^n\right)\int(a\sin(e+fx))^{m+n}\,dx$$

$$= \frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^{1+m}}{af(1+m+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int(a\sin(e+fx))^m(b\sin(e+fx))^n\,dx$$
$$= \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\sin(e}{f(1+m+n)}$$

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n*Tan[e + f*x])/(f*(1 +
 m + n))
```

Maple [F]

$$\int(a\sin(fx+e))^m\,(b\sin(fx+e))^n\,dx$$

```
[In] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
[Out] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\sin(fx+e))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(e+fx))^m\,(b\sin(e+fx))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))**m*(b*sin(f*x+e))**n,x)
[Out] Integral((a*sin(e + f*x))**m*(b*sin(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\sin(fx+e))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\sin(fx+e))^n\,dx$$

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,\left(b\,\sin\left(e+f\,x\right)\right)^n\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n, x)
```

3.42 $\int \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^3(a+bx)\sin(a+bx)\,dx=-\frac{\cos^4(a+bx)}{4b}$$

```
[Out] -1/4*cos(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2645, 30}

$$\int \cos^3(a+bx)\sin(a+bx)\,dx=-\frac{\cos^4(a+bx)}{4b}$$

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/4*Cos[a + b*x]^4/b
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 2645</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos^4(a+bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x],x]
[Out] -1/4*Cos[a + b*x]^4/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cos^4(bx+a)}{4b}$	14
default	$-\frac{\cos^4(bx+a)}{4b}$	14
risch	$-\frac{\cos(4bx+4a)}{32b}-\frac{\cos(2bx+2a)}{8b}$	30
parallelrisch	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$	45
norman	$\frac{\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$	50

```
[In] int(cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^4}{4\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
[Out] -1/4*cos(b*x + a)^4/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{\cos^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a),x)
[Out] Piecewise((-cos(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^4}{4\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
[Out] -1/4*cos(b*x + a)^4/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\sin(bx+a)^4 - 2\,\sin(bx+a)^2}{4\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
[Out] -1/4*(sin(b*x + a)^4 - 2*sin(b*x + a)^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos\left(a+b\,x\right)^4}{4\,b}$$

```
[In] int(cos(a + b*x)^3*sin(a + b*x),x)
[Out] -cos(a + b*x)^4/(4*b)
```

3.43 $\int \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -1/3*cos(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2645, 30}

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/3*Cos[a + b*x]^3/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos^3(a+bx)}{3b}$$

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x],x]
[Out] -1/3*Cos[a + b*x]^3/b
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cos^3(bx+a)}{3b}$	14
default	$-\frac{\cos^3(bx+a)}{3b}$	14
risch	$-\frac{\cos(bx+a)}{4b}-\frac{\cos(3bx+3a)}{12b}$	27
parallelrisch	$\frac{-2-6\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	36
norman	$\frac{-\frac{2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	39

```
[In] int(cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/3*cos(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
[Out] -1/3*cos(b*x + a)^3/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a),x)
[Out] Piecewise((-cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
[Out] -1/3*cos(b*x + a)^3/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] -1/3*cos(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos\left(a+b\,x\right)^3}{3\,b}$$

```
[In] int(cos(a + b*x)^2*sin(a + b*x),x)
[Out] -cos(a + b*x)^3/(3*b)
```

3.44 $\int \cos(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cos(a+bx)\sin(a+bx)\,dx = \frac{\sin^2(a+bx)}{2b}$$

```
[Out] 1/2*sin(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2644, 30}

$$\int \cos(a+bx)\sin(a+bx)\,dx = \frac{\sin^2(a+bx)}{2b}$$

```
[In] Int[Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] Sin[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, \sin(a+bx))}{b}$$
$$= \frac{\sin^2(a+bx)}{2b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cos(a+bx)\sin(a+bx)\,dx = \frac{1}{2}\left(-\frac{\cos(2a)\cos(2bx)}{2b} + \frac{\sin(2a)\sin(2bx)}{2b}\right)$$

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x],x]
[Out] (-1/2*(Cos[2*a]*Cos[2*b*x])/b + (Sin[2*a]*Sin[2*b*x])/(2*b))/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin^2(bx+a)}{2b}$	14
default	$\frac{\sin^2(bx+a)}{2b}$	14
risch	$-\frac{\cos(2bx+2a)}{4b}$	15
parallelrisch	$\frac{1-\cos(2bx+2a)}{4b}$	19
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	32

```
[In] int(cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*sin(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}^2}{2\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*cos(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \cos(a+bx)\sin(a+bx)\,dx = \begin{cases} \frac{\sin^2{(a+bx)}}{2b} & \text{for } b \neq 0 \\ x\sin{(a)}\cos{(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x)
[Out] Piecewise((sin(a + b*x)**2/(2*b), Ne(b, 0)), (x*sin(a)*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}^2}{2\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*cos(b*x + a)^2/b
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = \frac{\sin{(bx+a)}^2}{2\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] 1/2*sin(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \cos(a+bx)\sin(a+bx)\,dx=\begin{cases}\frac{x\,\sin(2\,a)}{2} & \text{if } b=0\\ -\frac{\cos(2\,a+2\,b\,x)}{4\,b} & \text{if } b\neq 0\end{cases}$$

```
[In] int(cos(a + b*x)*sin(a + b*x),x)
[Out] piecewise(b == 0, (x*sin(2*a))/2, b ~= 0, -cos(2*a + 2*b*x)/(4*b))
```

3.45 $\int \tan(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -ln(cos(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3556}

$$\int \tan(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{b}$$

```
[In] Int[Tan[a + b*x],x]
[Out] -(Log[Cos[a + b*x]]/b)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral}=-\frac{\log(\cos(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

```
[In] Integrate[Tan[a + b*x],x]
[Out] -(Log[Cos[a + b*x]]/b)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a))}{b}$	12
default	$\frac{\ln(\sec(bx+a))}{b}$	12
parallelrisch	$\frac{\ln\left(\sqrt{\sec^2(bx+a)}\right)}{b}$	16
norman	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
risch	$ix+\frac{2ia}{b}-\frac{\ln(e^{2i(bx+a)}+1)}{b}$	30

```
[In] int(sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*ln(sec(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \tan(a+bx)\,dx = -\frac{\log(-\cos(bx+a))}{b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] -log(-cos(b*x + a))/b
```

Sympy [F]

$$\int \tan(a+bx)\,dx = \int \sin(a+bx)\sec(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(a+bx)\,dx = -\frac{\log\left(-\sin\left(bx+a\right)^2+1\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*log(-sin(b*x + a)^2 + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(a+bx)\,dx = -\frac{\log\left(\frac{|\cos(bx+a)|}{|b|}\right)}{b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] -log(abs(cos(b*x + a))/abs(b))/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

```
[In] int(sin(a + b*x)/cos(a + b*x),x)
[Out] log(tan(a + b*x)^2 + 1)/(2*b)
```

3.46 $\int \sec(a + bx) \tan(a + bx) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \sec(a + bx) \tan(a + bx) \, dx = \frac{\sec(a + bx)}{b}$$

```
[Out] sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2686, 8}

$$\int \sec(a + bx) \tan(a + bx) \, dx = \frac{\sec(a + bx)}{b}$$

```
[In] Int[Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int 1\,dx, x, \sec(a+bx))}{b}$$
$$= \frac{\sec(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{\sec(a+bx)}{b}$$

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x],x]
[Out] Sec[a + b*x]/b
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sec(bx+a)}{b}$	11
default	$\frac{\sec(bx+a)}{b}$	11
norman	$-\frac{2}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	21
parallelrisch	$-\frac{2}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	21
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}$	28

```
[In] int(sec(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] sec(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
[Out] 1/(b*cos(b*x + a))
```

Sympy [F]

$$\int \sec(a+bx)\tan(a+bx)\,dx = \int \sin(a+bx)\sec^2(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
[Out] 1/(b*cos(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] 1/(b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \sec(a+bx)\tan(a+bx)\,dx = -\frac{2}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-1\right)}$$

```
[In] int(sin(a + b*x)/cos(a + b*x)^2,x)
[Out] -2/(b*(tan(a/2 + (b*x)/2)^2 - 1))
```

3.47 $\int \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec^2(a+bx)\tan(a+bx)\,dx=\frac{\sec^2(a+bx)}{2b}$$

```
[Out] 1/2*sec(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \sec^2(a+bx)\tan(a+bx)\,dx=\frac{\sec^2(a+bx)}{2b}$$

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}(\int x\,dx, x, \sec(a+bx))}{b}$$
$$= \frac{\sec^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{\sec^2(a+bx)}{2b}$$

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x],x]
[Out] Sec[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sec^2(bx+a)}{2b}$	14
default	$\frac{\sec^2(bx+a)}{2b}$	14
risch	$\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}$	28
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}$	32
parallelrisch	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^2}$	43

```
[In] int(sec(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*sec(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{1}{2\,b\cos\left(bx+a\right)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
[Out] 1/2/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \int \sin\left(a+bx\right)\sec^3\left(a+bx\right)dx$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = -\frac{1}{2\left(\sin\left(bx+a\right)^2-1\right)b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
[Out] -1/2/((sin(b*x + a)^2 - 1)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{1}{2\,b\cos\left(bx+a\right)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
[Out] 1/2/(b*cos(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{\tan(a+b\,x)^2}{2\,b}$$

```
[In] int(sin(a + b*x)/cos(a + b*x)^3,x)
[Out] tan(a + b*x)^2/(2*b)
```

3.48 $\int \sec^3(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec^3(a+bx)\tan(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}$$

```
[Out] 1/3*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \sec^3(a+bx)\tan(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(a+bx)\right)}{b} \\
&= \frac{\sec^3(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{\sec^3(a+bx)}{3b}$$

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x],x]
[Out] Sec[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sec^3(bx+a)}{3b}$	14
default	$\frac{\sec^3(bx+a)}{3b}$	14
risch	$\frac{8\,\mathrm{e}^{3i(bx+a)}}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3}$	28
norman	$\frac{-\frac{2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3}$	39
parallelrisch	$\frac{-2-6\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	47

```
[In] int(sec(b*x+a)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sec(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos{(bx+a)}^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="fricas")
[Out] 1/3/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \int \sin{(a+bx)}\sec^4{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos{(bx+a)}^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="maxima")
[Out] 1/3/(b*cos(b*x + a)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos{(bx+a)}^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="giac")
[Out] 1/3/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos\left(a+b\,x\right)^3}$$

```
[In] int(sin(a + b*x)/cos(a + b*x)^4,x)
[Out] 1/(3*b*cos(a + b*x)^3)
```

3.49 $\int \cos^7(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b} - \frac{3\sin^5(a+bx)}{5b} + \frac{3\sin^7(a+bx)}{7b} - \frac{\sin^9(a+bx)}{9b}$$

```
[Out] 1/3*sin(b*x+a)^3/b-3/5*sin(b*x+a)^5/b+3/7*sin(b*x+a)^7/b-1/9*sin(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 276}

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = -\frac{\sin^9(a+bx)}{9b} + \frac{3\sin^7(a+bx)}{7b} - \frac{3\sin^5(a+bx)}{5b} + \frac{\sin^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (3*Sin[a + b*x]^5)/(5*b) + (3*Sin[a + b*x]^7)/(7*b)
- Sin[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^2-3x^4+3x^6-x^8)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b} - \frac{3\sin^5(a+bx)}{5b} + \frac{3\sin^7(a+bx)}{7b} - \frac{\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \frac{(1606+1389\cos(2(a+bx))+330\cos(4(a+bx))+35\cos(6(a+bx)))\sin^3(a+bx)}{10080b}$$

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
[Out] ((1606 + 1389*Cos[2*(a + b*x)] + 330*Cos[4*(a + b*x)] + 35*Cos[6*(a + b*x)]
)*Sin[a + b*x]^3)/(10080*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{\frac{\left(\sin^{9}(bx+a)\right)}{9}-\frac{3\left(\sin^{7}(bx+a)\right)}{7}+\frac{3\left(\sin^{5}(bx+a)\right)}{5}-\frac{\left(\sin^{3}(bx+a)\right)}{3}}{b}$
default	$-\frac{\frac{\left(\sin^{9}(bx+a)\right)}{9}-\frac{3\left(\sin^{7}(bx+a)\right)}{7}+\frac{3\left(\sin^{5}(bx+a)\right)}{5}-\frac{\left(\sin^{3}(bx+a)\right)}{3}}{b}$
risch	$\frac{7\sin(bx+a)}{128b}-\frac{\sin(9bx+9a)}{2304b}-\frac{5\sin(7bx+7a)}{1792b}-\frac{\sin(5bx+5a)}{160b}$
parallelrisch	$-\frac{\left(\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)-3\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(35\cos(6bx+6a)+1389\cos(2bx+2a)+330\cos(4bx+4a)+1606)\left(\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+3\cos\left(\right.\right.}{20160b}$
norman	$\frac{\frac{8\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+\frac{632\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{2848\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{315b}+\frac{632\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{16\left(\tan^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{9}}$

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/9*sin(b*x+a)^9-3/7*sin(b*x+a)^7+3/5*sin(b*x+a)^5-1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(35\,\cos\left(bx+a\right)^{8}-5\,\cos\left(bx+a\right)^{6}-6\,\cos\left(bx+a\right)^{4}-8\,\cos\left(bx+a\right)^{2}-16\right)\sin\left(bx+a\right)}{315\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/315*(35*cos(b*x + a)^8 - 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x
 + a)^2 - 16)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{16\sin^{9}(a+bx)}{315b}+\frac{8\sin^{7}(a+bx)\cos^{2}(a+bx)}{35b}+\frac{2\sin^{5}(a+bx)\cos^{4}(a+bx)}{5b}+\frac{\sin^{3}(a+bx)\cos^{6}(a+bx)}{3b} & \text{for } b\neq 0 \\ x\sin^{2}(a)\cos^{7}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**2,x)
[Out] Piecewise((16*sin(a + b*x)**9/(315*b) + 8*sin(a + b*x)**7*cos(a + b*x)**2/(
35*b) + 2*sin(a + b*x)**5*cos(a + b*x)**4/(5*b) + sin(a + b*x)**3*cos(a + b
*x)**6/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = -\frac{35\,\sin(bx+a)^9 - 135\,\sin(bx+a)^7 + 189\,\sin(bx+a)^5 - 105\,\sin(bx+a)^3}{315\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/315*(35*sin(b*x + a)^9 - 135*sin(b*x + a)^7 + 189*sin(b*x + a)^5 - 105*s
in(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = -\frac{\sin(9\,bx+9\,a)}{2304\,b} - \frac{5\,\sin(7\,bx+7\,a)}{1792\,b} - \frac{\sin(5\,bx+5\,a)}{160\,b} + \frac{7\,\sin(bx+a)}{128\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2304*sin(9*b*x + 9*a)/b - 5/1792*sin(7*b*x + 7*a)/b - 1/160*sin(5*b*x +
5*a)/b + 7/128*sin(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \frac{-\frac{\sin(a+b\,x)^9}{9} + \frac{3\,\sin(a+b\,x)^7}{7} - \frac{3\,\sin(a+b\,x)^5}{5} + \frac{\sin(a+b\,x)^3}{3}}{b}$$

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^2,x)
```

```
[Out] (sin(a + b*x)^3/3 - (3*sin(a + b*x)^5)/5 + (3*sin(a + b*x)^7)/7 - sin(a + b
*x)^9/9)/b
```

3.50 $\int \cos^5(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b} - \frac{2\sin^5(a+bx)}{5b} + \frac{\sin^7(a+bx)}{7b}$$

```
[Out] 1/3*sin(b*x+a)^3/b-2/5*sin(b*x+a)^5/b+1/7*sin(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 276}

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^7(a+bx)}{7b} - \frac{2\sin^5(a+bx)}{5b} + \frac{\sin^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (2*Sin[a + b*x]^5)/(5*b) + Sin[a + b*x]^7/(7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b} - \frac{2\sin^5(a+bx)}{5b} + \frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{840b}$$

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(840*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^7(bx+a)\right)}{7}-\frac{2\left(\sin^5(bx+a)\right)}{5}+\frac{\left(\sin^3(bx+a)\right)}{3}}{b}$	36
default	$\frac{\frac{\left(\sin^7(bx+a)\right)}{7}-\frac{2\left(\sin^5(bx+a)\right)}{5}+\frac{\left(\sin^3(bx+a)\right)}{3}}{b}$	36
risch	$\frac{5\sin(bx+a)}{64b}-\frac{\sin(7bx+7a)}{448b}-\frac{3\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{192b}$	55
parallelrisch	$\frac{\left(-\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+3\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(157+15\cos(4bx+4a)+108\cos(2bx+2a))\left(\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+3\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{1680b}$	74
norman	$\frac{\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}+\frac{304\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{32\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}+\frac{8\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^7}$	98

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/7*sin(b*x+a)^7-2/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(15\cos\left(bx+a\right)^6 - 3\cos\left(bx+a\right)^4 - 4\cos\left(bx+a\right)^2 - 8\right)\sin\left(bx+a\right)}{105\,b}$$

[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{8\sin^7(a+bx)}{105b} + \frac{4\sin^5(a+bx)\cos^2(a+bx)}{15b} + \frac{\sin^3(a+bx)\cos^4(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos^5(a) & \text{otherwise} \end{cases}$$

[In] integrate(cos(b*x+a)**5*sin(b*x+a)**2,x)

[Out] Piecewise((8*sin(a + b*x)**7/(105*b) + 4*sin(a + b*x)**5*cos(a + b*x)**2/(15*b) + sin(a + b*x)**3*cos(a + b*x)**4/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**5, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{15\sin\left(bx+a\right)^7 - 42\sin\left(bx+a\right)^5 + 35\sin\left(bx+a\right)^3}{105\,b}$$

[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = -\frac{\sin(7\,bx+7\,a)}{448\,b} - \frac{3\,\sin(5\,bx+5\,a)}{320\,b} - \frac{\sin(3\,bx+3\,a)}{192\,b} + \frac{5\,\sin(bx+a)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/448*sin(7*b*x + 7*a)/b - 3/320*sin(5*b*x + 5*a)/b - 1/192*sin(3*b*x + 3*
a)/b + 5/64*sin(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3}{105\,b}$$

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^2,x)
[Out] (35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7)/(105*b)
```

3.51 $\int \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

```
[Out] 1/3*sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - Sin[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^2-x^4)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b} - \frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{(7+3\cos(2(a+bx)))\sin^3(a+bx)}{30b}$$

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
[Out] ((7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(30*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^5(bx+a)\right)}{5}+\frac{\left(\sin^3(bx+a)\right)}{3}}{b}$	26
default	$\frac{-\frac{\left(\sin^5(bx+a)\right)}{5}+\frac{\left(\sin^3(bx+a)\right)}{3}}{b}$	26
parallelrisch	$\frac{30\sin(bx+a)-3\sin(5bx+5a)-5\sin(3bx+3a)}{240b}$	37
risch	$\frac{\sin(bx+a)}{8b}-\frac{\sin(5bx+5a)}{80b}-\frac{\sin(3bx+3a)}{48b}$	41
norman	$\frac{\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}+\frac{8\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$	66

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(3\,\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{15\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{2\sin^5\left(a+bx\right)}{15b}+\frac{\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ x\sin^2\left(a\right)\cos^3\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Piecewise((2*sin(a + b*x)**5/(15*b) + sin(a + b*x)**3*cos(a + b*x)**2/(3*b)
, Ne(b, 0)), (x*sin(a)**2*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^5-5\,\sin\left(bx+a\right)^3}{15\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^5 - 5\,\sin\left(bx+a\right)^3}{15\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5\sin\left(a+b\,x\right)^3 - 3\sin\left(a+b\,x\right)^5}{15\,b}$$

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2,x)
[Out] (5*sin(a + b*x)^3 - 3*sin(a + b*x)^5)/(15*b)
```

3.52 $\int \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cos(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b}$$

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^2,x]
[Out] Sin[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin^3(bx+a)}{3b}$	14
default	$\frac{\sin^3(bx+a)}{3b}$	14
parallelrisch	$\frac{3\sin(bx+a)-\sin(3bx+3a)}{12b}$	26
risch	$\frac{\sin(bx+a)}{4b}-\frac{\sin(3bx+3a)}{12b}$	27
norman	$\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	32

```
[In] int(cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*sin(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{\sin^3\left(a+bx\right)}{3b} & \text{for}\ b\neq 0 \\ x\sin^2\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2,x)
[Out] Piecewise((sin(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(bx+a\right)^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*sin(b*x + a)^3/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(bx+a\right)^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/3*sin(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^3}{3\,b}$$

```
[In] int(cos(a + b*x)*sin(a + b*x)^2,x)
[Out] sin(a + b*x)^3/(3*b)
```

3.53 $\int \tan^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \tan^2(a+bx)\,dx = -x+\frac{\tan(a+bx)}{b}$$

```
[Out] -x+tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tan^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}-x$$

```
[In] Int[Tan[a + b*x]^2,x]
```

```
[Out] -x + Tan[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = \frac{\tan(a+bx)}{b} - \int 1\,dx$$
$$= -x + \frac{\tan(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \tan^2(a+bx)\,dx = -\frac{\arctan(\tan(a+bx))}{b} + \frac{\tan(a+bx)}{b}$$

```
[In] Integrate[Tan[a + b*x]^2,x]
[Out] -(ArcTan[Tan[a + b*x]]/b) + Tan[a + b*x]/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{\tan(bx+a)-bx-a}{b}$	19
default	$\frac{\tan(bx+a)-bx-a}{b}$	19
risch	$-x+\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	24
norman	$\frac{x-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}$	47
parallelrisch	$\frac{-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+bx-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	50

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(tan(b*x+a)-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \tan^2(a+bx)\,dx = -\frac{bx\cos(bx+a)-\sin(bx+a)}{b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F]

$$\int \tan^2(a+bx)\,dx = \int \sin^2(a+bx)\sec^2(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(a+bx)\,dx = -\frac{bx+a-\tan(bx+a)}{b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
[Out] -(b*x + a - tan(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(a+bx)\,dx = -\frac{bx+a-\tan(bx+a)}{b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
[Out] -(b*x + a - tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan^2(a+bx)\,dx=\frac{\tan(a+b\,x)}{b}-x$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^2,x)
[Out] tan(a + b*x)/b - x
```

3.54 $\int \sec^2(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}$$

```
[Out] 1/3*tan(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2687, 30}

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\tan^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}$$

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
[Out] Tan[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{\sin^3(bx+a)}{3b\cos(bx+a)^3}$	22
default	$\frac{\sin^3(bx+a)}{3b\cos(bx+a)^3}$	22
norman	$-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3}$	32
risch	$-\frac{2i\left(3\,e^{4i(bx+a)}+1\right)}{3b\left(e^{2i(bx+a)}+1\right)^3}$	33
parallelrisch	$-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	43

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3/b*sin(b*x+a)^3/cos(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \int \sin^2\left(a+bx\right)\sec^4\left(a+bx\right)\,dx$$

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*tan(b*x + a)^3/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/3*tan(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3}{3\,b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^4,x)
[Out] tan(a + b*x)^3/(3*b)
```

3.55 $\int \sec^4(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b} + \frac{\tan^5(a+bx)}{5b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+1/5*tan(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2687, 14}

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^5(a+bx)}{5b} + \frac{\tan^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^2(1+x^2)\, dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int (x^2+x^4)\, dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\tan^3(a+bx)}{3b} + \frac{\tan^5(a+bx)}{5b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$
\int \sec^4(a+bx)\tan^2(a+bx)\,dx = -\frac{2\tan(a+bx)}{15b} - \frac{\sec^2(a+bx)\tan(a+bx)}{15b} + \frac{\sec^4(a+bx)\tan(a+bx)}{5b}
$$

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
[Out] (-2*Tan[a + b*x])/(15*b) - (Sec[a + b*x]^2*Tan[a + b*x])/(15*b) + (Sec[a + b*x]^4*Tan[a + b*x])/(5*b)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{\sin^3(bx+a)}{5\cos(bx+a)^5}+\frac{2\left(\sin^3(bx+a)\right)}{15\cos(bx+a)^3}}{b}$	42
default	$\frac{\frac{\sin^3(bx+a)}{5\cos(bx+a)^5}+\frac{2\left(\sin^3(bx+a)\right)}{15\cos(bx+a)^3}}{b}$	42
risch	$-\frac{4i\left(15\,e^{6i(bx+a)}-5\,e^{4i(bx+a)}+5\,e^{2i(bx+a)}+1\right)}{15b\left(e^{2i(bx+a)}+1\right)^5}$	55
norman	$\frac{-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}-\frac{8\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}$	66
parallelrisch	$\frac{-\frac{8\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{16\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15}-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$	72

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] 1/b*(1/5*sin(b*x+a)^3/cos(b*x+a)^5+2/15*sin(b*x+a)^3/cos(b*x+a)^3)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(2\,\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2-3\right)\sin\left(bx+a\right)}{15\,b\cos\left(bx+a\right)^5}$$

[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")

[Out] -1/15*(2*cos(b*x + a)^4 + cos(b*x + a)^2 - 3)*sin(b*x + a)/(b*cos(b*x + a)^5)

Sympy [F]

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \int \sin^2\left(a+bx\right)\sec^6\left(a+bx\right)\,dx$$

[In] integrate(sec(b*x+a)**6*sin(b*x+a)**2,x)

[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**6, x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{3\,\tan\left(bx+a\right)^5+5\,\tan\left(bx+a\right)^3}{15\,b}$$

[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")

[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{3\,\tan\left(bx+a\right)^5+5\,\tan\left(bx+a\right)^3}{15\,b}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3\,\left(3\tan(a+b\,x)^2+5\right)}{15\,b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^6,x)
[Out] (tan(a + b*x)^3*(3*tan(a + b*x)^2 + 5))/(15*b)
```

3.56 $\int \sec^6(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b} + \frac{2\tan^5(a+bx)}{5b} + \frac{\tan^7(a+bx)}{7b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+2/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2687, 276}

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^7(a+bx)}{7b} + \frac{2\tan^5(a+bx)}{5b} + \frac{\tan^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (2*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^2(1+x^2)^2\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int (x^2+2x^4+x^6)\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\tan^3(a+bx)}{3b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = -\frac{8\tan(a+bx)}{105b}-\frac{4\sec^2(a+bx)\tan(a+bx)}{105b}-\frac{\sec^4(a+bx)\tan(a+bx)}{35b}+\frac{\sec^6(a+bx)\tan(a+bx)}{7b}$$

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
[Out] (-8*Tan[a + b*x])/(105*b) - (4*Sec[a + b*x]^2*Tan[a + b*x])/(105*b) - (Sec[
a + b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{\frac{\sin^3(bx+a)}{7\cos(bx+a)^7}+\frac{4\left(\sin^3(bx+a)\right)}{35\cos(bx+a)^5}+\frac{8\left(\sin^3(bx+a)\right)}{105\cos(bx+a)^3}}{b}$	60
default	$\frac{\frac{\sin^3(bx+a)}{7\cos(bx+a)^7}+\frac{4\left(\sin^3(bx+a)\right)}{35\cos(bx+a)^5}+\frac{8\left(\sin^3(bx+a)\right)}{105\cos(bx+a)^3}}{b}$	60
risch	$-\frac{16i\left(70\,e^{8i(bx+a)}-35\,e^{6i(bx+a)}+21\,e^{4i(bx+a)}+7\,e^{2i(bx+a)}+1\right)}{105b\left(e^{2i(bx+a)}+1\right)^7}$	66
parallelrisch	$-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(35\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+28\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+114\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+28\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+35\right)}{105b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	86
norman	$\frac{-\frac{8\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}-\frac{304\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{32\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}-\frac{8\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	98

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^3/cos(b*x+a)^7+4/35*sin(b*x+a)^3/cos(b*x+a)^5+8/105*sin
(b*x+a)^3/cos(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(8\cos\left(bx+a\right)^6+4\cos\left(bx+a\right)^4+3\cos\left(bx+a\right)^2-15\right)\sin\left(bx+a\right)}{105\,b\cos\left(bx+a\right)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(b*x + a)^6 + 4*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 15)*sin(b*
x + a)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{15\tan\left(bx+a\right)^7+42\tan\left(bx+a\right)^5+35\tan\left(bx+a\right)^3}{105\,b}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{15\,\tan\left(bx+a\right)^7+42\,\tan\left(bx+a\right)^5+35\,\tan\left(bx+a\right)^3}{105\,b}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3\,\left(15\tan(a+b\,x)^4+42\tan(a+b\,x)^2+35\right)}{105\,b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^8,x)
[Out] (tan(a + b*x)^3*(42*tan(a + b*x)^2 + 15*tan(a + b*x)^4 + 35))/(105*b)
```

3.57 $\int \sec^8(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx=\frac{\tan^3(a+bx)}{3b}+\frac{3\tan^5(a+bx)}{5b}+\frac{3\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+3/5*tan(b*x+a)^5/b+3/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 276}

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx=\frac{\tan^9(a+bx)}{9b}+\frac{3\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (3*Tan[a + b*x]^5)/(5*b) + (3*Tan[a + b*x]^7)/(7*b)
+ Tan[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^2(1+x^2)^3\,dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int (x^2+3x^4+3x^6+x^8)\,dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\tan^3(a+bx)}{3b}+\frac{3\tan^5(a+bx)}{5b}+\frac{3\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\begin{aligned} \int \sec^8(a+bx)\tan^2(a+bx)\,dx = &-\frac{16\tan(a+bx)}{315b}-\frac{8\sec^2(a+bx)\tan(a+bx)}{315b} \\ &-\frac{2\sec^4(a+bx)\tan(a+bx)}{105b} \\ &-\frac{\sec^6(a+bx)\tan(a+bx)}{63b}+\frac{\sec^8(a+bx)\tan(a+bx)}{9b} \end{aligned}$$

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
[Out] (-16*Tan[a + b*x])/(315*b) - (8*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) - (2*S
ec[a + b*x]^4*Tan[a + b*x])/(105*b) - (Sec[a + b*x]^6*Tan[a + b*x])/(63*b)
+ (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{32i\left(315\,\mathrm{e}^{10i(bx+a)}-189\,\mathrm{e}^{8i(bx+a)}+84\,\mathrm{e}^{6i(bx+a)}+36\,\mathrm{e}^{4i(bx+a)}+9\,\mathrm{e}^{2i(bx+a)}+1\right)}{315b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{9}}$
derivativedivides	$\frac{\frac{\sin^{3}(bx+a)}{9\cos(bx+a)^{9}}+\frac{2\left(\sin^{3}(bx+a)\right)}{21\cos(bx+a)^{7}}+\frac{8\left(\sin^{3}(bx+a)\right)}{105\cos(bx+a)^{5}}+\frac{16\left(\sin^{3}(bx+a)\right)}{315\cos(bx+a)^{3}}}{b}$
default	$\frac{\frac{\sin^{3}(bx+a)}{9\cos(bx+a)^{9}}+\frac{2\left(\sin^{3}(bx+a)\right)}{21\cos(bx+a)^{7}}+\frac{8\left(\sin^{3}(bx+a)\right)}{105\cos(bx+a)^{5}}+\frac{16\left(\sin^{3}(bx+a)\right)}{315\cos(bx+a)^{3}}}{b}$
parallelrisch	$-\frac{8\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(105\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+126\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+711\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+356\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+711\left(\tan^{4}\right.\right.}{315b\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{9}}$

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -32/315*I*(315*exp(10*I*(b*x+a))-189*exp(8*I*(b*x+a))+84*exp(6*I*(b*x+a))+3
6*exp(4*I*(b*x+a))+9*exp(2*I*(b*x+a))+1)/b/(exp(2*I*(b*x+a))+1)^9
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(16\,\cos\left(bx+a\right)^{8}+8\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+5\,\cos\left(bx+a\right)^{2}-35\right)\sin\left(bx+a\right)}{315\,b\cos\left(bx+a\right)^{9}}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 5*cos(b*x
 + a)^2 - 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{35\,\tan(bx+a)^9 + 135\,\tan(bx+a)^7 + 189\,\tan(bx+a)^5 + 105\,\tan(bx+a)^3}{315\,b}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{35\,\tan(bx+a)^9 + 135\,\tan(bx+a)^7 + 189\,\tan(bx+a)^5 + 105\,\tan(bx+a)^3}{315\,b}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{\tan(a+b\,x)^9}{9} + \frac{3\tan(a+b\,x)^7}{7} + \frac{3\tan(a+b\,x)^5}{5} + \frac{\tan(a+b\,x)^3}{3}}{b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^10,x)
```

```
[Out] (tan(a + b*x)^3/3 + (3*tan(a + b*x)^5)/5 + (3*tan(a + b*x)^7)/7 + tan(a + b
*x)^9/9)/b
```

3.58 $\int \cos^6(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx=\frac{5x}{128}+\frac{5\cos(a+bx)\sin(a+bx)}{128b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{\cos^5(a+bx)\sin(a+bx)}{48b}-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}$$

```
[Out] 5/128*x+5/128*cos(b*x+a)*sin(b*x+a)/b+5/192*cos(b*x+a)^3*sin(b*x+a)/b+1/48*
cos(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^7*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx=-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{5\sin(a+bx)\cos(a+bx)}{128b}+\frac{5x}{128}$$

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (5*x)/128 + (5*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (5*Cos[a + b*x]^3*Sin[a
 + b*x])/(192*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(48*b) - (Cos[a + b*x]^7*S
in[a + b*x])/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{1}{8}\int\cos^6(a+bx)\,dx\\
&= \frac{\cos^5(a+bx)\sin(a+bx)}{48b}-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{5}{48}\int\cos^4(a+bx)\,dx\\
&= \frac{5\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{\cos^5(a+bx)\sin(a+bx)}{48b}\\
&\quad-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{5}{64}\int\cos^2(a+bx)\,dx\\
&= \frac{5\cos(a+bx)\sin(a+bx)}{128b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{192b}\\
&\quad+\frac{\cos^5(a+bx)\sin(a+bx)}{48b}-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}+\frac{5\int 1\,dx}{128}\\
&= \frac{5x}{128}+\frac{5\cos(a+bx)\sin(a+bx)}{128b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{192b}\\
&\quad+\frac{\cos^5(a+bx)\sin(a+bx)}{48b}-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{120bx+48\sin(2(a+bx))-24\sin(4(a+bx))-16\sin(6(a+bx))-3\sin(8(a+bx))}{3072b}$$

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (120*b*x + 48*Sin[2*(a + b*x)] - 24*Sin[4*(a + b*x)] - 16*Sin[6*(a + b*x)]
- 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{120bx-3\sin(8bx+8a)-16\sin(6bx+6a)-24\sin(4bx+4a)+48\sin(2bx+2a)}{3072b}$
risch	$\frac{5x}{128}-\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{128b}+\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\left(\cos^7(bx+a)\right)\sin(bx+a)}{8}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{48}+\frac{5bx}{128}+\frac{5a}{128}}{b}$
default	$\frac{-\frac{\left(\cos^7(bx+a)\right)\sin(bx+a)}{8}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{48}+\frac{5bx}{128}+\frac{5a}{128}}{b}$
norman	$\frac{\frac{5x}{128}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{397\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{895\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{1765\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{1765\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{895\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right.}{192b}}{}$

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3072*(120*b*x-3*sin(8*b*x+8*a)-16*sin(6*b*x+6*a)-24*sin(4*b*x+4*a)+48*sin
(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{15\,bx - \left(48\,\cos\left(bx+a\right)^7 - 8\,\cos\left(bx+a\right)^5 - 10\,\cos\left(bx+a\right)^3 - 15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(15*b*x - (48*cos(b*x + a)^7 - 8*cos(b*x + a)^5 - 10*cos(b*x + a)^3 -
 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(80) = 160.

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{5x\sin^8(a+bx)}{128} + \frac{5x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{15x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{5x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{5x\cos^8(a+bx)}{128} + \frac{5\sin^7(
\\ x\sin^2(a)\cos^6(a) \end{cases}$$

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**8/128 + 5*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 15*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 5*x*sin(a + b*x)**2*cos(a + b*
x)**6/32 + 5*x*cos(a + b*x)**8/128 + 5*sin(a + b*x)**7*cos(a + b*x)/(128*b)
 + 55*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 73*sin(a + b*x)**3*cos(a +
b*x)**5/(384*b) - 5*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin
(a)**2*cos(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{64\,\sin\left(2\,bx+2\,a\right)^3 + 120\,bx + 120\,a - 3\,\sin\left(8\,bx+8\,a\right) - 24\,\sin\left(4\,bx+4\,a\right)}{3072\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3072*(64*sin(2*b*x + 2*a)^3 + 120*b*x + 120*a - 3*sin(8*b*x + 8*a) - 24*s
in(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{5}{128}\,x - \frac{\sin(8\,bx+8\,a)}{1024\,b} - \frac{\sin(6\,bx+6\,a)}{192\,b} - \frac{\sin(4\,bx+4\,a)}{128\,b} + \frac{\sin(2\,bx+2\,a)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/128*x - 1/1024*sin(8*b*x + 8*a)/b - 1/192*sin(6*b*x + 6*a)/b - 1/128*sin(
4*b*x + 4*a)/b + 1/64*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{5\,x}{128} + \frac{\frac{5\tan(a+b\,x)^7}{128} + \frac{55\tan(a+b\,x)^5}{384} + \frac{73\tan(a+b\,x)^3}{384} - \frac{5\tan(a+b\,x)}{128}}{b\,\left(\tan(a+b\,x)^8 + 4\tan(a+b\,x)^6 + 6\tan(a+b\,x)^4 + 4\tan(a+b\,x)^2 + 1\right)}$$

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^2,x)
```

```
[Out] (5*x)/128 + ((73*tan(a + b*x)^3)/384 - (5*tan(a + b*x))/128 + (55*tan(a + b
*x)^5)/384 + (5*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x)^
4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

3.59 $\int \cos^4(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx=\frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b}+\frac{\cos^3(a+bx)\sin(a+bx)}{24b}-\frac{\cos^5(a+bx)\sin(a+bx)}{6b}$$

```
[Out] 1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b+1/24*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos(
b*x+a)^5*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx=-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) + (Cos[a + b*x]^3*Sin[a + b*x])/(
24*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^5(a+bx)\sin(a+bx)}{6b} + \frac{1}{6}\int \cos^4(a+bx)\,dx \\
&= \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a+bx)}{6b} + \frac{1}{8}\int \cos^2(a+bx)\,dx \\
&= \frac{\cos(a+bx)\sin(a+bx)}{16b} + \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a+bx)}{6b} + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{\cos(a+bx)\sin(a+bx)}{16b} + \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = -\frac{-12bx - 3\sin(2(a+bx)) + 3\sin(4(a+bx)) + \sin(6(a+bx))}{192b}$$

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
[Out] -1/192*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{12bx-\sin(6bx+6a)-3\sin(4bx+4a)+3\sin(2bx+2a)}{192b}$
risch	$\frac{x}{16}-\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{64b}+\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\left(\cos^5(bx+a)\right)\sin(bx+a)}{6}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{24}+\frac{bx}{16}+\frac{a}{16}}{b}$
default	$\frac{-\frac{\left(\cos^5(bx+a)\right)\sin(bx+a)}{6}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{24}+\frac{bx}{16}+\frac{a}{16}}{b}$
norman	$\frac{\frac{x}{16}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{47\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{13\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{13\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{47\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{3x\left(\tan^2\right.}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\right.\right.}$

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/192*(12*b*x-sin(6*b*x+6*a)-3*sin(4*b*x+4*a)+3*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{3\,bx-\left(8\,\cos\left(bx+a\right)^5-2\,\cos\left(bx+a\right)^3-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/48*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^6(a+bx)}{16}+\frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{x\cos^6(a+bx)}{16}+\frac{\sin^5(a+bx)\cos(a+bx)}{16b}+\frac{\sin^3(a+bx)\cos}{6b} \\ x\sin^2(a)\cos^4(a) \end{cases}$$

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) + sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{4\,\sin\left(2\,bx+2\,a\right)^3+12\,bx+12\,a-3\,\sin\left(4\,bx+4\,a\right)}{192\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 12*b*x + 12*a - 3*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{1}{16}\,x-\frac{\sin\left(6\,bx+6\,a\right)}{192\,b}-\frac{\sin\left(4\,bx+4\,a\right)}{64\,b}+\frac{\sin\left(2\,bx+2\,a\right)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b + 1/64*sin(2*b*
x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{x}{16}-\frac{\frac{\sin(4\,a+4\,b\,x)}{64}-\frac{\sin(2\,a+2\,b\,x)}{64}+\frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^2,x)
```

```
[Out] x/16 - (sin(4*a + 4*b*x)/64 - sin(2*a + 2*b*x)/64 + sin(6*a + 6*b*x)/192)/b
```

3.60 $\int \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{x}{8}+\frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}$$

```
[Out] 1/8*x+1/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{\sin(a+bx)\cos(a+bx)}{8b}+\frac{x}{8}$$

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/8 + (Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(4*
b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{1}{4}\int\cos^2(a+bx)\,dx\\ &= \frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}+\frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{-4(a+bx)+\sin(4(a+bx))}{32b}$$

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
[Out] -1/32*(-4*(a + b*x) + Sin[4*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

method	result
risch	$\frac{x}{8}-\frac{\sin(4bx+4a)}{32b}$
parallelrisch	$\frac{4bx-\sin(4bx+4a)}{32b}$
derivativedivides	$\frac{-\frac{\left(\cos^3(bx+a)\right)\sin(bx+a)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}}{b}$
default	$\frac{-\frac{\left(\cos^3(bx+a)\right)\sin(bx+a)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}}{b}$
norman	$\frac{\frac{x}{8}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{7\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{7\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{bx}{2}+\right.\right.}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x-1/32/b*sin(4*b*x+4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{bx-\left(2\,\cos\left(bx+a\right)^3-\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(b*x - (2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\begin{cases}\frac{x\sin^4\left(a+bx\right)}{8}+\frac{x\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{4}+\frac{x\cos^4\left(a+bx\right)}{8}+\frac{\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{8b}-\frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{8b} & \text{for } b\neq 0\\ x\sin^2\left(a\right)\cos^2\left(a\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2,x)
[Out] Piecewise((x*sin(a + b*x)**4/8 + x*sin(a + b*x)**2*cos(a + b*x)**2/4 + x*co
s(a + b*x)**4/8 + sin(a + b*x)**3*cos(a + b*x)/(8*b) - sin(a + b*x)*cos(a +
 b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{4\,bx+4\,a-\sin\left(4\,bx+4\,a\right)}{32\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/32*(4*b*x + 4*a - sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{8}\,x - \frac{\sin\left(4\,bx+4\,a\right)}{32\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*b*x + 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{x}{8} - \frac{\frac{\tan(a+b\,x)}{8}-\frac{\tan(a+b\,x)^3}{8}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2,x)
[Out] x/8 - (tan(a + b*x)/8 - tan(a + b*x)^3/8)/(b*(2*tan(a + b*x)^2 + tan(a + b*
x)^4 + 1))
```

3.61 $\int \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a+bx)\,dx=\frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sin^2(a+bx)\,dx=\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = -\frac{-2(a+bx)+\sin(2(a+bx))}{4b}$$

```
[In] Integrate[Sin[a + b*x]^2,x]
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx-\sin(2bx+2a)}{4b}$	22
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

```
[In] int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/4/b*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = \frac{bx - \cos(bx+a)\sin(bx+a)}{2\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**2,x)
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \sin(2\,bx + 2\,a)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

```
[In] int(sin(a + b*x)^2,x)
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

3.62 $\int \sin(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sin(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+a))/b-sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2672, 327, 212}

$$\int \sin(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}\\ &= -\frac{\sin(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}$$

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x],x]
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	28
default	$\frac{-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	28
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\sin(bx+a)}{b}$	40
norman	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	64
risch	$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}$	67

```
[In] int(sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)-2\sin(bx+a)}{2b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 18.82 (sec) , antiderivative size = 3160, normalized size of antiderivative = 137.39

$$\int \sin(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Piecewise((log(tan(a + b*x) + sec(a + b*x))/b, Ne(b, 0)), (x*(tan(a)*sec(a)
 + sec(a)**2)/(tan(a) + sec(a)), True))/2 + 2*Piecewise((-sin(b*x)/b, Eq(a,
 pi/2)), (sin(b*x)/b, Eq(a, -pi/2)), (0, Eq(b, 0)), (-2*log(tan(b*x/2) - ta
n(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(
a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b
```

```
*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(a/2)/(tan(a/2)
 - 1) - 1/(tan(a/2) - 1))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) + 2*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(
a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a
/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b
*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)/(b*tan(a/2)**4
*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) -
 1/(tan(a/2) + 1))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 +
 b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*
x/2)**2 + b) + 2*log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1
))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)*
*2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2
) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)*tan(b*x/2)**2/(b*t
an(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 +
2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(a/2)/(tan(a
/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) + 2*tan(a/2)**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*ta
n(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/
2)**3*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2
)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)*ta
n(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan
(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2/(b*tan(a/2)**4*tan(
b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2
 + b*tan(b*x/2)**2 + b), True))*sin(a)*cos(a) + Piecewise((x/cos(a), Eq(b,
0)), (-log(tan(b*x/2))/b, Eq(a, pi/2)), (log(tan(b*x/2))/b, Eq(a, -pi/2)),
(log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))/b - log(tan(b
*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))/b, True))*cos(a)**2 - P
iecewise((x/cos(a), Eq(b, 0)), (-log(tan(b*x/2))/b, Eq(a, pi/2)), (log(tan(
b*x/2))/b, Eq(a, -pi/2)), (log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(ta
n(a/2) - 1))/b - log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1
))/b, True))/2 - 2*Piecewise((-log(tan(b*x/2))*tan(b*x/2)**2/(b*tan(b*x/2)*
*2 + b) - log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) - 2/(b*tan(b*x/2)**2 + b),
Eq(a, pi/2)), (log(tan(b*x/2))*tan(b*x/2)**2/(b*tan(b*x/2)**2 + b) + log(ta
n(b*x/2))/(b*tan(b*x/2)**2 + b) + 2/(b*tan(b*x/2)**2 + b), Eq(a, -pi/2)), (
x/cos(a), Eq(b, 0)), (4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a
/2) - 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/
2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
 b) + 4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/
2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*
x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) + tan(a
/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2
)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*ta
```

```
n(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) + tan(a/2)/(tan(a/2) +
1) - 1/(tan(a/2) + 1))*tan(a/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2
)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4
+ 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) -
4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**
2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)/(b*ta
n(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2
*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x
/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 +
b*tan(b*x/2)**2 + b), True))*cos(a)**2 + Piecewise((-log(tan(b*x/2))*tan(b*
x/2)**2/(b*tan(b*x/2)**2 + b) - log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) - 2/(
b*tan(b*x/2)**2 + b), Eq(a, pi/2)), (log(tan(b*x/2))*tan(b*x/2)**2/(b*tan(b
*x/2)**2 + b) + log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) + 2/(b*tan(b*x/2)**2
+ b), Eq(a, -pi/2)), (x/cos(a), Eq(b, 0)), (4*log(tan(b*x/2) - tan(a/2)/(ta
n(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/
(tan(a/2) - 1))*tan(a/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*
log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2*ta
n(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) +
 tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2/(b*tan(a/2)**4*tan
(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**
2 + b*tan(b*x/2)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/
2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b
*tan(b*x/2)**2 + b) - 4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/
2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
 b) - 4*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2
)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/
(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**
2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)-2\,\sin\left(bx+a\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log\left(|\sin\left(bx+a\right)+1|\right) - \log\left(|\sin\left(bx+a\right)-1|\right) - 2\,\sin\left(bx+a\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)) - 2*sin(b*x +
a))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b} - \frac{\sin\left(a+b\,x\right)}{b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x),x)
[Out] (2*atanh(tan(a/2 + (b*x)/2)))/b - sin(a + b*x)/b
```

3.63 $\int \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sec(a+bx)\tan^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(sin(b*x+a))/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \sec(a+bx)\tan^2(a+bx)\,dx=\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}$$

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[a + b*x]]/b + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sec(a+bx)\tan(a+bx)}{2b} - \frac{1}{2}\int \sec(a+bx)\,dx \\ &= -\frac{\operatorname{arctanh}(\sin(a+bx))}{2b} + \frac{\sec(a+bx)\tan(a+bx)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{2b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}$$

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^2,x]
[Out] -1/2*ArcTanh[Sin[a + b*x]]/b + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{\sin^3(bx+a)}{2\cos(bx+a)^2}+\frac{\sin(bx+a)}{2}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	48
default	$\frac{\frac{\sin^3(bx+a)}{2\cos(bx+a)^2}+\frac{\sin(bx+a)}{2}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	48
risch	$-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^2}-\frac{\ln\left(e^{i(bx+a)}+i\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}-i\right)}{2b}$	78
parallelrisch	$\frac{(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-1-\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+2\sin(bx+a)}{2b(1+\cos(2bx+2a))}$	78
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$	81

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sin(b*x+a)^3/cos(b*x+a)^2+1/2*sin(b*x+a)-1/2*ln(sec(b*x+a)+tan(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^2\log(\sin(bx+a)+1) - \cos(bx+a)^2\log(-\sin(bx+a)+1) - 2\sin(bx+a)}{4\,b\cos(bx+a)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) - 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = \int \sin^2(a+bx)\sec^3(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + \log(\sin(bx+a)+1) - \log(\sin(bx+a)-1)}{4\,b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin
(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + \log\left(|\sin\left(bx+a\right)+1|\right) - \log\left(|\sin\left(bx+a\right)-1|\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(abs(sin(b*x + a) + 1)) - lo
g(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)} - \frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^3,x)
```

```
[Out] (tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3)/(b*(tan(a/2 + (b*x)/2)^4 - 2*ta
n(a/2 + (b*x)/2)^2 + 1)) - atanh(tan(a/2 + (b*x)/2))/b
```

3.64 $\int \sec^3(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{8b} - \frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

```
[Out] -1/8*arctanh(sin(b*x+a))/b-1/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)^3*tan
(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2691, 3853, 3855}

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{8b} + \frac{\tan(a+bx)\sec^3(a+bx)}{4b} - \frac{\tan(a+bx)\sec(a+bx)}{8b}$$

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/8*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sec^3(a+bx)\tan(a+bx)}{4b} - \frac{1}{4}\int \sec^3(a+bx)\,dx \\ &= -\frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b} - \frac{1}{8}\int \sec(a+bx)\,dx \\ &= -\frac{\operatorname{arctanh}(\sin(a+bx))}{8b} - \frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{8b} - \frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
[Out] -1/8*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\sin^3(bx+a)}{4\cos(bx+a)^4}+\frac{\sin^3(bx+a)}{8\cos(bx+a)^2}+\frac{\sin(bx+a)}{8}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{\sin^3(bx+a)}{4\cos(bx+a)^4}+\frac{\sin^3(bx+a)}{8\cos(bx+a)^2}+\frac{\sin(bx+a)}{8}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$\frac{i\left(e^{7i(bx+a)}-7\,e^{5i(bx+a)}+7\,e^{3i(bx+a)}-e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4}-\frac{\ln\left(e^{i(bx+a)}+i\right)}{8b}+\frac{\ln\left(e^{i(bx+a)}-i\right)}{8b}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{7\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{7\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{8b}$
parallelrisch	$\frac{(\cos(4bx+4a)+4\cos(2bx+2a)+3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-\cos(4bx+4a)-4\cos(2bx+2a)-3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+14\,\mathrm{si}}{8b(\cos(4bx+4a)+4\cos(2bx+2a)+3)}$

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*sin(b*x+a)^3/cos(b*x+a)^4+1/8*sin(b*x+a)^3/cos(b*x+a)^2+1/8*sin(b*
x+a)-1/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^4\log(\sin(bx+a)+1)-\cos(bx+a)^4\log(-\sin(bx+a)+1)+2\left(\cos(bx+a)^2-2\right)\sin(b}{16\,b\cos(bx+a)^4}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^4*log(sin(b*x + a) + 1) - cos(b*x + a)^4*log(-sin(b*x +
 a) + 1) + 2*(cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \int \sin^2(a+bx)\sec^5(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(\sin(bx+a)^3+\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1} - \log(\sin(bx+a)+1) + \log(\sin(bx+a)-1)}{16\,b}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/16*(2*(sin(b*x + a)^3 + sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a)^2
+ 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{4\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)}{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4} - \log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)+2\right|\right) + \log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)-2\right|\right)}{32\,b}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/32*(4*(1/sin(b*x + a) + sin(b*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2
- 4) - log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) + log(abs(1/sin(b*x + a)
 + sin(b*x + a) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4}+\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4}}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8-4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6+6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)} - \frac{\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{4\,b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^5,x)
```

```
[Out] (tan(a/2 + (b*x)/2)/4 + (7*tan(a/2 + (b*x)/2)^3)/4 + (7*tan(a/2 + (b*x)/2)^
5)/4 + tan(a/2 + (b*x)/2)^7/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (b*
x)/2)^2 - 4*tan(a/2 + (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8 + 1)) - atanh(tan(a
/2 + (b*x)/2))/(4*b)
```

3.65 $\int \sec^5(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

```
[Out] -1/16*arctanh(sin(b*x+a))/b-1/16*sec(b*x+a)*tan(b*x+a)/b-1/24*sec(b*x+a)^3*
tan(b*x+a)/b+1/6*sec(b*x+a)^5*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2691, 3853, 3855}

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} + \frac{\tan(a+bx)\sec^5(a+bx)}{6b} - \frac{\tan(a+bx)\sec^3(a+bx)}{24b} - \frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -1/16*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sec^5(a+bx)\tan(a+bx)}{6b} - \frac{1}{6}\int \sec^5(a+bx)\,dx \\ &= -\frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b} - \frac{1}{8}\int \sec^3(a+bx)\,dx \\ &= -\frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} \\ &\quad + \frac{\sec^5(a+bx)\tan(a+bx)}{6b} - \frac{1}{16}\int \sec(a+bx)\,dx \\ &= -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} \\ &\quad - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -1/16*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\sin^{3}(bx+a)}{6\cos(bx+a)^{6}}+\frac{\sin^{3}(bx+a)}{8\cos(bx+a)^{4}}+\frac{\sin^{3}(bx+a)}{16\cos(bx+a)^{2}}+\frac{\sin(bx+a)}{16}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{\frac{\sin^{3}(bx+a)}{6\cos(bx+a)^{6}}+\frac{\sin^{3}(bx+a)}{8\cos(bx+a)^{4}}+\frac{\sin^{3}(bx+a)}{16\cos(bx+a)^{2}}+\frac{\sin(bx+a)}{16}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$\frac{i\left(3\,\mathrm{e}^{11i(bx+a)}+17\,\mathrm{e}^{9i(bx+a)}-114\,\mathrm{e}^{7i(bx+a)}+114\,\mathrm{e}^{5i(bx+a)}-17\,\mathrm{e}^{3i(bx+a)}-3\,\mathrm{e}^{i(bx+a)}\right)}{24b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{6}}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{16b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\right.}{16b}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{47\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{13\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{13\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{47\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{6}}+\frac{\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{16}$
parallelrisch	$\frac{(3\cos(6bx+6a)+18\cos(4bx+4a)+45\cos(2bx+2a)+30)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-45\cos(2bx+2a)-18\cos(4bx+4a)-3\cos(}{48b(\cos(6bx+6a)+6\cos(4bx+4a)+15\cos(2bx+}$

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*sin(b*x+a)^3/cos(b*x+a)^6+1/8*sin(b*x+a)^3/cos(b*x+a)^4+1/16*sin(b
*x+a)^3/cos(b*x+a)^2+1/16*sin(b*x+a)-1/16*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \sec^{5}(a+bx)\tan^{2}(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^{6}\log\left(\sin\left(bx+a\right)+1\right)-3\,\cos\left(bx+a\right)^{6}\log\left(-\sin\left(bx+a\right)+1\right)+2\left(3\,\cos\left(bx+a\right)^{4}+2\,\mathrm{c}\right.}{96\,b\cos\left(bx+a\right)^{6}}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^4 + 2*cos(b*x + a)^2 - 8)*sin(b*x + a))/(b
*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{96\,b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3} - 3\log\left(|\sin\left(bx+a\right)+1|\right) + 3\log\left(|\sin\left(bx+a\right)-1|\right)}{96\,b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.33

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{8}+\frac{47\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{24}+\frac{13\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{4}+\frac{13\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{4}+\frac{47\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}-6\tan\left(\frac{a}{2}\right.\right.}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^7,x)
```

```
[Out] (tan(a/2 + (b*x)/2)/8 + (47*tan(a/2 + (b*x)/2)^3)/24 + (13*tan(a/2 + (b*x)/
2)^5)/4 + (13*tan(a/2 + (b*x)/2)^7)/4 + (47*tan(a/2 + (b*x)/2)^9)/24 + tan(
a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 + (b*x)/2)^4 - 6*tan(a/2 + (b*x)/2)^2 -
 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2 + (b*x)/2)^8 - 6*tan(a/2 + (b*x)/2)^1
0 + tan(a/2 + (b*x)/2)^12 + 1)) - atanh(tan(a/2 + (b*x)/2))/(8*b)
```

3.66 $\int \cos^5(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx=-\frac{\cos^6(a+bx)}{6b}+\frac{\cos^8(a+bx)}{8b}$$

```
[Out] -1/6*cos(b*x+a)^6/b+1/8*cos(b*x+a)^8/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {2645, 14}

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx=\frac{\cos^8(a+bx)}{8b}-\frac{\cos^6(a+bx)}{6b}$$

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] -1/6*Cos[a + b*x]^6/b + Cos[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^5(1-x^2)\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int (x^5-x^7)\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\cos^6(a+bx)}{6b}+\frac{\cos^8(a+bx)}{8b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{3072b}$$

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^8(bx+a)\right)}{8}-\frac{\left(\cos^6(bx+a)\right)}{6}}{b}$	26
default	$\frac{\frac{\left(\cos^8(bx+a)\right)}{8}-\frac{\left(\cos^6(bx+a)\right)}{6}}{b}$	26
parallelrisch	$\frac{3\cos(8bx+8a)+73-12\cos(4bx+4a)-72\cos(2bx+2a)+8\cos(6bx+6a)}{3072b}$	52
risch	$\frac{\cos(8bx+8a)}{1024b}+\frac{\cos(6bx+6a)}{384b}-\frac{\cos(4bx+4a)}{256b}-\frac{3\cos(2bx+2a)}{128b}$	58
norman	$\frac{\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{16\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{40\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^8}$	98

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/8*cos(b*x+a)^8-1/6*cos(b*x+a)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\cos\left(bx+a\right)^8-4\,\cos\left(bx+a\right)^6}{24\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/24*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2{(a+bx)}\cos^6{(a+bx)}}{6b} - \frac{\cos^8{(a+bx)}}{24b} & \text{for}\ b \neq 0 \\ x\sin^3{(a)}\cos^5{(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**3,x)
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**6/(6*b) - cos(a + b*x)**8/(24*b),
 Ne(b, 0)), (x*sin(a)**3*cos(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\sin\left(bx+a\right)^8-8\,\sin\left(bx+a\right)^6+6\,\sin\left(bx+a\right)^4}{24\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/24*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(bx+a)^8}{8\,b} - \frac{\cos(bx+a)^6}{6\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/8*cos(b*x + a)^8/b - 1/6*cos(b*x + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(a+b\,x)^6\,\left(3\cos(a+b\,x)^2-4\right)}{24\,b}$$

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^3,x)
[Out] (cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(24*b)
```

3.67 $\int \cos^4(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx=-\frac{\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}$$

```
[Out] -1/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 14}

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx=\frac{\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -1/5*Cos[a + b*x]^5/b + Cos[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^4(1-x^2)\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int (x^4-x^6)\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{\cos^5(a+bx)(-9+5\cos(2(a+bx)))}{70b}$$

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
[Out] (Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(70*b)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^7(bx+a)\right)}{7}-\frac{\left(\cos^5(bx+a)\right)}{5}}{b}$	26
default	$\frac{\frac{\left(\cos^7(bx+a)\right)}{7}-\frac{\left(\cos^5(bx+a)\right)}{5}}{b}$	26
parallelrisch	$\frac{-105\cos(bx+a)+7\cos(5bx+5a)-35\cos(3bx+3a)-128+5\cos(7bx+7a)}{2240b}$	49
risch	$-\frac{3\cos(bx+a)}{64b}+\frac{\cos(7bx+7a)}{448b}+\frac{\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{64b}$	55
norman	$\frac{-\frac{4\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4}{35b}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+\frac{8\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^7}$	103

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/7*cos(b*x+a)^7-1/5*cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{5\,\cos\left(bx+a\right)^7 - 7\,\cos\left(bx+a\right)^5}{35\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2\left(a+bx\right)\cos^5\left(a+bx\right)}{5b} - \frac{2\cos^7\left(a+bx\right)}{35b} & \text{for}\ b \neq 0 \\ x\sin^3\left(a\right)\cos^4\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**3,x)
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 2*cos(a + b*x)**7/(35*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{5\,\cos\left(bx+a\right)^7 - 7\,\cos\left(bx+a\right)^5}{35\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(bx+a)^7}{7\,b} - \frac{\cos(bx+a)^5}{5\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/7*cos(b*x + a)^7/b - 1/5*cos(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = -\frac{7\cos(a+b\,x)^5 - 5\cos(a+b\,x)^7}{35\,b}$$

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^3,x)
[Out] -(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7)/(35*b)
```

3.68 $\int \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

```
[Out] 1/4*sin(b*x+a)^4/b-1/6*sin(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b) - Sin[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^3(1-x^2)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^3-x^5)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b} - \frac{\sin^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{1}{8}\left(-\frac{3\cos(2(a+bx))}{8b} + \frac{\cos(6(a+bx))}{24b}\right)$$

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
[Out] ((-3*Cos[2*(a + b*x)])/(8*b) + Cos[6*(a + b*x)]/(24*b))/8
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^6(bx+a)\right)}{6}+\frac{\left(\sin^4(bx+a)\right)}{4}}{b}$	26
default	$\frac{-\frac{\left(\sin^6(bx+a)\right)}{6}+\frac{\left(\sin^4(bx+a)\right)}{4}}{b}$	26
parallelrisch	$\frac{\cos(6bx+6a)-9\cos(2bx+2a)+8}{192b}$	28
risch	$\frac{\cos(6bx+6a)}{192b}-\frac{3\cos(2bx+2a)}{64b}$	30
norman	$\frac{\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^6}$	66

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/6*sin(b*x+a)^6+1/4*sin(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{2\,\cos\left(bx+a\right)^6 - 3\,\cos\left(bx+a\right)^4}{12\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/12*(2*cos(b*x + a)^6 - 3*cos(b*x + a)^4)/b
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2\left(a+bx\right)\cos^4\left(a+bx\right)}{4b} - \frac{\cos^6\left(a+bx\right)}{12b} & \text{for } b \neq 0 \\ x\sin^3\left(a\right)\cos^3\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - cos(a + b*x)**6/(12*b),
 Ne(b, 0)), (x*sin(a)**3*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^6 - 3\,\sin\left(bx+a\right)^4}{12\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^6 - 3\,\sin\left(bx+a\right)^4}{12\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\cos\left(a+b\,x\right)^4\,\left(\cos\left(a+b\,x\right)^2-1\right)}{4\,b} - \frac{\cos\left(a+b\,x\right)^6}{12\,b}$$

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3,x)
[Out] (cos(a + b*x)^4*(cos(a + b*x)^2 - 1))/(4*b) - cos(a + b*x)^6/(12*b)
```

3.69 $\int \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 14}

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -1/3*Cos[a + b*x]^3/b + Cos[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int (x^2-x^4)\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\cos^3(a+bx)(-7+3\cos(2(a+bx)))}{30b}$$

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
[Out] (Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(30*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^5(bx+a)\right)}{5}-\frac{\left(\cos^3(bx+a)\right)}{3}}{b}$	26
default	$\frac{\frac{\left(\cos^5(bx+a)\right)}{5}-\frac{\left(\cos^3(bx+a)\right)}{3}}{b}$	26
parallelrisch	$\frac{-32-30\cos(bx+a)-5\cos(3bx+3a)+3\cos(5bx+5a)}{240b}$	38
risch	$-\frac{\cos(bx+a)}{8b}+\frac{\cos(5bx+5a)}{80b}-\frac{\cos(3bx+3a)}{48b}$	41
norman	$\frac{-\frac{4}{15b}-\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$	71

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*cos(b*x+a)^5-1/3*cos(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\cos(bx+a)^5 - 5\cos(bx+a)^3}{15\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2\cos^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sin^3(a)\cos^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*cos(a + b*x)**5/(15*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\cos(bx+a)^5 - 5\cos(bx+a)^3}{15\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\cos{(bx+a)}^5}{5\,b} - \frac{\cos{(bx+a)}^3}{3\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/5*cos(b*x + a)^5/b - 1/3*cos(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{5\cos{(a+b\,x)}^3 - 3\cos{(a+b\,x)}^5}{15\,b}$$

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3,x)
[Out] -(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5)/(15*b)
```

3.70 $\int \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}$$

```
[Out] 1/4*sin(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}$$

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\sin^4(a+bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^3,x]
[Out] Sin[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin^4(bx+a)}{4b}$	14
default	$\frac{\sin^4(bx+a)}{4b}$	14
parallelrisch	$\frac{3+\cos(4bx+4a)-4\cos(2bx+2a)}{32b}$	28
risch	$\frac{\cos(4bx+4a)}{32b}-\frac{\cos(2bx+2a)}{8b}$	30
norman	$\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$	32

```
[In] int(cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*sin(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\cos\left(bx+a\right)^4 - 2\,\cos\left(bx+a\right)^2}{4\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \begin{cases} \frac{\sin^4\left(a+bx\right)}{4b} & \text{for}\ b \neq 0 \\ x\sin^3\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3,x)
[Out] Piecewise((sin(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\sin\left(bx+a\right)^4}{4\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*sin(b*x + a)^4/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\sin\left(bx+a\right)^4}{4\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/4*sin(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin\left(a+b\,x\right)^4}{4\,b}$$

```
[In] int(cos(a + b*x)*sin(a + b*x)^3,x)
[Out] sin(a + b*x)^4/(4*b)
```

3.71 $\int \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sin^2(a+bx)\tan(a+bx)\,dx=\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] 1/2*cos(b*x+a)^2/b-ln(cos(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sin^2(a+bx)\tan(a+bx)\,dx=\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/(2*b) - Log[Cos[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, \cos(a+bx)\right)}{b} \\
&= \frac{\cos^2(a+bx)}{2b} - \frac{\log(\cos(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$
\int \sin^2(a+bx)\tan(a+bx)\,dx = -\frac{-\frac{1}{2}\cos^2(a+bx)+\log(\cos(a+bx))}{b}
$$

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x],x]
[Out] -((-1/2*Cos[a + b*x]^2 + Log[Cos[a + b*x]])/b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^2(bx+a)\right)}{2}-\ln(\cos(bx+a))}{b}$	25
default	$\frac{-\frac{\left(\sin^2(bx+a)\right)}{2}-\ln(\cos(bx+a))}{b}$	25
risch	$ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}+\frac{2ia}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$	58
parallelrisch	$\frac{-4\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-4\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+4\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1+\cos(2bx+2a)}{4b}$	59
norman	$-\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}+\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	85

```
[In] int(sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2*sin(b*x+a)^2-ln(cos(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos(bx+a)^2 - 2\log(-\cos(bx+a))}{2b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^2 - 2*log(-cos(b*x + a)))/b
```

Sympy [F]

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \int \sin^3(a+bx)\sec(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Integral(sin(a + b*x)**3*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = -\frac{\sin(bx+a)^2 + \log\left(\sin(bx+a)^2 - 1\right)}{2b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos(bx+a)^2 - \log\left(\frac{\cos(bx+a)^2}{b^2}\right)}{2b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*(cos(b*x + a)^2 - log(cos(b*x + a)^2/b^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos{(a+b\,x)}^2+\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x),x)
[Out] (log(tan(a + b*x)^2 + 1) + cos(a + b*x)^2)/(2*b)
```

3.72 $\int \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sin(a+bx)\tan^2(a+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] cos(b*x+a)/b+sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \sin(a+bx)\tan^2(a+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^2,x]
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

method	result	size
parallelrisch	$\frac{\cos(2bx+2a)+4\cos(bx+a)+3}{2b\cos(bx+a)}$	33
norman	$-\frac{4}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	36
derivativedivides	$\frac{\frac{\sin^4(bx+a)}{\cos(bx+a)}+\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{b}$	40
default	$\frac{\frac{\sin^4(bx+a)}{\cos(bx+a)}+\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{b}$	40
risch	$\frac{e^{3i(bx+a)}+7\cos(bx+a)+5i\sin(bx+a)}{2b\left(e^{2i(bx+a)}+1\right)}$	46

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/b*(cos(2*b*x+2*a)+4*cos(b*x+a)+3)/cos(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos{(bx+a)}^2+1}{b\cos{(bx+a)}}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
[Out] (cos(b*x + a)^2 + 1)/(b*cos(b*x + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{1}{\cos(bx+a)}+\cos{(bx+a)}}{b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
[Out] (1/cos(b*x + a) + cos(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos{(bx+a)}}{b}+\frac{1}{b\cos{(bx+a)}}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
[Out] cos(b*x + a)/b + 1/(b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = -\frac{4}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-1\right)}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^2,x)
[Out] -4/(b*(tan(a/2 + (b*x)/2)^4 - 1))
```

3.73 $\int \tan^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tan^3(a+bx)\,dx=\frac{\log(\cos(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}$$

```
[Out] ln(cos(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \tan^3(a+bx)\,dx=\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

```
[In] Int[Tan[a + b*x]^3,x]
```

```
[Out] Log[Cos[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^2(a+bx)}{2b} - \int \tan(a+bx)\,dx \\ &= \frac{\log(\cos(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan^3(a+bx)\,dx = \frac{2\log(\cos(a+bx)) + \tan^2(a+bx)}{2b}$$

[In] Integrate[Tan[a + b*x]^3,x]

[Out] (2*Log[Cos[a + b*x]] + Tan[a + b*x]^2)/(2*b)

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{\left(\tan^2(bx+a)\right)}{2}+\ln(\cos(bx+a))}{b}$
default	$\frac{\frac{\left(\tan^2(bx+a)\right)}{2}+\ln(\cos(bx+a))}{b}$
risch	$-ix - \frac{2ia}{b} + \frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2} + \frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2} + \frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b} + \frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b} - \frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$
parallelrisch	$\frac{(-2\cos(2bx+2a)-2)\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(2\cos(2bx+2a)+2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(2\cos(2bx+2a)+2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right.}{2b(1+\cos(2bx+2a))}$

[In] int(sec(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b*(1/2*tan(b*x+a)^2+ln(cos(b*x+a)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \tan^3(a+bx)\,dx = \frac{2\cos(bx+a)^2\log(-\cos(bx+a))+1}{2\,b\cos(bx+a)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(2*cos(b*x + a)^2*log(-cos(b*x + a)) + 1)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tan^3(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2-1}-\log\left(\sin(bx+a)^2-1\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \tan^3(a+bx)\,dx = \frac{\log\left(\frac{\cos(bx+a)^2}{b^2}\right)}{2\,b} - \frac{\cos(bx+a)^2-1}{2\,b\cos(bx+a)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*log(cos(b*x + a)^2/b^2)/b - 1/2*(cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(a+bx)\,dx = -\frac{\ln\left(\tan(a+b\,x)^2+1\right)-\tan(a+b\,x)^2}{2\,b}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^3,x)
[Out] -(log(tan(a + b*x)^2 + 1) - tan(a + b*x)^2)/(2*b)
```

3.74 $\int \sec(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[Out] -sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2686}

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int (-1+x^2)\,dx,x,\sec(a+bx)\right)}{b}\\&=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^3,x]
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^3(bx+a)\right)}{3}-\sec(bx+a)}{b}$	24
default	$\frac{\frac{\left(\sec^3(bx+a)\right)}{3}-\sec(bx+a)}{b}$	24
norman	$\frac{-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3}$	39
parallelrisch	$\frac{\frac{4}{3}-4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	47
risch	$-\frac{2\left(3\,\mathrm{e}^{5i(bx+a)}+2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3}$	53

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*sec(b*x+a)^3-sec(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=-\frac{3\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{\cos\left(a+b\,x\right)^2-\frac{1}{3}}{b\cos\left(a+b\,x\right)^3}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^4,x)
[Out] -(cos(a + b*x)^2 - 1/3)/(b*cos(a + b*x)^3)
```

3.75 $\int \sec^2(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}$$

```
[Out] 1/4*tan(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}$$

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] Tan[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{\tan^4(a+bx)}{4b}$$

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
[Out] Tan[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^4(bx+a)\right)}{4}-\frac{\left(\sec^2(bx+a)\right)}{2}}{b}$	26
default	$\frac{\frac{\left(\sec^4(bx+a)\right)}{4}-\frac{\left(\sec^2(bx+a)\right)}{2}}{b}$	26
norman	$\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4}$	32
risch	$-\frac{2\left(e^{6i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^4}$	38
parallelrisch	$\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^4}$	43

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4*sec(b*x+a)^4-1/2*sec(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = -\frac{2\cos{(bx+a)}^2-1}{4\,b\cos{(bx+a)}^4}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{2\sin{(bx+a)}^2-1}{4\left(\sin{(bx+a)}^4-2\sin{(bx+a)}^2+1\right)b}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/((sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = -\frac{2\cos{(bx+a)}^2-1}{4\,b\cos{(bx+a)}^4}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^4}{4\,b}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^5,x)
[Out] tan(a + b*x)^4/(4*b)
```

3.76 $\int \sec^3(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

```
[Out] -1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx=\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] -1/3*Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^2(-1+x^2)\, dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (-x^2+x^4)\, dx, x, \sec(a+bx)\right)}{b}\\ &= -\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^3(a+bx)\, dx = -\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
[Out] -1/3*Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^5(bx+a)\right)}{5}-\frac{\left(\sec^3(bx+a)\right)}{3}}{b}$	26
default	$\frac{\frac{\left(\sec^5(bx+a)\right)}{5}-\frac{\left(\sec^3(bx+a)\right)}{3}}{b}$	26
risch	$-\frac{8\left(5\,e^{7i(bx+a)}-2\,e^{5i(bx+a)}+5\,e^{3i(bx+a)}\right)}{15b\left(e^{2i(bx+a)}+1\right)^5}$	53
norman	$\frac{\frac{4}{15b}-\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}$	71
parallelrisch	$\frac{\frac{4}{15}-4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$	73

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*sec(b*x+a)^5-1/3*sec(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos\left(bx+a\right)^2-3}{15\,b\cos\left(bx+a\right)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos\left(bx+a\right)^2-3}{15\,b\cos\left(bx+a\right)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos\left(bx+a\right)^2-3}{15\,b\cos\left(bx+a\right)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^2}{3}-\frac{1}{5}}{b\cos\left(a+b\,x\right)^5}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^6,x)
```

```
[Out] -(cos(a + b*x)^2/3 - 1/5)/(b*cos(a + b*x)^5)
```

3.77 $\int \sec^4(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[Out] -1/4*sec(b*x+a)^4/b+1/6*sec(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = \frac{\sec^6(a+bx)}{6b}-\frac{\sec^4(a+bx)}{4b}$$

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

```
[Out] -1/4*Sec[a + b*x]^4/b + Sec[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^3(-1+x^2)\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (-x^3+x^5)\,dx, x, \sec(a+bx)\right)}{b}\\ &= -\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
[Out] -1/4*Sec[a + b*x]^4/b + Sec[a + b*x]^6/(6*b)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^6(bx+a)\right)}{6}-\frac{\left(\sec^4(bx+a)\right)}{4}}{b}$	26
default	$\frac{\frac{\left(\sec^6(bx+a)\right)}{6}-\frac{\left(\sec^4(bx+a)\right)}{4}}{b}$	26
risch	$-\frac{4\left(3\,\mathrm{e}^{8i(bx+a)}-2\,\mathrm{e}^{6i(bx+a)}+3\,\mathrm{e}^{4i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^6}$	53
parallelrisch	$\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\right)}{3b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}$	60
norman	$\frac{\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{8\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}$	66

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/6*sec(b*x+a)^6-1/4*sec(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-2}{12\,b\cos\left(bx+a\right)^6}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^2-1}{12\left(\sin\left(bx+a\right)^6-3\,\sin\left(bx+a\right)^4+3\,\sin\left(bx+a\right)^2-1\right)b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/12*(3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*
x + a)^2 - 1)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-2}{12\,b\cos\left(bx+a\right)^6}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx=\frac{\tan(a+b\,x)^4\,\left(2\tan(a+b\,x)^2+3\right)}{12\,b}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^7,x)
[Out] (tan(a + b*x)^4*(2*tan(a + b*x)^2 + 3))/(12*b)
```

3.78 $\int \sec^5(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

```
[Out] -1/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx=\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

```
[Out] -1/5*Sec[a + b*x]^5/b + Sec[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^4(-1+x^2)\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (-x^4+x^6)\,dx, x, \sec(a+bx)\right)}{b}\\ &= -\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
[Out] -1/5*Sec[a + b*x]^5/b + Sec[a + b*x]^7/(7*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^7(bx+a)\right)}{7}-\frac{\left(\sec^5(bx+a)\right)}{5}}{b}$	26
default	$\frac{\frac{\left(\sec^7(bx+a)\right)}{7}-\frac{\left(\sec^5(bx+a)\right)}{5}}{b}$	26
risch	$-\frac{32\left(7\,e^{9i(bx+a)}-6\,e^{7i(bx+a)}+7\,e^{5i(bx+a)}\right)}{35b\left(e^{2i(bx+a)}+1\right)^7}$	53
parallelrisch	$\frac{\frac{4}{35}-4\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{8\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	88
norman	$\frac{-\frac{4\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4}{35b}-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{4\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	103

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/7*sec(b*x+a)^7-1/5*sec(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos\left(a+b\,x\right)^2-5}{35\,b\cos\left(a+b\,x\right)^7}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^8,x)
[Out] -(7*cos(a + b*x)^2 - 5)/(35*b*cos(a + b*x)^7)
```

3.79 $\int \sec^6(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^6(a+bx)}{6b}+\frac{\sec^8(a+bx)}{8b}$$

```
[Out] -1/6*sec(b*x+a)^6/b+1/8*sec(b*x+a)^8/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 14}

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx=\frac{\sec^8(a+bx)}{8b}-\frac{\sec^6(a+bx)}{6b}$$

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

```
[Out] -1/6*Sec[a + b*x]^6/b + Sec[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^5(-1+x^2)\ dx, x, \sec(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int (-x^5+x^7)\ dx, x, \sec(a+bx)\right)}{b} \\ &= -\frac{\sec^6(a+bx)}{6b} + \frac{\sec^8(a+bx)}{8b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^6(a+bx)}{6b} + \frac{\sec^8(a+bx)}{8b}$$

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
[Out] -1/6*Sec[a + b*x]^6/b + Sec[a + b*x]^8/(8*b)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\left(\frac{\sec^8(bx+a)}{8}\right)-\left(\frac{\sec^6(bx+a)}{6}\right)}{b}$	26
default	$\frac{\left(\frac{\sec^8(bx+a)}{8}\right)-\left(\frac{\sec^6(bx+a)}{6}\right)}{b}$	26
risch	$-\frac{32\left(e^{10i(bx+a)}-e^{8i(bx+a)}+e^{6i(bx+a)}\right)}{3b\left(e^{2i(bx+a)}+1\right)^8}$	49
parallelrisch	$\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(3\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\right)}{3b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^8}$	86
norman	$\frac{\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{16\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{16\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{40\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^8}$	98

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/8*sec(b*x+a)^8-1/6*sec(b*x+a)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = -\frac{4\,\cos\left(bx+a\right)^2-3}{24\,b\cos\left(bx+a\right)^8}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \frac{4\,\sin\left(bx+a\right)^2-1}{24\,\left(\sin\left(bx+a\right)^8-4\,\sin\left(bx+a\right)^6+6\,\sin\left(bx+a\right)^4-4\,\sin\left(bx+a\right)^2+1\right)b}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/24*(4*sin(b*x + a)^2 - 1)/((sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x
 + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = -\frac{4\,\cos\left(bx+a\right)^2-3}{24\,b\cos\left(bx+a\right)^8}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^4\,\left(3\tan(a+b\,x)^4+8\tan(a+b\,x)^2+6\right)}{24\,b}$$

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^9,x)
[Out] (tan(a + b*x)^4*(8*tan(a + b*x)^2 + 3*tan(a + b*x)^4 + 6))/(24*b)
```

3.80 $\int \cos^7(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx=\frac{\sin^5(a+bx)}{5b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{3b}-\frac{\sin^{11}(a+bx)}{11b}$$

```
[Out] 1/5*sin(b*x+a)^5/b-3/7*sin(b*x+a)^7/b+1/3*sin(b*x+a)^9/b-1/11*sin(b*x+a)^11
/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 276}

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx=-\frac{\sin^{11}(a+bx)}{11b}+\frac{\sin^9(a+bx)}{3b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - (3*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(3*b) - Si
n[a + b*x]^11/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^4(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^4-3x^6+3x^8-x^{10})\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{3b}-\frac{\sin^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{(3042+3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx)))\sin^5(a+bx)}{36960b}$$

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(36960*b)
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{\frac{\left(\sin^{11}(bx+a)\right)}{11}-\frac{\left(\sin^9(bx+a)\right)}{3}+\frac{3\left(\sin^7(bx+a)\right)}{7}-\frac{\left(\sin^5(bx+a)\right)}{5}}{b}$
default	$-\frac{\frac{\left(\sin^{11}(bx+a)\right)}{11}-\frac{\left(\sin^9(bx+a)\right)}{3}+\frac{3\left(\sin^7(bx+a)\right)}{7}-\frac{\left(\sin^5(bx+a)\right)}{5}}{b}$
risch	$\frac{7\sin(bx+a)}{512b}+\frac{\sin(11bx+11a)}{11264b}+\frac{\sin(9bx+9a)}{3072b}-\frac{\sin(7bx+7a)}{7168b}-\frac{11\sin(5bx+5a)}{5120b}-\frac{\sin(3bx+3a)}{512b}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(105\cos(6bx+6a)+3335\cos(2bx+2a)+910\cos(4bx+4a)+3042)\left(\cos\right.}{295680b}$

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(1/11*sin(b*x+a)^11-1/3*sin(b*x+a)^9+3/7*sin(b*x+a)^7-1/5*sin(b*x+a)^5
)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{\left(105\,\cos\left(bx+a\right)^{10} - 140\,\cos\left(bx+a\right)^{8} + 5\,\cos\left(bx+a\right)^{6} + 6\,\cos\left(bx+a\right)^{4} + 8\,\cos\left(bx+a\right)^{2} + 16\right)\sin\left(bx\right.}{1155\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos
(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{16\sin^{11}(a+bx)}{1155b} + \frac{8\sin^{9}(a+bx)\cos^{2}(a+bx)}{105b} + \frac{6\sin^{7}(a+bx)\cos^{4}(a+bx)}{35b} + \frac{\sin^{5}(a+bx)\cos^{6}(a+bx)}{5b} & \text{for } b \neq 0 \\ x\sin^{4}(a)\cos^{7}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**11/(1155*b) + 8*sin(a + b*x)**9*cos(a + b*x)**2
/(105*b) + 6*sin(a + b*x)**7*cos(a + b*x)**4/(35*b) + sin(a + b*x)**5*cos(a
 + b*x)**6/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = -\frac{105\,\sin\left(bx+a\right)^{11} - 385\,\sin\left(bx+a\right)^{9} + 495\,\sin\left(bx+a\right)^{7} - 231\,\sin\left(bx+a\right)^{5}}{1155\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 - 23
1*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(11\,bx+11\,a)}{11264\,b} + \frac{\sin(9\,bx+9\,a)}{3072\,b} - \frac{\sin(7\,bx+7\,a)}{7168\,b} - \frac{11\,\sin(5\,bx+5\,a)}{5120\,b} - \frac{\sin(3\,bx+3\,a)}{512\,b} + \frac{7\,\sin(bx+a)}{512\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/11264*sin(11*b*x + 11*a)/b + 1/3072*sin(9*b*x + 9*a)/b - 1/7168*sin(7*b*x
 + 7*a)/b - 11/5120*sin(5*b*x + 5*a)/b - 1/512*sin(3*b*x + 3*a)/b + 7/512*s
in(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{-\frac{\sin(a+b\,x)^{11}}{11} + \frac{\sin(a+b\,x)^9}{3} - \frac{3\sin(a+b\,x)^7}{7} + \frac{\sin(a+b\,x)^5}{5}}{b}$$

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^4,x)
```

```
[Out] (sin(a + b*x)^5/5 - (3*sin(a + b*x)^7)/7 + sin(a + b*x)^9/3 - sin(a + b*x)^
11/11)/b
```

3.81 $\int \cos^5(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx=\frac{\sin^5(a+bx)}{5b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{9b}$$

```
[Out] 1/5*sin(b*x+a)^5/b-2/7*sin(b*x+a)^7/b+1/9*sin(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 276}

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx=\frac{\sin^9(a+bx)}{9b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
[Out] Sin[a + b*x]^5/(5*b) - (2*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^4(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\sin^5(a+bx)}{5b} - \frac{2\sin^7(a+bx)}{7b} + \frac{\sin^9(a+bx)}{9b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$
\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{(249+220\cos(2(a+bx))+35\cos(4(a+bx)))\sin^5(a+bx)}{2520b}
$$

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
[Out] ((249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(2520*b
)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{(\sin^9(bx+a))}{9}-\frac{2(\sin^7(bx+a))}{7}+\frac{(\sin^5(bx+a))}{5}}{b}$
default	$\frac{\frac{(\sin^9(bx+a))}{9}-\frac{2(\sin^7(bx+a))}{7}+\frac{(\sin^5(bx+a))}{5}}{b}$
risch	$\frac{3\sin(bx+a)}{128b}+\frac{\sin(9bx+9a)}{2304b}+\frac{\sin(7bx+7a)}{1792b}-\frac{\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{192b}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(249+35\cos(4bx+4a)+220\cos(2bx+2a))\left(\cos\left(\frac{5bx}{2}+\frac{5a}{2}\right)+5\cos\left(\frac{3bx}{2}\right.\right.}{20160b}$
norman	$\frac{\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{384\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}+\frac{6976\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{315b}-\frac{384\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}+\frac{32\left(\tan^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^9}$

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/9*sin(b*x+a)^9-2/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{\left(35\,\cos\left(bx+a\right)^8-50\,\cos\left(bx+a\right)^6+3\,\cos\left(bx+a\right)^4+4\,\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{315\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x
 + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{8\sin^9(a+bx)}{315b}+\frac{4\sin^7(a+bx)\cos^2(a+bx)}{35b}+\frac{\sin^5(a+bx)\cos^4(a+bx)}{5b} & \text{for } b\neq 0 \\ x\sin^4(a)\cos^5(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**4,x)
[Out] Piecewise((8*sin(a + b*x)**9/(315*b) + 4*sin(a + b*x)**7*cos(a + b*x)**2/(3
5*b) + sin(a + b*x)**5*cos(a + b*x)**4/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a
)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{35\,\sin\left(bx+a\right)^9-90\,\sin\left(bx+a\right)^7+63\,\sin\left(bx+a\right)^5}{315\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(9\,bx+9\,a)}{2304\,b} + \frac{\sin(7\,bx+7\,a)}{1792\,b} - \frac{\sin(5\,bx+5\,a)}{320\,b} - \frac{\sin(3\,bx+3\,a)}{192\,b} + \frac{3\,\sin(bx+a)}{128\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/2304*sin(9*b*x + 9*a)/b + 1/1792*sin(7*b*x + 7*a)/b - 1/320*sin(5*b*x + 5
*a)/b - 1/192*sin(3*b*x + 3*a)/b + 3/128*sin(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{35\sin(a+b\,x)^9 - 90\sin(a+b\,x)^7 + 63\sin(a+b\,x)^5}{315\,b}$$

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^4,x)
```

```
[Out] (63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9)/(315*b)
```

3.82 $\int \cos^3(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx=\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

```
[Out] 1/5*sin(b*x+a)^5/b-1/7*sin(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx=\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - Sin[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^4(1-x^2)\ dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int (x^4-x^6)\ dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{\sin^5(a+bx)}{5b} - \frac{\sin^7(a+bx)}{7b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{(9+5\cos(2(a+bx)))\sin^5(a+bx)}{70b}$$

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
[Out] ((9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(70*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\left(\sin^7(bx+a)\right)}{7}+\frac{\left(\sin^5(bx+a)\right)}{5}}{b}$
default	$\frac{-\frac{\left(\sin^7(bx+a)\right)}{7}+\frac{\left(\sin^5(bx+a)\right)}{5}}{b}$
risch	$\frac{3\sin(bx+a)}{64b}+\frac{\sin(7bx+7a)}{448b}-\frac{\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{64b}$
norman	$\frac{\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{192\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}+\frac{32\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^7}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(9+5\cos(2bx+2a))\left(\cos\left(\frac{5bx}{2}+\frac{5a}{2}\right)+5\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{560b}$

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{\left(5\,\cos\left(bx+a\right)^{6}-8\,\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{35\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{2\sin^{7}\left(a+bx\right)}{35b}+\frac{\sin^{5}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{5b} & \text{for}\ b\neq 0 \\ x\sin^{4}\left(a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**4,x)
[Out] Piecewise((2*sin(a + b*x)**7/(35*b) + sin(a + b*x)**5*cos(a + b*x)**2/(5*b)
, Ne(b, 0)), (x*sin(a)**4*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = -\frac{5\,\sin\left(bx+a\right)^{7}-7\,\sin\left(bx+a\right)^{5}}{35\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = -\frac{5\,\sin(bx+a)^7 - 7\,\sin(bx+a)^5}{35\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{7\sin(a+b\,x)^5 - 5\sin(a+b\,x)^7}{35\,b}$$

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^4,x)
[Out] (7*sin(a + b*x)^5 - 5*sin(a + b*x)^7)/(35*b)
```

3.83 $\int \cos(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}$$

```
[Out] 1/5*sin(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin^5(a+bx)}{5b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}$$

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^4,x]
[Out] Sin[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin^5(bx+a)}{5b}$	14
default	$\frac{\sin^5(bx+a)}{5b}$	14
norman	$\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$	32
parallelrisch	$\frac{10\sin(bx+a)+\sin(5bx+5a)-5\sin(3bx+3a)}{80b}$	35
risch	$\frac{\sin(bx+a)}{8b}+\frac{\sin(5bx+5a)}{80b}-\frac{\sin(3bx+3a)}{16b}$	41

```
[In] int(cos(b*x+a)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/5*sin(b*x+a)^5/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^4 - 2\cos\left(bx+a\right)^2 + 1\right)\sin\left(bx+a\right)}{5\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{\sin^5(a+bx)}{5b} & \text{for } b \neq 0 \\ x\sin^4(a)\cos(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**4,x)
[Out] Piecewise((sin(a + b*x)**5/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(bx+a\right)^5}{5\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/5*sin(b*x + a)^5/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(bx+a\right)^5}{5\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/5*sin(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^5}{5\,b}$$

```
[In] int(cos(a + b*x)*sin(a + b*x)^4,x)
[Out] sin(a + b*x)^5/(5*b)
```

3.84 $\int \sin^2(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx=-\frac{3x}{2}+\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}$$

```
[Out] -3/2*x+3/2*tan(b*x+a)/b-1/2*sin(b*x+a)^2*tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 327, 209}

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx=\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}-\frac{3x}{2}$$

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
[Out] (-3*x)/2 + (3*Tan[a + b*x])/(2*b) - (Sin[a + b*x]^2*Tan[a + b*x])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}\\
&= -\frac{\sin^2(a+bx)\tan(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}\\
&= \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}\\
&= -\frac{3x}{2} + \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{-6(a+bx)+\sin(2(a+bx))+4\tan(a+bx)}{4b}$$

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
[Out] (-6*(a + b*x) + Sin[2*(a + b*x)] + 4*Tan[a + b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{-12bx\cos(bx+a)+9\sin(bx+a)+\sin(3bx+3a)}{8b\cos(bx+a)}$	42
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{\cos(bx+a)}+\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	54
default	$\frac{\frac{\sin^5(bx+a)}{\cos(bx+a)}+\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	54
risch	$-\frac{3x}{2}-\frac{i e^{2i(bx+a)}}{8b}+\frac{i e^{-2i(bx+a)}}{8b}+\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	54
norman	$\frac{\frac{3x}{2}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{3x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{3x\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	124

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/8*(-12*b*x*cos(b*x+a)+9*sin(b*x+a)+sin(3*b*x+3*a))/b/cos(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,bx\cos\left(bx+a\right)-\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{2\,b\cos\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
[Out] -1/2*(3*b*x*cos(b*x + a) - (cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x +
a))
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \int \sin^4\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**4,x)
[Out] Integral(sin(a + b*x)**4*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,bx+3\,a-\frac{\tan(bx+a)}{\tan(bx+a)^2+1}-2\,\tan\left(bx+a\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,bx+3\,a-\frac{\tan(bx+a)}{\tan(bx+a)^2+1}-2\,\tan\left(bx+a\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)\,\sin(a+b\,x)}{2}+\frac{\sin(a+b\,x)}{\cos(a+b\,x)}}{b}-\frac{3\,x}{2}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^2,x)
[Out] ((cos(a + b*x)*sin(a + b*x))/2 + sin(a + b*x)/cos(a + b*x))/b - (3*x)/2
```

3.85 $\int \tan^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tan^4(a+bx)\,dx = x-\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] x-tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tan^4(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}+x$$

```
[In] Int[Tan[a + b*x]^4,x]
```

```
[Out] x - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^3(a+bx)}{3b} - \int \tan^2(a+bx)\,dx \\
&= -\frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b} + \int 1\,dx \\
&= x - \frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tan^4(a+bx)\,dx = \frac{\arctan(\tan(a+bx))}{b} - \frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b}$$

```
[In] Integrate[Tan[a + b*x]^4,x]
[Out] ArcTan[Tan[a + b*x]]/b - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\left(\tan^3(bx+a)\right)}{3}-\tan(bx+a)+bx+a}{b}$
default	$\frac{\frac{\left(\tan^3(bx+a)\right)}{3}-\tan(bx+a)+bx+a}{b}$
risch	$x-\frac{4i\left(3\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}+2\right)}{3b\left(e^{2i(bx+a)}+1\right)^3}$
norman	$\frac{x\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-x+\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{20\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{2\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3}$
parallelrisch	$\frac{3\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb-9\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+6\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb-20\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3bx+6\tan\left(\right.}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*tan(b*x+a)^3-tan(b*x+a)+b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \tan^4(a+bx)\,dx = \frac{3\,bx\cos\left(bx+a\right)^3 - \left(4\,\cos\left(bx+a\right)^2 - 1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(3*b*x*cos(b*x + a)^3 - (4*cos(b*x + a)^2 - 1)*sin(b*x + a))/(b*cos(b*x
 + a)^3)
```

Sympy [F]

$$\int \tan^4(a+bx)\,dx = \int \sin^4\left(a+bx\right)\sec^4\left(a+bx\right)dx$$

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**4,x)
[Out] Integral(sin(a + b*x)**4*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3 + 3\,bx + 3\,a - 3\,\tan\left(bx+a\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3 + 3\,bx + 3\,a - 3\,\tan\left(bx+a\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tan^4(a+bx)\,dx = x - \frac{\tan(a+b\,x) - \frac{\tan(a+b\,x)^3}{3}}{b}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^4,x)
[Out] x - (tan(a + b*x) - tan(a + b*x)^3/3)/b
```

3.86 $\int \sec^2(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}$$

```
[Out] 1/5*tan(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}$$

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan^5(a+bx)}{5b}$$

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
[Out] Tan[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{\sin^5(bx+a)}{5b\cos(bx+a)^5}$	22
default	$\frac{\sin^5(bx+a)}{5b\cos(bx+a)^5}$	22
norman	$-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}$	32
parallelrisch	$-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$	43
risch	$\frac{2i\left(5\,e^{8i(bx+a)}+10\,e^{4i(bx+a)}+1\right)}{5b\left(e^{2i(bx+a)}+1\right)^5}$	44

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/5/b*sin(b*x+a)^5/cos(b*x+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^4 - 2\,\cos\left(bx+a\right)^2 + 1\right)\sin\left(bx+a\right)}{5\,b\cos\left(bx+a\right)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^5}{5\,b}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/5*tan(b*x + a)^5/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^5}{5\,b}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/5*tan(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\tan(a+b\,x)^5}{5\,b}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^6,x)
[Out] tan(a + b*x)^5/(5*b)
```

3.87 $\int \sec^4(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}$$

```
[Out] 1/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 14}

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^4(1+x^2)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^4+x^6)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{2\tan(a+bx)}{35b}+\frac{\sec^2(a+bx)\tan(a+bx)}{35b} - \frac{8\sec^4(a+bx)\tan(a+bx)}{35b}+\frac{\sec^6(a+bx)\tan(a+bx)}{7b}$$

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
[Out] (2*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(35*b) - (8*Sec[a +
 b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{7\cos(bx+a)^7}+\frac{2\left(\sin^5(bx+a)\right)}{35\cos(bx+a)^5}}{b}$	42
default	$\frac{\frac{\sin^5(bx+a)}{7\cos(bx+a)^7}+\frac{2\left(\sin^5(bx+a)\right)}{35\cos(bx+a)^5}}{b}$	42
parallelrisch	$-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(7\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+6\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+7\right)}{35b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	60
norman	$\frac{-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{192\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{32\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	66
risch	$\frac{4i\left(35\,e^{10i(bx+a)}-35\,e^{8i(bx+a)}+70\,e^{6i(bx+a)}-14\,e^{4i(bx+a)}+7\,e^{2i(bx+a)}+1\right)}{35b\left(e^{2i(bx+a)}+1\right)^7}$	77

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^5/cos(b*x+a)^7+2/35*sin(b*x+a)^5/cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{\left(2\cos(bx+a)^6+\cos(bx+a)^4-8\cos(bx+a)^2+5\right)\sin(bx+a)}{35\,b\cos(bx+a)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(b*x + a)^6 + cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 5)*sin(b*x + a
)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{5\tan(bx+a)^7+7\tan(bx+a)^5}{35\,b}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{5\,\tan\left(bx+a\right)^7+7\,\tan\left(bx+a\right)^5}{35\,b}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(a+b\,x)^5\,\left(5\tan(a+b\,x)^2+7\right)}{35\,b}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^8,x)
[Out] (tan(a + b*x)^5*(5*tan(a + b*x)^2 + 7))/(35*b)
```

3.88 $\int \sec^6(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}$$

```
[Out] 1/5*tan(b*x+a)^5/b+2/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 276}

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^9(a+bx)}{9b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + (2*Tan[a + b*x]^7)/(7*b) + Tan[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^4(1+x^2)^2\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^4+2x^6+x^8)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{8\tan(a+bx)}{315b}+\frac{4\sec^2(a+bx)\tan(a+bx)}{315b}+\frac{\sec^4(a+bx)\tan(a+bx)}{105b}-\frac{10\sec^6(a+bx)\tan(a+bx)}{63b}+\frac{\sec^8(a+bx)\tan(a+bx)}{9b}$$

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
[Out] (8*Tan[a + b*x])/(315*b) + (4*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) + (Sec[a
 + b*x]^4*Tan[a + b*x])/(105*b) - (10*Sec[a + b*x]^6*Tan[a + b*x])/(63*b) +
 (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

method	result	si
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{9\cos(bx+a)^9}+\frac{4\left(\sin^5(bx+a)\right)}{63\cos(bx+a)^7}+\frac{8\left(\sin^5(bx+a)\right)}{315\cos(bx+a)^5}}{b}$	60
default	$\frac{\frac{\sin^5(bx+a)}{9\cos(bx+a)^9}+\frac{4\left(\sin^5(bx+a)\right)}{63\cos(bx+a)^7}+\frac{8\left(\sin^5(bx+a)\right)}{315\cos(bx+a)^5}}{b}$	60
parallelrisch	$-\frac{32\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(63\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+108\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+218\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+108\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+63\right)}{315b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^9}$	86
risch	$\frac{16i\left(210\,e^{12i(bx+a)}-315\,e^{10i(bx+a)}+441\,e^{8i(bx+a)}-126\,e^{6i(bx+a)}+36\,e^{4i(bx+a)}+9\,e^{2i(bx+a)}+1\right)}{315b\left(e^{2i(bx+a)}+1\right)^9}$	88

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^5/cos(b*x+a)^9+4/63*sin(b*x+a)^5/cos(b*x+a)^7+8/315*sin
(b*x+a)^5/cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{\left(8\cos\left(bx+a\right)^8+4\cos\left(bx+a\right)^6+3\cos\left(bx+a\right)^4-50\cos\left(bx+a\right)^2+35\right)\sin\left(bx+a\right)}{315\,b\cos\left(bx+a\right)^9}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(b*x + a)^8 + 4*cos(b*x + a)^6 + 3*cos(b*x + a)^4 - 50*cos(b*x
+ a)^2 + 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{35\tan\left(bx+a\right)^9+90\tan\left(bx+a\right)^7+63\tan\left(bx+a\right)^5}{315\,b}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{35\,\tan\left(bx+a\right)^9 + 90\,\tan\left(bx+a\right)^7 + 63\,\tan\left(bx+a\right)^5}{315\,b}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(a+b\,x)^5\,\left(35\tan(a+b\,x)^4+90\tan(a+b\,x)^2+63\right)}{315\,b}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^10,x)
[Out] (tan(a + b*x)^5*(90*tan(a + b*x)^2 + 35*tan(a + b*x)^4 + 63))/(315*b)
```

3.89 $\int \cos^6(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3x}{256}+\frac{3\cos(a+bx)\sin(a+bx)}{256b}+\frac{\cos^3(a+bx)\sin(a+bx)}{128b}+\frac{\cos^5(a+bx)\sin(a+bx)}{160b}-\frac{3\cos^7(a+bx)\sin(a+bx)}{80b}-\frac{\cos^7(a+bx)\sin^3(a+bx)}{10b}$$

```
[Out] 3/256*x+3/256*cos(b*x+a)*sin(b*x+a)/b+1/128*cos(b*x+a)^3*sin(b*x+a)/b+1/160
*cos(b*x+a)^5*sin(b*x+a)/b-3/80*cos(b*x+a)^7*sin(b*x+a)/b-1/10*cos(b*x+a)^7
*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = -\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}-\frac{3\sin(a+bx)\cos^7(a+bx)}{80b}+\frac{\sin(a+bx)\cos^5(a+bx)}{160b}+\frac{\sin(a+bx)\cos^3(a+bx)}{128b}+\frac{3\sin(a+bx)\cos(a+bx)}{256b}+\frac{3x}{256}$$

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[a + b*x]*Sin[a + b*x])/(256*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(128*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(160*b) - (3*Cos[a + b*x]^7*
Sin[a + b*x])/(80*b) - (Cos[a + b*x]^7*Sin[a + b*x]^3)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{80}\int \cos^6(a+bx)\,dx \\
&= \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} \\
&\quad - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{1}{32}\int \cos^4(a+bx)\,dx \\
&= \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} \\
&\quad - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{128}\int \cos^2(a+bx)\,dx \\
&= \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} \\
&\quad - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3\int 1\,dx}{256} \\
&= \frac{3x}{256} + \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} \\
&\quad + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))}{10240b}$$

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(10240*b)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{120bx+2\sin(10bx+10a)+5\sin(8bx+8a)-10\sin(6bx+6a)-40\sin(4bx+4a)+20\sin(2bx+2a)}{10240b}$
risch	$\frac{3x}{256}+\frac{\sin(10bx+10a)}{5120b}+\frac{\sin(8bx+8a)}{2048b}-\frac{\sin(6bx+6a)}{1024b}-\frac{\sin(4bx+4a)}{256b}+\frac{\sin(2bx+2a)}{512b}$
derivativedivides	$\frac{-\frac{\left(\sin^3(bx+a)\right)\left(\cos^7(bx+a)\right)}{10}-\frac{3\left(\cos^7(bx+a)\right)\sin(bx+a)}{80}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{160}+\frac{3bx}{256}+\frac{3a}{256}}{b}$
default	$\frac{-\frac{\left(\sin^3(bx+a)\right)\left(\cos^7(bx+a)\right)}{10}-\frac{3\left(\cos^7(bx+a)\right)\sin(bx+a)}{80}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{160}+\frac{3bx}{256}+\frac{3a}{256}}{b}$

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/10240*(120*b*x+2*sin(10*b*x+10*a)+5*sin(8*b*x+8*a)-10*sin(6*b*x+6*a)-40*s
in(4*b*x+4*a)+20*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{15\,bx+\left(128\,\cos\left(bx+a\right)^9-176\,\cos\left(bx+a\right)^7+8\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+15\,\cos\left(bx+a\right)\right)\sin\left(\right.}{1280\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^
5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^{10}(a+bx)}{256} + \frac{15x\sin^8(a+bx)\cos^2(a+bx)}{256} + \frac{15x\sin^6(a+bx)\cos^4(a+bx)}{128} + \frac{15x\sin^4(a+bx)\cos^6(a+bx)}{128} + \frac{15x\sin^2(a+bx)\cos^8(a\cdot}{256} \\ x\sin^4(a)\cos^6(a) \end{cases}$$

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**10/256 + 15*x*sin(a + b*x)**8*cos(a + b*x)**2/
256 + 15*x*sin(a + b*x)**6*cos(a + b*x)**4/128 + 15*x*sin(a + b*x)**4*cos(a
 + b*x)**6/128 + 15*x*sin(a + b*x)**2*cos(a + b*x)**8/256 + 3*x*cos(a + b*x
)**10/256 + 3*sin(a + b*x)**9*cos(a + b*x)/(256*b) + 7*sin(a + b*x)**7*cos(
a + b*x)**3/(128*b) + sin(a + b*x)**5*cos(a + b*x)**5/(10*b) - 7*sin(a + b*
x)**3*cos(a + b*x)**7/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**9/(256*b), Ne(
b, 0)), (x*sin(a)**4*cos(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{32\sin(2bx+2a)^5 + 120bx + 120a + 5\sin(8bx+8a) - 40\sin(4bx+4a)}{10240b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10240*(32*sin(2*b*x + 2*a)^5 + 120*b*x + 120*a + 5*sin(8*b*x + 8*a) - 40*
sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3}{256}\,x + \frac{\sin\left(10\,bx+10\,a\right)}{5120\,b} + \frac{\sin\left(8\,bx+8\,a\right)}{2048\,b} - \frac{\sin\left(6\,bx+6\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx+4\,a\right)}{256\,b} + \frac{\sin\left(2\,bx+2\,a\right)}{512\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*b*x + 10*a)/b + 1/2048*sin(8*b*x + 8*a)/b - 1/1024*
sin(6*b*x + 6*a)/b - 1/256*sin(4*b*x + 4*a)/b + 1/512*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3\,x}{256} + \frac{\frac{3\tan(a+b\,x)^9}{256}+\frac{7\tan(a+b\,x)^7}{128}+\frac{\tan(a+b\,x)^5}{10}-\frac{7\tan(a+b\,x)^3}{128}-\frac{3\tan(a+b\,x)}{256}}{b\,\left(\tan\left(a+b\,x\right)^{10}+5\tan\left(a+b\,x\right)^{8}+10\tan\left(a+b\,x\right)^{6}+10\tan\left(a+b\,x\right)^{4}+5\tan\left(a+b\,x\right)^{2}+1\right)}$$

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^4,x)
```

```
[Out] (3*x)/256 + (tan(a + b*x)^5/10 - (7*tan(a + b*x)^3)/128 - (3*tan(a + b*x))/
256 + (7*tan(a + b*x)^7)/128 + (3*tan(a + b*x)^9)/256)/(b*(5*tan(a + b*x)^2
 + 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^
10 + 1))
```

3.90 $\int \cos^4(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx=\frac{3x}{128}+\frac{3\cos(a+bx)\sin(a+bx)}{128b}+\frac{\cos^3(a+bx)\sin(a+bx)}{64b}-\frac{\cos^5(a+bx)\sin(a+bx)}{16b}-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}$$

```
[Out] 3/128*x+3/128*cos(b*x+a)*sin(b*x+a)/b+1/64*cos(b*x+a)^3*sin(b*x+a)/b-1/16*c
os(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^5*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx=-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}-\frac{\sin(a+bx)\cos^5(a+bx)}{16b}+\frac{\sin(a+bx)\cos^3(a+bx)}{64b}+\frac{3\sin(a+bx)\cos(a+bx)}{128b}+\frac{3x}{128}$$

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(64*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^5*Sin[
a + b*x]^3)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}+\frac{3}{8}\int\cos^4(a+bx)\sin^2(a+bx)\,dx\\
&= -\frac{\cos^5(a+bx)\sin(a+bx)}{16b}-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}+\frac{1}{16}\int\cos^4(a+bx)\,dx\\
&= \frac{\cos^3(a+bx)\sin(a+bx)}{64b}-\frac{\cos^5(a+bx)\sin(a+bx)}{16b}\\
&\quad-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}+\frac{3}{64}\int\cos^2(a+bx)\,dx\\
&= \frac{3\cos(a+bx)\sin(a+bx)}{128b}+\frac{\cos^3(a+bx)\sin(a+bx)}{64b}\\
&\quad-\frac{\cos^5(a+bx)\sin(a+bx)}{16b}-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}+\frac{3\int 1\,dx}{128}\\
&= \frac{3x}{128}+\frac{3\cos(a+bx)\sin(a+bx)}{128b}+\frac{\cos^3(a+bx)\sin(a+bx)}{64b}\\
&\quad-\frac{\cos^5(a+bx)\sin(a+bx)}{16b}-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{24(a+bx)-8\sin(4(a+bx))+\sin(8(a+bx))}{1024b}$$

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
[Out] (24*(a + b*x) - 8*Sin[4*(a + b*x)] + Sin[8*(a + b*x)])/(1024*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

method	result
parallelrisch	$\frac{24bx+\sin(8bx+8a)-8\sin(4bx+4a)}{1024b}$
risch	$\frac{3x}{128}+\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(4bx+4a)}{128b}$
derivativedivides	$\frac{-\frac{\left(\cos^5(bx+a)\right)\left(\sin^3(bx+a)\right)}{8}-\frac{\left(\cos^5(bx+a)\right)\sin(bx+a)}{16}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{64}+\frac{3bx}{128}+\frac{3a}{128}}{b}$
default	$\frac{-\frac{\left(\cos^5(bx+a)\right)\left(\sin^3(bx+a)\right)}{8}-\frac{\left(\cos^5(bx+a)\right)\sin(bx+a)}{16}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{64}+\frac{3bx}{128}+\frac{3a}{128}}{b}$
norman	$\frac{3x}{128}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{23\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{333\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}-\frac{671\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{671\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}-\frac{333\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{64b}$

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/1024*(24*b*x+sin(8*b*x+8*a)-8*sin(4*b*x+4*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{3\,bx+\left(16\,\cos\left(bx+a\right)^7-24\,\cos\left(bx+a\right)^5+2\,\cos\left(bx+a\right)^3+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{128\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/128*(3*b*x + (16*cos(b*x + a)^7 - 24*cos(b*x + a)^5 + 2*cos(b*x + a)^3 +
3*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^8(a+bx)}{128} + \frac{3x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{9x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{3x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{3x\cos^8(a+bx)}{128} + \frac{3\sin^7(a+ \\ x\sin^4(a)\cos^4(a) \end{cases}$$

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**8/128 + 3*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 9*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 3*x*sin(a + b*x)**2*cos(a + b*x
)**6/32 + 3*x*cos(a + b*x)**8/128 + 3*sin(a + b*x)**7*cos(a + b*x)/(128*b)
+ 11*sin(a + b*x)**5*cos(a + b*x)**3/(128*b) - 11*sin(a + b*x)**3*cos(a + b
*x)**5/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin(
a)**4*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{24\,bx + 24\,a + \sin(8\,bx + 8\,a) - 8\,\sin(4\,bx + 4\,a)}{1024\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(24*b*x + 24*a + sin(8*b*x + 8*a) - 8*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{3}{128}\,x + \frac{\sin(8\,bx + 8\,a)}{1024\,b} - \frac{\sin(4\,bx + 4\,a)}{128\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/128*sin(4*b*x + 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx$$
$$=\frac{3\,x}{128}-\frac{-\frac{3\tan(a+b\,x)^7}{128}-\frac{11\tan(a+b\,x)^5}{128}+\frac{11\tan(a+b\,x)^3}{128}+\frac{3\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8+4\tan\left(a+b\,x\right)^6+6\tan\left(a+b\,x\right)^4+4\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^4,x)
```

```
[Out] (3*x)/128 - ((3*tan(a + b*x))/128 + (11*tan(a + b*x)^3)/128 - (11*tan(a + b
*x)^5)/128 - (3*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x)^
4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

3.91 $\int \cos^2(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}$$

```
[Out] 1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b-1/8*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos(b
*x+a)^3*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2648, 2715, 8}

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = -\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b} + \frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(
8*b) - (Cos[a + b*x]^3*Sin[a + b*x]^3)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b} + \frac{1}{2}\int \cos^2(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx)}{6b} + \frac{1}{8}\int \cos^2(a+bx)\,dx \\
&= \frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx)}{6b} + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{12bx - 3\sin(2(a+bx)) - 3\sin(4(a+bx)) + \sin(6(a+bx))}{192b}$$

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(192*
b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{12bx+\sin(6bx+6a)-3\sin(4bx+4a)-3\sin(2bx+2a)}{192b}$
risch	$\frac{x}{16}+\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{64b}-\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\left(\cos^3(bx+a)\right)\left(\sin^3(bx+a)\right)}{6}-\frac{\left(\cos^3(bx+a)\right)\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
default	$\frac{-\frac{\left(\cos^3(bx+a)\right)\left(\sin^3(bx+a)\right)}{6}-\frac{\left(\cos^3(bx+a)\right)\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
norman	$\frac{\frac{x}{16}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{17\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{19\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{19\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{17\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{3x\left(\tan^2\left(\right.\right.}{8}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}$

[In] int(cos(b*x+a)^2*sin(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] 1/192*(12*b*x+sin(6*b*x+6*a)-3*sin(4*b*x+4*a)-3*sin(2*b*x+2*a))/b

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{3\,bx+\left(8\,\cos\left(bx+a\right)^5-14\,\cos\left(bx+a\right)^3+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")

[Out] 1/48*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(58) = 116.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{x\sin^6(a+bx)}{16}+\frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{x\cos^6(a+bx)}{16}+\frac{\sin^5(a+bx)\cos(a+bx)}{16b}-\frac{\sin^3(a+bx)\cos^3}{6b} \\ x\sin^4(a)\cos^2(a) \end{cases}$$

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) - sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = -\frac{4\,\sin\left(2\,bx+2\,a\right)^3-12\,bx-12\,a+3\,\sin\left(4\,bx+4\,a\right)}{192\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*b*x + 2*a)^3 - 12*b*x - 12*a + 3*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{1}{16}\,x+\frac{\sin\left(6\,bx+6\,a\right)}{192\,b}-\frac{\sin\left(4\,bx+4\,a\right)}{64\,b}-\frac{\sin\left(2\,bx+2\,a\right)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b - 1/64*sin(2*b*
x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{x}{16}-\frac{\frac{\sin(2\,a+2\,b\,x)}{64}+\frac{\sin(4\,a+4\,b\,x)}{64}-\frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^4,x)
```

```
[Out] x/16 - (sin(2*a + 2*b*x)/64 + sin(4*a + 4*b*x)/64 - sin(6*a + 6*b*x)/192)/b
```

3.92 $\int \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sin^4(a+bx)\,dx = \frac{3x}{8} - \frac{3\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

```
[Out] 3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\int \sin^4(a+bx)\,dx = -\frac{\sin^3(a+bx)\cos(a+bx)}{4b} - \frac{3\sin(a+bx)\cos(a+bx)}{8b} + \frac{3x}{8}$$

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b} + \frac{3}{4}\int \sin^2(a+bx)\,dx \\
&= -\frac{3\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos(a+bx)\sin^3(a+bx)}{4b} + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} - \frac{3\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos(a+bx)\sin^3(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12(a+bx) - 8\sin(2(a+bx)) + \sin(4(a+bx))}{32b}$$

```
[In] Integrate[Sin[a + b*x]^4,x]
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sin(4bx+4a)-8\sin(2bx+2a)}{32b}$
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}-\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{11\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{11\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x(}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

```
[In] int(sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/32*(12*b*x+sin(4*b*x+4*a)-8*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^4(a+bx)\,dx = \frac{3\,bx + \left(2\,\cos\left(bx+a\right)^3 - 5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^4(a+bx)}{8} + \frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{3x\cos^4(a+bx)}{8} - \frac{5\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**4,x)
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(4\,bx + 4\,a\right) - 8\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sin^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \sin^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{5\tan(a+b\,x)^3}{8} + \frac{3\tan(a+b\,x)}{8}}{b\,\left(\tan\left(a+b\,x\right)^4 + 2\tan\left(a+b\,x\right)^2 + 1\right)}$$

```
[In] int(sin(a + b*x)^4,x)
[Out] (3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
+ tan(a + b*x)^4 + 1))
```

3.93 $\int \sin^3(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

```
[Out] arctanh(sin(b*x+a))/b-sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2672, 308, 212}

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin^3(a+bx)}{3b} - \frac{\sin(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(a+bx)\right)}{b} \\
&= -\frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x],x]
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^{3}(bx+a)\right)}{3}-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	38
default	$\frac{-\frac{\left(\sin^{3}(bx+a)\right)}{3}-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	38
parallelrisch	$\frac{12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-15\sin(bx+a)+\sin(3bx+3a)}{12b}$	52
risch	$\frac{5i e^{i(bx+a)}}{8b}-\frac{5i e^{-i(bx+a)}}{8b}+\frac{\ln(e^{i(bx+a)}+i)}{b}-\frac{\ln(e^{i(bx+a)}-i)}{b}+\frac{\sin(3bx+3a)}{12b}$	81
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{20\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{2\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{3}}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	98

```
[In] int(sec(b*x+a)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3*sin(b*x+a)^3-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{2\left(\cos\left(bx+a\right)^2-4\right)\sin\left(bx+a\right)+3\,\log\left(\sin\left(bx+a\right)+1\right)-3\,\log\left(-\sin\left(bx+a\right)+1\right)}{6\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/6*(2*(cos(b*x + a)^2 - 4)*sin(b*x + a) + 3*log(sin(b*x + a) + 1) - 3*log(
-sin(b*x + a) + 1))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^3 - 3\,\log\left(\sin\left(bx+a\right)+1\right) + 3\,\log\left(\sin\left(bx+a\right)-1\right) + 6\,\sin\left(bx+a\right)}{6\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)
+ 6*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^3 - 3\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 3\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right) + 6\,\sin\left(bx+a\right)}{6\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x +
 a) - 1)) + 6*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\right)}{b} - \frac{5\,\sin\left(a+b\,x\right)}{4\,b} + \frac{\sin\left(3\,a+3\,b\,x\right)}{12\,b}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x),x)
[Out] (2*atanh(sin(a/2 + (b*x)/2)/cos(a/2 + (b*x)/2)))/b - (5*sin(a + b*x))/(4*b)
 + sin(3*a + 3*b*x)/(12*b)
```

3.94 $\int \sin(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b}$$

```
[Out] -3/2*arctanh(sin(b*x+a))/b+3/2*sin(b*x+a)/b+1/2*sin(b*x+a)*tan(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2672, 294, 327, 212}

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b}$$

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sin[a + b*x])/(2*b) + (Sin[a + b*x]*T
an[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b} \\
&= \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b} \\
&= -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{3\sec(a+bx)\tan(a+bx)}{2b} - \frac{\sin(a+bx)\tan^2(a+bx)}{b}$$

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(2*b) - (S
in[a + b*x]*Tan[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^3(bx+a)\right)}{2}+\frac{3\sin(bx+a)}{2}-\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	58
default	$\frac{\frac{\sin^5(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^3(bx+a)\right)}{2}+\frac{3\sin(bx+a)}{2}-\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	58
parallelrisch	$\frac{(3\cos(2bx+2a)+3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-3\cos(2bx+2a)-3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+3\sin(bx+a)+\sin(3bx+3a)}{2b(1+\cos(2bx+2a))}$	89
risch	$-\frac{i\,\mathrm{e}^{i(bx+a)}}{2b}+\frac{i\,\mathrm{e}^{-i(bx+a)}}{2b}-\frac{i\left(\mathrm{e}^{3i(bx+a)}-\mathrm{e}^{i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{2b}-\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{2b}$	108
norman	$\frac{\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{3\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$	114

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*sin(b*x+a)^5/cos(b*x+a)^2+1/2*sin(b*x+a)^3+3/2*sin(b*x+a)-3/2*ln(s
ec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \sin(a+bx)\tan^3(a+bx)\,dx =$$
$$-\frac{3\,\cos\left(bx+a\right)^2\log\left(\sin\left(bx+a\right)+1\right)-3\,\cos\left(bx+a\right)^2\log\left(-\sin\left(bx+a\right)+1\right)-2\left(2\,\cos\left(bx+a\right)^2+1\right)\,\mathrm{s}}{4\,b\cos\left(bx+a\right)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^2*log(-sin(b*
x + a) + 1) - 2*(2*cos(b*x + a)^2 + 1)*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = \int \sin^4(a+bx)\sec^3(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**4,x)
[Out] Integral(sin(a + b*x)**4*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + 3\log(\sin(bx+a)+1) - 3\log(\sin(bx+a)-1) - 4\sin(bx+a)}{4b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(sin(b*x + a) + 1) - 3*log
(sin(b*x + a) - 1) - 4*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + 3\log(|\sin(bx+a)+1|) - 3\log(|\sin(bx+a)-1|) - 4\sin(bx+a)}{4b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(abs(sin(b*x + a) + 1)) -
3*log(abs(sin(b*x + a) - 1)) - 4*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b} - \frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(-\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^3,x)
```

```
[Out] - (3*atanh(tan(a/2 + (b*x)/2)))/b - (3*tan(a/2 + (b*x)/2) - 2*tan(a/2 + (b*
x)/2)^3 + 3*tan(a/2 + (b*x)/2)^5)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b*x
)/2)^4 - tan(a/2 + (b*x)/2)^6 - 1))
```

3.95 $\int \sec(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \sec(a+bx)\tan^4(a+bx)\,dx=\frac{3\text{arctanh}(\sin(a+bx))}{8b}-\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec(a+bx)\tan^3(a+bx)}{4b}$$

```
[Out] 3/8*arctanh(sin(b*x+a))/b-3/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)*tan(b*
x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \sec(a+bx)\tan^4(a+bx)\,dx=\frac{3\text{arctanh}(\sin(a+bx))}{8b}+\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) - (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]*Tan[a + b*x]^3)/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sec(a+bx)\tan^3(a+bx)}{4b} - \frac{3}{4}\int \sec(a+bx)\tan^2(a+bx)\,dx \\ &= -\frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b} + \frac{3}{8}\int \sec(a+bx)\,dx \\ &= \frac{3\text{arctanh}(\sin(a+bx))}{8b} - \frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{8b} + \frac{3\sec(a+bx)\tan(a+bx)}{8b} - \frac{3\sec^3(a+bx)\tan(a+bx)}{4b} + \frac{\sec(a+bx)\tan^3(a+bx)}{b}$$

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^4,x]
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) - (3*
Sec[a + b*x]^3*Tan[a + b*x])/(4*b) + (Sec[a + b*x]*Tan[a + b*x]^3)/b
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{4\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{8\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{8}-\frac{3\sin(bx+a)}{8}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{\sin^5(bx+a)}{4\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{8\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{8}-\frac{3\sin(bx+a)}{8}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$\frac{i\left(5\,e^{7i(bx+a)}-3\,e^{5i(bx+a)}+3\,e^{3i(bx+a)}-5\,e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4}+\frac{3\ln\left(e^{i(bx+a)}+i\right)}{8b}-\frac{3\ln\left(e^{i(bx+a)}-i\right)}{8b}$
norman	$\frac{-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{11\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{11\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{3\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{8b}$
parallelrisch	$\frac{(-12\cos(2bx+2a)-3\cos(4bx+4a)-9)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(12\cos(2bx+2a)+3\cos(4bx+4a)+9)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-}{8b(\cos(4bx+4a)+4\cos(2bx+2a)+3)}$

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*sin(b*x+a)^5/cos(b*x+a)^4-1/8*sin(b*x+a)^5/cos(b*x+a)^2-1/8*sin(b*
x+a)^3-3/8*sin(b*x+a)+3/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{3\cos(bx+a)^4\log(\sin(bx+a)+1)-3\cos(bx+a)^4\log(-\sin(bx+a)+1)-2\left(5\cos(bx+a)^2-2\right)\mathrm{si}}{16\,b\cos(bx+a)^4}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b*
x + a) + 1) - 2*(5*cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}+3\log\left(\sin\left(bx+a\right)+1\right)-3\log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")

[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) + 3*log(sin(b*x + a) + 1) - 3*log(sin(b*x + a) - 1))/b

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")

[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 3*log(abs(sin(b*x + a) + 1)) - 3*log(abs(sin(b*x + a) - 1)))/b

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{4\,b} - \frac{\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{4}-\frac{11\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{4}-\frac{11\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{4}+\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{4}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^5,x)
```

```
[Out] (3*atanh(tan(a/2 + (b*x)/2)))/(4*b) - ((3*tan(a/2 + (b*x)/2))/4 - (11*tan(a
/2 + (b*x)/2)^3)/4 - (11*tan(a/2 + (b*x)/2)^5)/4 + (3*tan(a/2 + (b*x)/2)^7)
/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (b*x)/2)^2 - 4*tan(a/2 + (b*x)
/2)^6 + tan(a/2 + (b*x)/2)^8 + 1))
```

3.96 $\int \sec^3(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{16b} + \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b-1/8*sec(b*x+a)^3*ta
n(b*x+a)/b+1/6*sec(b*x+a)^3*tan(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2691, 3853, 3855}

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{16b} + \frac{\tan^3(a+bx)\sec^3(a+bx)}{6b} - \frac{\tan(a+bx)\sec^3(a+bx)}{8b} + \frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
+ b*x]^3*Tan[a + b*x])/(8*b) + (Sec[a + b*x]^3*Tan[a + b*x]^3)/(6*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b} - \frac{1}{2}\int \sec^3(a+bx)\tan^2(a+bx)\,dx\\ &= -\frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b} + \frac{1}{8}\int \sec^3(a+bx)\,dx\\ &= \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{8b}\\ &\quad + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b} + \frac{1}{16}\int \sec(a+bx)\,dx\\ &= \frac{\operatorname{arctanh}(\sin(a+bx))}{16b} + \frac{\sec(a+bx)\tan(a+bx)}{16b}\\ &\quad - \frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \sec^3(a+bx)\tan^4(a+bx)\,dx &= \frac{\operatorname{arctanh}(\sin(a+bx))}{16b} + \frac{\sec(a+bx)\tan(a+bx)}{16b}\\ &\quad + \frac{\sec^3(a+bx)\tan(a+bx)}{24b} - \frac{\sec^5(a+bx)\tan(a+bx)}{6b}\\ &\quad + \frac{\sec^3(a+bx)\tan^3(a+bx)}{3b}\end{aligned}$$

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) + (Sec[a
+ b*x]^3*Tan[a + b*x])/(24*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (Sec[
a + b*x]^3*Tan[a + b*x]^3)/(3*b)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{6\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{24\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{48\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{48}-\frac{\sin(bx+a)}{16}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{\frac{\sin^5(bx+a)}{6\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{24\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{48\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{48}-\frac{\sin(bx+a)}{16}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$-\frac{i\left(3\,\mathrm{e}^{11i(bx+a)}-47\,\mathrm{e}^{9i(bx+a)}+78\,\mathrm{e}^{7i(bx+a)}-78\,\mathrm{e}^{5i(bx+a)}+47\,\mathrm{e}^{3i(bx+a)}-3\,\mathrm{e}^{i(bx+a)}\right)}{24b\left(\mathrm{e}^{2i(bx+a)}+1\right)^6}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{16b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-\right.}{16b}$
norman	$\frac{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{17\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{19\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{19\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{17\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}-\frac{\ln\left(\tan\left(\frac{b}{}\right.\right.}{1}$
parallelrisch	$\frac{(-45\cos(2bx+2a)-18\cos(4bx+4a)-3\cos(6bx+6a)-30)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(3\cos(6bx+6a)+18\cos(4bx+4a)+45\cos(}{48b(\cos(6bx+6a)+6\cos(4bx+4a)+15\cos(2bx+}$

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*sin(b*x+a)^5/cos(b*x+a)^6+1/24*sin(b*x+a)^5/cos(b*x+a)^4-1/48*sin(
b*x+a)^5/cos(b*x+a)^2-1/48*sin(b*x+a)^3-1/16*sin(b*x+a)+1/16*ln(sec(b*x+a)+
tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx$$
$$=\frac{3\cos(bx+a)^6\log(\sin(bx+a)+1)-3\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(3\cos(bx+a)^4-14\,\mathrm{co}\right.}{96\,b\cos(bx+a)^6}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b*
x + a) + 1) + 2*(3*cos(b*x + a)^4 - 14*cos(b*x + a)^2 + 8)*sin(b*x + a))/(b
*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\,\left(3\,\sin(bx+a)^5+8\,\sin(bx+a)^3-3\,\sin(bx+a)\right)}{\sin(bx+a)^6-3\,\sin(bx+a)^4+3\,\sin(bx+a)^2-1}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)}{96\,b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\,\left(3\,\sin(bx+a)^5+8\,\sin(bx+a)^3-3\,\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3}-3\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{96\,b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/
b
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.27

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8\,b} + \frac{-\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{8}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{24}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{4}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{4}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}}{24}-\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}-6\tan\left(\right.\right.}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^7,x)
```

```
[Out] atanh(tan(a/2 + (b*x)/2))/(8*b) + ((17*tan(a/2 + (b*x)/2)^3)/24 - tan(a/2 +
 (b*x)/2)/8 + (19*tan(a/2 + (b*x)/2)^5)/4 + (19*tan(a/2 + (b*x)/2)^7)/4 + (
17*tan(a/2 + (b*x)/2)^9)/24 - tan(a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 + (b*
x)/2)^4 - 6*tan(a/2 + (b*x)/2)^2 - 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2 + (
b*x)/2)^8 - 6*tan(a/2 + (b*x)/2)^10 + tan(a/2 + (b*x)/2)^12 + 1))
```

3.97 $\int \sec^5(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b}$$

```
[Out] 3/128*arctanh(sin(b*x+a))/b+3/128*sec(b*x+a)*tan(b*x+a)/b+1/64*sec(b*x+a)^3
*tan(b*x+a)/b-1/16*sec(b*x+a)^5*tan(b*x+a)/b+1/8*sec(b*x+a)^5*tan(b*x+a)^3/
b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2691, 3853, 3855}

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{128b} + \frac{\tan^3(a+bx)\sec^5(a+bx)}{8b} - \frac{\tan(a+bx)\sec^5(a+bx)}{16b} + \frac{\tan(a+bx)\sec^3(a+bx)}{64b} + \frac{3\tan(a+bx)\sec(a+bx)}{128b}$$

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b) +
 (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(16*b
) + (Sec[a + b*x]^5*Tan[a + b*x]^3)/(8*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} - \frac{3}{8}\int \sec^5(a+bx)\tan^2(a+bx)\,dx \\
&= -\frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} + \frac{1}{16}\int \sec^5(a+bx)\,dx \\
&= \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} \\
&\quad + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} + \frac{3}{64}\int \sec^3(a+bx)\,dx \\
&= \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} \\
&\quad + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} + \frac{3}{128}\int \sec(a+bx)\,dx \\
&= \frac{3\text{arctanh}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} \\
&\quad - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{128b}+\frac{3\sec(a+bx)\tan(a+bx)}{128b}+\frac{\sec^3(a+bx)\tan(a+bx)}{64b}+\frac{\sec^5(a+bx)\tan(a+bx)}{80b}-\frac{3\sec^7(a+bx)\tan(a+bx)}{40b}+\frac{\sec^5(a+bx)\tan^3(a+bx)}{5b}$$

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b) +
 (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(80*b
) - (3*Sec[a + b*x]^7*Tan[a + b*x])/(40*b) + (Sec[a + b*x]^5*Tan[a + b*x]^3
)/(5*b)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\sin^5(bx+a)}{8\cos(bx+a)^8}+\frac{\sin^5(bx+a)}{16\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{64\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{128\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{128}-\frac{3\sin(bx+a)}{128}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{128}}{b}$
default	$\frac{\frac{\sin^5(bx+a)}{8\cos(bx+a)^8}+\frac{\sin^5(bx+a)}{16\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{64\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{128\cos(bx+a)^2}-\frac{\left(\sin^3(bx+a)\right)}{128}-\frac{3\sin(bx+a)}{128}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{128}}{b}$
risch	$-\frac{i\left(3\,\mathrm{e}^{15i(bx+a)}+23\,\mathrm{e}^{13i(bx+a)}-333\,\mathrm{e}^{11i(bx+a)}+671\,\mathrm{e}^{9i(bx+a)}-671\,\mathrm{e}^{7i(bx+a)}+333\,\mathrm{e}^{5i(bx+a)}-23\,\mathrm{e}^{3i(bx+a)}-3\,\mathrm{e}^{i(bx+a)}\right)}{64b\left(\mathrm{e}^{2i(bx+a)}+1\right)^8}$
norman	$\frac{-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{23\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{333\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{671\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{671\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{333\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^8}$
parallelrisch	$\frac{(-168\cos(2bx+2a)-84\cos(4bx+4a)-24\cos(6bx+6a)-3\cos(8bx+8a)-105)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(168\cos(2bx+2a)+8}{128b(\cos(8bx+8a)+8\cos}$

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*sin(b*x+a)^5/cos(b*x+a)^8+1/16*sin(b*x+a)^5/cos(b*x+a)^6+1/64*sin(
b*x+a)^5/cos(b*x+a)^4-1/128*sin(b*x+a)^5/cos(b*x+a)^2-1/128*sin(b*x+a)^3-3/
128*sin(b*x+a)+3/128*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\,\cos(bx+a)^8\log(\sin(bx+a)+1)-3\,\cos(bx+a)^8\log(-\sin(bx+a)+1)+2\left(3\,\cos(bx+a)^6+2\,\cos\right.}{256\,b\cos(bx+a)^8}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/256*(3*cos(b*x + a)^8*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^8*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^6 + 2*cos(b*x + a)^4 - 24*cos(b*x + a)^2 +
 16)*sin(b*x + a))/(b*cos(b*x + a)^8)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\left(3\,\sin(bx+a)^7-11\,\sin(bx+a)^5-11\,\sin(bx+a)^3+3\,\sin(bx+a)\right)}{\sin(bx+a)^8-4\,\sin(bx+a)^6+6\,\sin(bx+a)^4-4\,\sin(bx+a)^2+1}-3\,\log(\sin(bx+a)+1)+3\,\log(\sin(bx+a)-1)}{256\,b}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/256*(2*(3*sin(b*x + a)^7 - 11*sin(b*x + a)^5 - 11*sin(b*x + a)^3 + 3*sin
(b*x + a))/(sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*
x + a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{4\left(3\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^3-\frac{20}{\sin(bx+a)}-20\sin(bx+a)\right)}{\left(\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4\right)^2} - 3\log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)+2\right|\right) + 3\log\left(\left|\frac{1}{\sin(bx+a)}+\right.\right.}{512\,b}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*(4*(3*(1/sin(b*x + a) + sin(b*x + a))^3 - 20/sin(b*x + a) - 20*sin(b
*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2 - 4)^2 - 3*log(abs(1/sin(b*x +
a) + sin(b*x + a) + 2)) + 3*log(abs(1/sin(b*x + a) + sin(b*x + a) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.31

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{64\,b} + \frac{-\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{15}}{64}+\frac{23\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{13}}{64}+\frac{333\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{64}+\frac{671\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{64}+\frac{671\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{64}+\frac{333}{}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{16}-8\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{14}+28\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-56\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+70\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-56\right.}$$

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^9,x)
```

```
[Out] (3*atanh(tan(a/2 + (b*x)/2)))/(64*b) + ((23*tan(a/2 + (b*x)/2)^3)/64 - (3*t
an(a/2 + (b*x)/2))/64 + (333*tan(a/2 + (b*x)/2)^5)/64 + (671*tan(a/2 + (b*x
)/2)^7)/64 + (671*tan(a/2 + (b*x)/2)^9)/64 + (333*tan(a/2 + (b*x)/2)^11)/64
 + (23*tan(a/2 + (b*x)/2)^13)/64 - (3*tan(a/2 + (b*x)/2)^15)/64)/(b*(28*tan
(a/2 + (b*x)/2)^4 - 8*tan(a/2 + (b*x)/2)^2 - 56*tan(a/2 + (b*x)/2)^6 + 70*t
an(a/2 + (b*x)/2)^8 - 56*tan(a/2 + (b*x)/2)^10 + 28*tan(a/2 + (b*x)/2)^12 -
 8*tan(a/2 + (b*x)/2)^14 + tan(a/2 + (b*x)/2)^16 + 1))
```

3.98 $\int \cos^7(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx=-\frac{\cos^8(a+bx)}{8b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^{12}(a+bx)}{12b}$$

```
[Out] -1/8*cos(b*x+a)^8/b+1/5*cos(b*x+a)^10/b-1/12*cos(b*x+a)^12/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2645, 272, 45}

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx=-\frac{\cos^{12}(a+bx)}{12b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^8(a+bx)}{8b}$$

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

```
[Out] -1/8*Cos[a + b*x]^8/b + Cos[a + b*x]^10/(5*b) - Cos[a + b*x]^12/(12*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^7 (1-x^2)^2 \, dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int (1-x)^2 x^3 \, dx, x, \cos^2(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int (x^3 - 2x^4 + x^5) \, dx, x, \cos^2(a+bx)\right)}{2b} \\
&= -\frac{\cos^8(a+bx)}{8b} + \frac{\cos^{10}(a+bx)}{5b} - \frac{\cos^{12}(a+bx)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{600\cos(2(a+bx)) + 75\cos(4(a+bx)) - 100\cos(6(a+bx)) - 30\cos(8(a+bx)) + 12\cos(10(a+bx))}{122880b}$$

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
[Out] -1/122880*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)
] - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	siz
derivativedivides	$-\frac{\frac{\left(\sin^{12}(bx+a)\right)}{12}-\frac{3\left(\sin^{10}(bx+a)\right)}{10}+\frac{3\left(\sin^{8}(bx+a)\right)}{8}-\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	47
default	$-\frac{\frac{\left(\sin^{12}(bx+a)\right)}{12}-\frac{3\left(\sin^{10}(bx+a)\right)}{10}+\frac{3\left(\sin^{8}(bx+a)\right)}{8}-\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	47
parallelrisch	$\frac{30\cos(8bx+8a)+100\cos(6bx+6a)-600\cos(2bx+2a)-75\cos(4bx+4a)-5\cos(12bx+12a)-12\cos(10bx+10a)+562}{122880b}$	74
risch	$-\frac{\cos(12bx+12a)}{24576b}-\frac{\cos(10bx+10a)}{10240b}+\frac{\cos(8bx+8a)}{4096b}+\frac{5\cos(6bx+6a)}{6144b}-\frac{5\cos(4bx+4a)}{8192b}-\frac{5\cos(2bx+2a)}{1024b}$	86

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/12*sin(b*x+a)^12-3/10*sin(b*x+a)^10+3/8*sin(b*x+a)^8-1/6*sin(b*x+a)
^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{10\,\cos\left(bx+a\right)^{12}-24\,\cos\left(bx+a\right)^{10}+15\,\cos\left(bx+a\right)^{8}}{120\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/120*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^8(a+bx)}{8b}-\frac{\sin^2(a+bx)\cos^{10}(a+bx)}{20b}-\frac{\cos^{12}(a+bx)}{120b} & \text{for } b\neq 0 \\ x\sin^5(a)\cos^7(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**8/(8*b) - sin(a + b*x)**2*cos(a +
 b*x)**10/(20*b) - cos(a + b*x)**12/(120*b), Ne(b, 0)), (x*sin(a)**5*cos(a)
**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{10\,\sin\left(bx+a\right)^{12}-36\,\sin\left(bx+a\right)^{10}+45\,\sin\left(bx+a\right)^{8}-20\,\sin\left(bx+a\right)^{6}}{120\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/120*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*si
n(b*x + a)^6)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.85

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos\left(12\,bx+12\,a\right)}{24576\,b}-\frac{\cos\left(10\,bx+10\,a\right)}{10240\,b}+\frac{\cos\left(8\,bx+8\,a\right)}{4096\,b}+\frac{5\,\cos\left(6\,bx+6\,a\right)}{6144\,b}-\frac{5\,\cos\left(4\,bx+4\,a\right)}{8192\,b}-\frac{5\,\cos\left(2\,bx+2\,a\right)}{1024\,b}$$

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
[Out] -1/24576*cos(12*b*x + 12*a)/b - 1/10240*cos(10*b*x + 10*a)/b + 1/4096*cos(8
*b*x + 8*a)/b + 5/6144*cos(6*b*x + 6*a)/b - 5/8192*cos(4*b*x + 4*a)/b - 5/1
024*cos(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{{\cos\left(a+b\,x\right)}^8\,\left(10\,{\cos\left(a+b\,x\right)}^4-24\,{\cos\left(a+b\,x\right)}^2+15\right)}{120\,b}$$

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^5,x)
[Out] -(cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(120*b)
```

3.99 $\int \cos^6(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^{11}(a+bx)}{11b}$$

```
[Out] -1/7*cos(b*x+a)^7/b+2/9*cos(b*x+a)^9/b-1/11*cos(b*x+a)^11/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 276}

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^{11}(a+bx)}{11b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^7(a+bx)}{7b}$$

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

```
[Out] -1/7*Cos[a + b*x]^7/b + (2*Cos[a + b*x]^9)/(9*b) - Cos[a + b*x]^11/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^6(1-x^2)^2\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int (x^6-2x^8+x^{10})\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^{11}(a+bx)}{11b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = \frac{\cos^7(a+bx)(-365+364\cos(2(a+bx))-63\cos(4(a+bx)))}{5544b}$$

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
[Out] (Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(5544*
b)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\frac{\left(\cos^{11}(bx+a)\right)}{11}-\frac{2\left(\cos^{9}(bx+a)\right)}{9}+\frac{\left(\cos^{7}(bx+a)\right)}{7}}{b}$
default	$-\frac{\frac{\left(\cos^{11}(bx+a)\right)}{11}-\frac{2\left(\cos^{9}(bx+a)\right)}{9}+\frac{\left(\cos^{7}(bx+a)\right)}{7}}{b}$
parallelrisch	$\frac{-8192-63\cos(11bx+11a)-77\cos(9bx+9a)-6930\cos(bx+a)+693\cos(5bx+5a)-2310\cos(3bx+3a)+495\cos(7bx+7a)}{709632b}$
risch	$-\frac{5\cos(bx+a)}{512b}-\frac{\cos(11bx+11a)}{11264b}-\frac{\cos(9bx+9a)}{9216b}+\frac{5\cos(7bx+7a)}{7168b}+\frac{\cos(5bx+5a)}{1024b}-\frac{5\cos(3bx+3a)}{1536b}$

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/11*cos(b*x+a)^11-2/9*cos(b*x+a)^9+1/7*cos(b*x+a)^7)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(bx+a)^{11} - 154\cos(bx+a)^9 + 99\cos(bx+a)^7}{693\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^7(a+bx)}{7b} - \frac{4\sin^2(a+bx)\cos^9(a+bx)}{63b} - \frac{8\cos^{11}(a+bx)}{693b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^6(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**7/(7*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**9/(63*b) - 8*cos(a + b*x)**11/(693*b), Ne(b, 0)), (x*sin(a)**5*cos
(a)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(bx+a)^{11} - 154\cos(bx+a)^9 + 99\cos(bx+a)^7}{693\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos(11\,bx+11\,a)}{11264\,b} - \frac{\cos(9\,bx+9\,a)}{9216\,b} + \frac{5\,\cos(7\,bx+7\,a)}{7168\,b} + \frac{\cos(5\,bx+5\,a)}{1024\,b} - \frac{5\,\cos(3\,bx+3\,a)}{1536\,b} - \frac{5\,\cos(bx+a)}{512\,b}$$

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/11264*cos(11*b*x + 11*a)/b - 1/9216*cos(9*b*x + 9*a)/b + 5/7168*cos(7*b*
x + 7*a)/b + 1/1024*cos(5*b*x + 5*a)/b - 5/1536*cos(3*b*x + 3*a)/b - 5/512*
cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(a+b\,x)^{11} - 154\cos(a+b\,x)^9 + 99\cos(a+b\,x)^7}{693\,b}$$

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^5,x)
```

```
[Out] -(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11)/(693*b)
```

3.100 $\int \cos^5(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b} - \frac{\sin^8(a+bx)}{4b} + \frac{\sin^{10}(a+bx)}{10b}$$

```
[Out] 1/6*sin(b*x+a)^6/b-1/4*sin(b*x+a)^8/b+1/10*sin(b*x+a)^10/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2644, 272, 45}

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^{10}(a+bx)}{10b} - \frac{\sin^8(a+bx)}{4b} + \frac{\sin^6(a+bx)}{6b}$$

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(4*b) + Sin[a + b*x]^10/(10*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^5(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (1-x)^2x^2\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int (x^2-2x^3+x^4)\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^{10}(a+bx)}{10b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{1}{32}\left(-\frac{5\cos(2(a+bx))}{16b}+\frac{5\cos(6(a+bx))}{96b}-\frac{\cos(10(a+bx))}{160b}\right)$$

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
[Out] ((-5*Cos[2*(a + b*x)])/(16*b) + (5*Cos[6*(a + b*x)])/(96*b) - Cos[10*(a + b
*x)]/(160*b))/32
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^{10}(bx+a)\right)}{10}-\frac{\left(\sin^{8}(bx+a)\right)}{4}+\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	36
default	$\frac{\frac{\left(\sin^{10}(bx+a)\right)}{10}-\frac{\left(\sin^{8}(bx+a)\right)}{4}+\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	36
parallelrisch	$\frac{128+25\cos(6bx+6a)-150\cos(2bx+2a)-3\cos(10bx+10a)}{15360b}$	41
risch	$-\frac{\cos(10bx+10a)}{5120b}+\frac{5\cos(6bx+6a)}{3072b}-\frac{5\cos(2bx+2a)}{512b}$	44

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/10*sin(b*x+a)^10-1/4*sin(b*x+a)^8+1/6*sin(b*x+a)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{6\,\cos\left(bx+a\right)^{10}-15\,\cos\left(bx+a\right)^{8}+10\,\cos\left(bx+a\right)^{6}}{60\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/60*(6*cos(b*x + a)^10 - 15*cos(b*x + a)^8 + 10*cos(b*x + a)^6)/b
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^6(a+bx)}{6b}-\frac{\sin^2(a+bx)\cos^8(a+bx)}{12b}-\frac{\cos^{10}(a+bx)}{60b} & \text{for } b\neq 0 \\ x\sin^5(a)\cos^5(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**6/(6*b) - sin(a + b*x)**2*cos(a +
 b*x)**8/(12*b) - cos(a + b*x)**10/(60*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**
5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{6\sin(bx+a)^{10} - 15\sin(bx+a)^8 + 10\sin(bx+a)^6}{60\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/60*(6*sin(b*x + a)^10 - 15*sin(b*x + a)^8 + 10*sin(b*x + a)^6)/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos(10\,bx+10\,a)}{5120\,b} + \frac{5\cos(6\,bx+6\,a)}{3072\,b} - \frac{5\cos(2\,bx+2\,a)}{512\,b}$$

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
[Out] -1/5120*cos(10*b*x + 10*a)/b + 5/3072*cos(6*b*x + 6*a)/b - 5/512*cos(2*b*x
+ 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^{10}}{10} - \frac{\cos(a+b\,x)^8}{4} + \frac{\cos(a+b\,x)^6}{6}}{b}$$

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^5,x)
[Out] -(cos(a + b*x)^6/6 - cos(a + b*x)^8/4 + cos(a + b*x)^10/10)/b
```

3.101 $\int \cos^4(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^9(a+bx)}{9b}$$

```
[Out] -1/5*cos(b*x+a)^5/b+2/7*cos(b*x+a)^7/b-1/9*cos(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 276}

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^9(a+bx)}{9b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

```
[Out] -1/5*Cos[a + b*x]^5/b + (2*Cos[a + b*x]^7)/(7*b) - Cos[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x^4(1-x^2)^2\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int (x^4-2x^6+x^8)\ dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^9(a+bx)}{9b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx=\frac{\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{2520b}$$

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(2520*
b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\frac{\left(\cos^9(bx+a)\right)}{9}-\frac{2\left(\cos^7(bx+a)\right)}{7}+\frac{\left(\cos^5(bx+a)\right)}{5}}{b}$
default	$-\frac{\frac{\left(\cos^9(bx+a)\right)}{9}-\frac{2\left(\cos^7(bx+a)\right)}{7}+\frac{\left(\cos^5(bx+a)\right)}{5}}{b}$
parallelrisch	$\frac{-2048-35\cos(9bx+9a)-1890\cos(bx+a)+252\cos(5bx+5a)-420\cos(3bx+3a)+45\cos(7bx+7a)}{80640b}$
risch	$-\frac{3\cos(bx+a)}{128b}-\frac{\cos(9bx+9a)}{2304b}+\frac{\cos(7bx+7a)}{1792b}+\frac{\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{192b}$
norman	$\frac{-\frac{16}{315b}-\frac{112\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{32\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{16\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{64\left(\tan^4\left(\frac{bx}{2}\right.\right.}{35b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^9}$

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/9*cos(b*x+a)^9-2/7*cos(b*x+a)^7+1/5*cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\,\cos{(bx+a)}^9 - 90\,\cos{(bx+a)}^7 + 63\,\cos{(bx+a)}^5}{315\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^5(a+bx)}{5b} - \frac{4\sin^2(a+bx)\cos^7(a+bx)}{35b} - \frac{8\cos^9(a+bx)}{315b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^4(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**5/(5*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**7/(35*b) - 8*cos(a + b*x)**9/(315*b), Ne(b, 0)), (x*sin(a)**5*cos(
a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\,\cos{(bx+a)}^9 - 90\,\cos{(bx+a)}^7 + 63\,\cos{(bx+a)}^5}{315\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos(9\,bx+9\,a)}{2304\,b} + \frac{\cos(7\,bx+7\,a)}{1792\,b} + \frac{\cos(5\,bx+5\,a)}{320\,b} - \frac{\cos(3\,bx+3\,a)}{192\,b} - \frac{3\,\cos(bx+a)}{128\,b}$$

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/2304*cos(9*b*x + 9*a)/b + 1/1792*cos(7*b*x + 7*a)/b + 1/320*cos(5*b*x +
5*a)/b - 1/192*cos(3*b*x + 3*a)/b - 3/128*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\cos(a+b\,x)^9 - 90\cos(a+b\,x)^7 + 63\cos(a+b\,x)^5}{315\,b}$$

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^5,x)
```

```
[Out] -(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9)/(315*b)
```

3.102 $\int \cos^3(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b} - \frac{\sin^8(a+bx)}{8b}$$

```
[Out] 1/6*sin(b*x+a)^6/b-1/8*sin(b*x+a)^8/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 14}

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b} - \frac{\sin^8(a+bx)}{8b}$$

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^5(1-x^2)\ dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^5-x^7)\ dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{3072b}$$

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(3072*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\left(\sin^{8}(bx+a)\right)}{8}+\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	26
default	$\frac{-\frac{\left(\sin^{8}(bx+a)\right)}{8}+\frac{\left(\sin^{6}(bx+a)\right)}{6}}{b}$	26
parallelrisch	$\frac{12\cos(4bx+4a)-3\cos(8bx+8a)-72\cos(2bx+2a)+55+8\cos(6bx+6a)}{3072b}$	52
risch	$-\frac{\cos(8bx+8a)}{1024b}+\frac{\cos(6bx+6a)}{384b}+\frac{\cos(4bx+4a)}{256b}-\frac{3\cos(2bx+2a)}{128b}$	58
norman	$\frac{\frac{32\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{32\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{32\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{8}}$	66

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/8*sin(b*x+a)^8+1/6*sin(b*x+a)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\cos(bx+a)^8 - 8\cos(bx+a)^6 + 6\cos(bx+a)^4}{24\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/24*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^4(a+bx)}{4b} - \frac{\sin^2(a+bx)\cos^6(a+bx)}{6b} - \frac{\cos^8(a+bx)}{24b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**4/(4*b) - sin(a + b*x)**2*cos(a +
 b*x)**6/(6*b) - cos(a + b*x)**8/(24*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**3,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\sin(bx+a)^8 - 4\sin(bx+a)^6}{24\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^8 - 4\,\sin\left(bx+a\right)^6}{24\,b}$$

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{4\sin\left(a+b\,x\right)^6 - 3\sin\left(a+b\,x\right)^8}{24\,b}$$

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^5,x)
[Out] (4*sin(a + b*x)^6 - 3*sin(a + b*x)^8)/(24*b)
```

3.103 $\int \cos^2(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx=-\frac{\cos^3(a+bx)}{3b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^7(a+bx)}{7b}$$

```
[Out] -1/3*cos(b*x+a)^3/b+2/5*cos(b*x+a)^5/b-1/7*cos(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 276}

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx=-\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
[Out] -1/3*Cos[a + b*x]^3/b + (2*Cos[a + b*x]^5)/(5*b) - Cos[a + b*x]^7/(7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^2(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = \frac{\cos^3(a+bx)(-157+108\cos(2(a+bx))-15\cos(4(a+bx)))}{840b}$$

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(840*b
)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\frac{\left(\cos^7(bx+a)\right)}{7}-\frac{2\left(\cos^5(bx+a)\right)}{5}+\frac{\left(\cos^3(bx+a)\right)}{3}}{b}$	37
default	$-\frac{\frac{\left(\cos^7(bx+a)\right)}{7}-\frac{2\left(\cos^5(bx+a)\right)}{5}+\frac{\left(\cos^3(bx+a)\right)}{3}}{b}$	37
parallelrisch	$\frac{-512-525\cos(bx+a)-35\cos(3bx+3a)+63\cos(5bx+5a)-15\cos(7bx+7a)}{6720b}$	49
risch	$-\frac{5\cos(bx+a)}{64b}-\frac{\cos(7bx+7a)}{448b}+\frac{3\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{192b}$	55
norman	$\frac{-\frac{16}{105b}-\frac{32\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}-\frac{16\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+\frac{16\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^7}$	87

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/b*(1/7*cos(b*x+a)^7-2/5*cos(b*x+a)^5+1/3*cos(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\,\cos\left(bx+a\right)^7 - 42\,\cos\left(bx+a\right)^5 + 35\,\cos\left(bx+a\right)^3}{105\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^3(a+bx)}{3b} - \frac{4\sin^2(a+bx)\cos^5(a+bx)}{15b} - \frac{8\cos^7(a+bx)}{105b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**3/(3*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**5/(15*b) - 8*cos(a + b*x)**7/(105*b), Ne(b, 0)), (x*sin(a)**5*cos(
a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\,\cos\left(bx+a\right)^7 - 42\,\cos\left(bx+a\right)^5 + 35\,\cos\left(bx+a\right)^3}{105\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos(7\,bx+7\,a)}{448\,b} + \frac{3\,\cos(5\,bx+5\,a)}{320\,b} - \frac{\cos(3\,bx+3\,a)}{192\,b} - \frac{5\,\cos(bx+a)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/448*cos(7*b*x + 7*a)/b + 3/320*cos(5*b*x + 5*a)/b - 1/192*cos(3*b*x + 3*
a)/b - 5/64*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\cos(a+b\,x)^7 - 42\cos(a+b\,x)^5 + 35\cos(a+b\,x)^3}{105\,b}$$

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^5,x)
```

```
[Out] -(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7)/(105*b)
```

3.104 $\int \cos(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b}$$

```
[Out] 1/6*sin(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2644, 30}

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b}$$

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^5\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin^6(a+bx)}{6b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin^6(a+bx)}{6b}$$

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^5,x]
[Out] Sin[a + b*x]^6/(6*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin^6(bx+a)}{6b}$	14
default	$\frac{\sin^6(bx+a)}{6b}$	14
norman	$\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^6}$	32
parallelrisch	$\frac{6\cos(4bx+4a)+10-15\cos(2bx+2a)-\cos(6bx+6a)}{192b}$	41
risch	$-\frac{\cos(6bx+6a)}{192b}+\frac{\cos(4bx+4a)}{32b}-\frac{5\cos(2bx+2a)}{64b}$	44

```
[In] int(cos(b*x+a)*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/6*sin(b*x+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos\left(bx+a\right)^6 - 3\,\cos\left(bx+a\right)^4 + 3\,\cos\left(bx+a\right)^2}{6\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/6*(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \begin{cases} \frac{\sin^6(a+bx)}{6b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**5,x)
[Out] Piecewise((sin(a + b*x)**6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin\left(bx+a\right)^6}{6\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/6*sin(b*x + a)^6/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin\left(bx+a\right)^6}{6\,b}$$

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
[Out] 1/6*sin(b*x + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin{(a+b\,x)}^6}{6\,b}$$

```
[In] int(cos(a + b*x)*sin(a + b*x)^5,x)
[Out] sin(a + b*x)^6/(6*b)
```

3.105 $\int \sin^4(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sin^4(a+bx)\tan(a+bx)\,dx=\frac{\cos^2(a+bx)}{b}-\frac{\cos^4(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] cos(b*x+a)^2/b-1/4*cos(b*x+a)^4/b-ln(cos(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 272, 45}

$$\int \sin^4(a+bx)\tan(a+bx)\,dx=-\frac{\cos^4(a+bx)}{4b}+\frac{\cos^2(a+bx)}{b}-\frac{\log(\cos(a+bx))}{b}$$

```
[In] Int[Sin[a + b*x]^4*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/b - Cos[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x} \, dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x} \, dx, x, \cos^2(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right) \, dx, x, \cos^2(a+bx)\right)}{2b} \\
&= \frac{\cos^2(a+bx)}{b} - \frac{\cos^4(a+bx)}{4b} - \frac{\log(\cos(a+bx))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{-\cos^2(a+bx)+\frac{1}{4}\cos^4(a+bx)+\log(\cos(a+bx))}{b}$$

```
[In] Integrate[Sin[a + b*x]^4*Tan[a + b*x],x]
[Out] -((-Cos[a + b*x]^2 + Cos[a + b*x]^4/4 + Log[Cos[a + b*x]])/b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{\left(\sin^{4}(bx+a)\right)}{4}-\frac{\left(\sin^{2}(bx+a)\right)}{2}-\ln(\cos(bx+a))}{b}$
default	$\frac{-\frac{\left(\sin^{4}(bx+a)\right)}{4}-\frac{\left(\sin^{2}(bx+a)\right)}{2}-\ln(\cos(bx+a))}{b}$
risch	$ix+\frac{3\,\mathrm{e}^{2i(bx+a)}}{16b}+\frac{3\,\mathrm{e}^{-2i(bx+a)}}{16b}+\frac{2ia}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}-\frac{\cos(4bx+4a)}{32b}$
parallelrisch	$\frac{-32\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-32\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+32\ln\left(\sec^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-11-\cos(4bx+4a)+12\cos(2bx+2a)}{32b}$
norman	$\frac{-\frac{2\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{4}}+\frac{\ln\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{b}$

```
[In] int(sec(b*x+a)*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/4*sin(b*x+a)^4-1/2*sin(b*x+a)^2-ln(cos(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\cos\left(bx+a\right)^{4}-4\,\cos\left(bx+a\right)^{2}+4\,\log\left(-\cos\left(bx+a\right)\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(cos(b*x + a)^4 - 4*cos(b*x + a)^2 + 4*log(-cos(b*x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\sin\left(bx+a\right)^4 + 2\,\sin\left(bx+a\right)^2 + 2\,\log\left(\sin\left(bx+a\right)^2 - 1\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/4*(sin(b*x + a)^4 + 2*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2 - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(38) = 76.

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.65

$$\int \sin^4(a+bx)\tan(a+bx)\,dx =$$

$$-\frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-\frac{20\,(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{20\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right)^2} - 2\,\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right) + 2\,\log}{4\,b}$$

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
[Out] -1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b
*x + a) + 1))^2 - 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - 20*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (
cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)^2 - 2*log(abs(-(cos(b*x + a) + 1)
/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) + 2*log(a
bs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} + \frac{\tan(a+b\,x)^2+\frac{3}{4}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x),x)
[Out] log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2 + 3/4)/(b*(2*tan(a + b*x)^2
 + tan(a + b*x)^4 + 1))
```

3.106 $\int \sin^3(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{2\cos(a+bx)}{b} - \frac{\cos^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b}$$

```
[Out] 2*cos(b*x+a)/b-1/3*cos(b*x+a)^3/b+sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos^3(a+bx)}{3b} + \frac{2\cos(a+bx)}{b} + \frac{\sec(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] (2*Cos[a + b*x])/b - Cos[a + b*x]^3/(3*b) + Sec[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{2\cos(a+bx)}{b}-\frac{\cos^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx=\frac{7\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{12b}+\frac{\sec(a+bx)}{b}$$

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
[Out] (7*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) + Sec[a + b*x]/b
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

method	result	size
parallelrisch	$\frac{20\cos(2bx+2a)+45-\cos(4bx+4a)+64\cos(bx+a)}{24b\cos(bx+a)}$	46
derivativedivides	$\frac{\frac{\sin^6(bx+a)}{\cos(bx+a)}+\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{b}$	50
default	$\frac{\frac{\sin^6(bx+a)}{\cos(bx+a)}+\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{b}$	50
norman	$\frac{-\frac{16}{3b}-\frac{32\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	54
risch	$\frac{7\,e^{i(bx+a)}}{8b}+\frac{7\,e^{-i(bx+a)}}{8b}+\frac{2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)}-\frac{\cos(3bx+3a)}{12b}$	71

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/24/b*(20*cos(2*b*x+2*a)+45-cos(4*b*x+4*a)+64*cos(b*x+a))/cos(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^4 - 6\cos(bx+a)^2 - 3}{3\,b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/3*(cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 3)/(b*cos(b*x + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**5,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^3 - \frac{3}{\cos(bx+a)} - 6\cos(bx+a)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/3*(cos(b*x + a)^3 - 3/cos(b*x + a) - 6*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(35) = 70$.

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.68

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + \frac{\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{(\cos(a+b\,x)+1)^3\,(\cos(a+b\,x)-3)}{3\,b\,\cos(a+b\,x)}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^2,x)
[Out] -((cos(a + b*x) + 1)^3*(cos(a + b*x) - 3))/(3*b*cos(a + b*x))
```

3.107 $\int \sin^2(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx=-\frac{\cos^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}$$

```
[Out] -1/2*cos(b*x+a)^2/b+2*ln(cos(b*x+a))/b+1/2*sec(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {2670, 272, 45}

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx=-\frac{\cos^2(a+bx)}{2b}+\frac{\sec^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}$$

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] -1/2*Cos[a + b*x]^2/b + (2*Log[Cos[a + b*x]])/b + Sec[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \cos^2(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \cos^2(a+bx)\right)}{2b}\\
&= -\frac{\cos^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \frac{4\log(\cos(a+bx))+\sec^2(a+bx)+\sin^2(a+bx)}{2b}$$

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
[Out] (4*Log[Cos[a + b*x]] + Sec[a + b*x]^2 + Sin[a + b*x]^2)/(2*b)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\sin^6(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^4(bx+a)\right)}{2}+\sin^2(bx+a)+2\ln(\cos(bx+a))}{b}$
default	$\frac{\frac{\sin^6(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^4(bx+a)\right)}{2}+\sin^2(bx+a)+2\ln(\cos(bx+a))}{b}$
risch	$-2ix-\frac{e^{2i(bx+a)}}{8b}-\frac{e^{-2i(bx+a)}}{8b}-\frac{4ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{2\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}+\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}+\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{2\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{b}$
parallelrisch	$\frac{5+16(-1-\cos(2bx+2a))\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+16(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+16(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\right.\right.}{8b(1+\cos(2bx+2a))}$

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sin(b*x+a)^6/cos(b*x+a)^2+1/2*sin(b*x+a)^4+sin(b*x+a)^2+2*ln(cos(b
*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = -\frac{2\cos(bx+a)^4-8\cos(bx+a)^2\log(-\cos(bx+a))-\cos(bx+a)^2-2}{4\,b\cos(bx+a)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(2*cos(b*x + a)^4 - 8*cos(b*x + a)^2*log(-cos(b*x + a)) - cos(b*x + a)
^2 - 2)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \frac{\sin(bx+a)^2 - \frac{1}{\sin(bx+a)^2-1} + 2\log\left(\sin(bx+a)^2-1\right)}{2b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/2*(sin(b*x + a)^2 - 1/(sin(b*x + a)^2 - 1) + 2*log(sin(b*x + a)^2 - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(39) = 78.

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.23

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx =$$
$$-\frac{\frac{4\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-4} + \log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right) - \log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right.\right.}{b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
[Out] -(4*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x +
a) + 1))/(((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(
b*x + a) + 1))^2 - 4) + log(abs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - log(abs(-(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = -\frac{\ln\left(\tan(a+b\,x)^2+1\right) + \frac{\cos(a+b\,x)^2}{2} - \frac{\tan(a+b\,x)^2}{2}}{b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^3,x)
[Out] -(log(tan(a + b*x)^2 + 1) + cos(a + b*x)^2/2 - tan(a + b*x)^2/2)/b
```

3.108 $\int \sin(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sin(a+bx)\tan^4(a+bx)\,dx=-\frac{\cos(a+bx)}{b}-\frac{2\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[Out] -cos(b*x+a)/b-2*sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 276}

$$\int \sin(a+bx)\tan^4(a+bx)\,dx=-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^4,x]
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}-\frac{2\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\cos(a+bx)}{b}-\frac{2\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^4,x]
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

method	result	size
norman	$\frac{\frac{16}{3b}-\frac{32\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3}$	54
parallelrisch	$\frac{-36\cos(2bx+2a)-25-3\cos(4bx+4a)-48\cos(bx+a)-16\cos(3bx+3a)}{6b(\cos(3bx+3a)+3\cos(bx+a))}$	69
derivativedivides	$\frac{\frac{\sin^6(bx+a)}{3\cos(bx+a)^3}-\frac{\sin^6(bx+a)}{\cos(bx+a)}-\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{b}$	70
default	$\frac{\frac{\sin^6(bx+a)}{3\cos(bx+a)^3}-\frac{\sin^6(bx+a)}{\cos(bx+a)}-\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{b}$	70
risch	$-\frac{3\,\mathrm{e}^{7i(bx+a)}+36\,\mathrm{e}^{5i(bx+a)}+50\,\mathrm{e}^{3i(bx+a)}+39\cos(bx+a)+33i\sin(bx+a)}{6b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3}$	70

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] (16/3/b-32/3/b*tan(1/2*b*x+1/2*a)^2)/(1+tan(1/2*b*x+1/2*a)^2)/(tan(1/2*b*x+
1/2*a)^2-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^4+6\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/3*(3*cos(b*x + a)^4 + 6*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{6\,\cos(bx+a)^2-1}{\cos(bx+a)^3}+3\,\cos\left(bx+a\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/3*((6*cos(b*x + a)^2 - 1)/cos(b*x + a)^3 + 3*cos(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(36) = 72.

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = \frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\frac{\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{3\cos\left(a+b\,x\right)^4+6\cos\left(a+b\,x\right)^2-1}{3\,b\cos\left(a+b\,x\right)^3}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^4,x)
[Out] -(6*cos(a + b*x)^2 + 3*cos(a + b*x)^4 - 1)/(3*b*cos(a + b*x)^3)
```

3.109 $\int \tan^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tan^5(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b} - \frac{\tan^2(a+bx)}{2b} + \frac{\tan^4(a+bx)}{4b}$$

```
[Out] -ln(cos(b*x+a))/b-1/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 3556}

$$\int \tan^5(a+bx)\,dx = \frac{\tan^4(a+bx)}{4b} - \frac{\tan^2(a+bx)}{2b} - \frac{\log(\cos(a+bx))}{b}$$

```
[In] Int[Tan[a + b*x]^5,x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) - Tan[a + b*x]^2/(2*b) + Tan[a + b*x]^4/(4*b)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^4(a+bx)}{4b} - \int \tan^3(a+bx)\,dx \\
&= -\frac{\tan^2(a+bx)}{2b} + \frac{\tan^4(a+bx)}{4b} + \int \tan(a+bx)\,dx \\
&= -\frac{\log(\cos(a+bx))}{b} - \frac{\tan^2(a+bx)}{2b} + \frac{\tan^4(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \tan^5(a+bx)\,dx = -\frac{4\log(\cos(a+bx)) + 2\tan^2(a+bx) - \tan^4(a+bx)}{4b}$$

```
[In] Integrate[Tan[a + b*x]^5,x]
[Out] -1/4*(4*Log[Cos[a + b*x]] + 2*Tan[a + b*x]^2 - Tan[a + b*x]^4)/b
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{(\tan^4(bx+a))}{4}-\frac{(\tan^2(bx+a))}{2}-\ln(\cos(bx+a))}{b}$
default	$\frac{\frac{(\tan^4(bx+a))}{4}-\frac{(\tan^2(bx+a))}{2}-\ln(\cos(bx+a))}{b}$
risch	$ix+\frac{2ia}{b}-\frac{4(e^{6i(bx+a)}+e^{4i(bx+a)}+e^{2i(bx+a)})}{b(e^{2i(bx+a)}+1)^4}-\frac{\ln(e^{2i(bx+a)}+1)}{b}$
norman	$\frac{-\frac{2(\tan^2(\frac{bx}{2}+\frac{a}{2}))}{b}-\frac{2(\tan^6(\frac{bx}{2}+\frac{a}{2}))}{b}+\frac{8(\tan^4(\frac{bx}{2}+\frac{a}{2}))}{b}}{(\tan^2(\frac{bx}{2}+\frac{a}{2})-1)^4}+\frac{\ln(1+\tan^2(\frac{bx}{2}+\frac{a}{2}))}{b}-\frac{\ln(\tan(\frac{bx}{2}+\frac{a}{2})-1)}{b}-\frac{\ln(\tan(}{}$
parallelrisch	$\frac{(4\cos(4bx+4a)+16\cos(2bx+2a)+12)\ln(\sec^2(\frac{bx}{2}+\frac{a}{2}))+(-16\cos(2bx+2a)-4\cos(4bx+4a)-12)\ln(\tan(\frac{bx}{2}+\frac{a}{2})-1)}{4b(\cos(4bx+4a)+4\cos(2bx+}$

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4*tan(b*x+a)^4-1/2*tan(b*x+a)^2-ln(cos(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(a+bx)\,dx = -\frac{4\cos(bx+a)^4\log(-\cos(bx+a)) + 4\cos(bx+a)^2 - 1}{4\,b\cos(bx+a)^4}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(4*cos(b*x + a)^4*log(-cos(b*x + a)) + 4*cos(b*x + a)^2 - 1)/(b*cos(b*
x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \tan^5(a+bx)\,dx = \frac{\frac{4\sin(bx+a)^2-3}{\sin(bx+a)^4-2\sin(bx+a)^2+1} - 2\log\left(\sin(bx+a)^2-1\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/4*((4*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) - 2*log
(sin(b*x + a)^2 - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(39) = 78$.

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.26

$$\int \tan^5(a+bx)\,dx = \frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2+\frac{20\,(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{20\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right)^2}+2\,\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)-2\,\log\left(\ldots\right.}{4\,b}$$

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*
x + a) + 1))^2 + 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) + 20*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)^2 + 2*log(abs(-(cos(b*x + a) + 1)/
(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - 2*log(ab
s(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a)
 + 1) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \tan^5(a+bx)\,dx = \frac{\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2}-\frac{\tan(a+b\,x)^2}{2}+\frac{\tan(a+b\,x)^4}{4}}{b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^5,x)
```

```
[Out] (log(tan(a + b*x)^2 + 1)/2 - tan(a + b*x)^2/2 + tan(a + b*x)^4/4)/b
```

3.110 $\int \sec(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(a+bx)}{b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

```
[Out] sec(b*x+a)/b-2/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 200}

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^5(a+bx)}{5b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b}$$

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (-1+x^2)^2 \, dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int (1-2x^2+x^4) \, dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\sec(a+bx)}{b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(a+bx)}{b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^5(bx+a)\right)}{5}-\frac{2\left(\sec^3(bx+a)\right)}{3}+\sec(bx+a)}{b}$	32
default	$\frac{\frac{\left(\sec^5(bx+a)\right)}{5}-\frac{2\left(\sec^3(bx+a)\right)}{3}+\sec(bx+a)}{b}$	32
norman	$\frac{-\frac{16}{15b}+\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{32\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}$	55
parallelrisch	$\frac{-\frac{16}{15}-\frac{32\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$	60
risch	$\frac{2\,e^{9i(bx+a)}+\frac{8\,e^{7i(bx+a)}}{3}+\frac{116\,e^{5i(bx+a)}}{15}+\frac{8\,e^{3i(bx+a)}}{3}+2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^5}$	75

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/5*sec(b*x+a)^5-2/3*sec(b*x+a)^3+sec(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos(bx+a)^4 - 10\cos(bx+a)^2 + 3}{15\,b\cos(bx+a)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos(bx+a)^4 - 10\cos(bx+a)^2 + 3}{15\,b\cos(bx+a)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{16\left(\frac{5(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{10(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
[Out] 16/15*(5*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 10*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos\left(a+b\,x\right)^4-10\cos\left(a+b\,x\right)^2+3}{15\,b\cos\left(a+b\,x\right)^5}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^6,x)
[Out] (15*cos(a + b*x)^4 - 10*cos(a + b*x)^2 + 3)/(15*b*cos(a + b*x)^5)
```

3.111 $\int \sec^2(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}$$

```
[Out] 1/6*tan(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}$$

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^5\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\tan^6(a+bx)}{6b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{\tan^6(a+bx)}{6b}$$

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
[Out] Tan[a + b*x]^6/(6*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

method	result	size
norman	$\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}$	32
parallelrisch	$\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}$	32
derivativedivides	$\frac{\frac{\left(\sec^6(bx+a)\right)}{6}-\frac{\left(\sec^4(bx+a)\right)}{2}+\frac{\left(\sec^2(bx+a)\right)}{2}}{b}$	36
default	$\frac{\frac{\left(\sec^6(bx+a)\right)}{6}-\frac{\left(\sec^4(bx+a)\right)}{2}+\frac{\left(\sec^2(bx+a)\right)}{2}}{b}$	36
risch	$\frac{2\,e^{10i(bx+a)}+\frac{20\,e^{6i(bx+a)}}{3}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^6}$	53

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 32/3/b*tan(1/2*b*x+1/2*a)^6/(tan(1/2*b*x+1/2*a)^2-1)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{3\cos(bx+a)^4 - 3\cos(bx+a)^2 + 1}{6\,b\cos(bx+a)^6}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/6*(3*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.93

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = -\frac{3\sin(bx+a)^4 - 3\sin(bx+a)^2 + 1}{6\left(\sin(bx+a)^6 - 3\sin(bx+a)^4 + 3\sin(bx+a)^2 - 1\right)b}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/6*(3*sin(b*x + a)^4 - 3*sin(b*x + a)^2 + 1)/((sin(b*x + a)^6 - 3*sin(b*x
 + a)^4 + 3*sin(b*x + a)^2 - 1)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(13) = 26$.

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.20

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = -\frac{32\left(\cos(bx+a)-1\right)^3}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^6\left(\cos(bx+a)+1\right)^3}$$

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
[Out] -32/3*(cos(b*x + a) - 1)^3/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6
*(cos(b*x + a) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6}{6\,b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^7,x)
[Out] tan(a + b*x)^6/(6*b)
```

3.112 $\int \sec^3(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^3(a+bx)}{3b} - \frac{2\sec^5(a+bx)}{5b} + \frac{\sec^7(a+bx)}{7b}$$

```
[Out] 1/3*sec(b*x+a)^3/b-2/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 276}

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^7(a+bx)}{7b} - \frac{2\sec^5(a+bx)}{5b} + \frac{\sec^3(a+bx)}{3b}$$

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \sec(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sec(a+bx)\right)}{b}$$

$$= \frac{\sec^3(a+bx)}{3b} - \frac{2\sec^5(a+bx)}{5b} + \frac{\sec^7(a+bx)}{7b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^3(a+bx)}{3b} - \frac{2\sec^5(a+bx)}{5b} + \frac{\sec^7(a+bx)}{7b}$$

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^7(bx+a)\right)}{7}-\frac{2\left(\sec^5(bx+a)\right)}{5}+\frac{\left(\sec^3(bx+a)\right)}{3}}{b}$	36
default	$\frac{\frac{\left(\sec^7(bx+a)\right)}{7}-\frac{2\left(\sec^5(bx+a)\right)}{5}+\frac{\left(\sec^3(bx+a)\right)}{3}}{b}$	36
risch	$\frac{\frac{8\,e^{11i(bx+a)}}{3}-\frac{32\,e^{9i(bx+a)}}{15}+\frac{304\,e^{7i(bx+a)}}{35}-\frac{32\,e^{5i(bx+a)}}{15}+\frac{8\,e^{3i(bx+a)}}{3}}{b\left(e^{2i(bx+a)}+1\right)^7}$	75
parallelrisch	$\frac{-\frac{16}{105}-\frac{32\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{16\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{16\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}+\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15}}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	75
norman	$\frac{-\frac{16}{105b}-\frac{16\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{16\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{15b}-\frac{16\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{32\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^7}$	87

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/7*sec(b*x+a)^7-2/5*sec(b*x+a)^5+1/3*sec(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{35\,\cos\left(bx+a\right)^4 - 42\,\cos\left(bx+a\right)^2 + 15}{105\,b\cos\left(bx+a\right)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{35\,\cos\left(bx+a\right)^4 - 42\,\cos\left(bx+a\right)^2 + 15}{105\,b\cos\left(bx+a\right)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(40) = 80$.

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.52

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{16\left(\frac{7\,(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{21\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{35\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{70\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + 1\right)}{105\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^7}$$

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 21*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 35*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 70*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx=\frac{35\cos\left(a+b\,x\right)^4-42\cos\left(a+b\,x\right)^2+15}{105\,b\cos\left(a+b\,x\right)^7}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^8,x)
```

```
[Out] (35*cos(a + b*x)^4 - 42*cos(a + b*x)^2 + 15)/(105*b*cos(a + b*x)^7)
```

3.113 $\int \sec^4(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}+\frac{\tan^8(a+bx)}{8b}$$

```
[Out] 1/6*tan(b*x+a)^6/b+1/8*tan(b*x+a)^8/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 14}

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^8(a+bx)}{8b}+\frac{\tan^6(a+bx)}{6b}$$

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b) + Tan[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^5(1+x^2)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int (x^5+x^7)\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^6(a+bx)}{6b}+\frac{\tan^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^4(a+bx)}{4b}-\frac{\sec^6(a+bx)}{3b}+\frac{\sec^8(a+bx)}{8b}$$

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^4/(4*b) - Sec[a + b*x]^6/(3*b) + Sec[a + b*x]^8/(8*b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{\frac{\sec^8(bx+a)}{8}-\frac{\sec^6(bx+a)}{3}+\frac{\sec^4(bx+a)}{4}}{b}$	36
default	$\frac{\frac{\sec^8(bx+a)}{8}-\frac{\sec^6(bx+a)}{3}+\frac{\sec^4(bx+a)}{4}}{b}$	36
parallelrisch	$\frac{\frac{32\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{32\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{32\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^8}$	55
norman	$\frac{\frac{32\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{32\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{32\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^8}$	66
risch	$\frac{4\,e^{12i(bx+a)}-\frac{16\,e^{10i(bx+a)}}{3}+\frac{40\,e^{8i(bx+a)}}{3}-\frac{16\,e^{6i(bx+a)}}{3}+4\,e^{4i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^8}$	75

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/8*sec(b*x+a)^8-1/3*sec(b*x+a)^6+1/4*sec(b*x+a)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{6\,\cos(bx+a)^4 - 8\,\cos(bx+a)^2 + 3}{24\,b\cos(bx+a)^8}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/24*(6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^8)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{6\,\sin(bx+a)^4 - 4\,\sin(bx+a)^2 + 1}{24\,\left(\sin(bx+a)^8 - 4\,\sin(bx+a)^6 + 6\,\sin(bx+a)^4 - 4\,\sin(bx+a)^2 + 1\right)b}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/24*(6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)/((sin(b*x + a)^8 - 4*sin(b*x
 + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(27) = 54$.

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.00

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = -\frac{32\left(\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^8}$$

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="giac")
[Out] -32/3*((cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - (cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 + (cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5)/(b*((cos(b*x
 + a) - 1)/(cos(b*x + a) + 1) + 1)^8)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6\,\left(3\tan(a+b\,x)^2+4\right)}{24\,b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^9,x)
[Out] (tan(a + b*x)^6*(3*tan(a + b*x)^2 + 4))/(24*b)
```

3.114 $\int \sec^5(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^5(a+bx)}{5b} - \frac{2\sec^7(a+bx)}{7b} + \frac{\sec^9(a+bx)}{9b}$$

```
[Out] 1/5*sec(b*x+a)^5/b-2/7*sec(b*x+a)^7/b+1/9*sec(b*x+a)^9/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 276}

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^9(a+bx)}{9b} - \frac{2\sec^7(a+bx)}{7b} + \frac{\sec^5(a+bx)}{5b}$$

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^4(-1+x^2)^2\, dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int (x^4-2x^6+x^8)\, dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\sec^5(a+bx)}{5b} - \frac{2\sec^7(a+bx)}{7b} + \frac{\sec^9(a+bx)}{9b}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^5(a+bx)\, dx = \frac{\sec^5(a+bx)}{5b} - \frac{2\sec^7(a+bx)}{7b} + \frac{\sec^9(a+bx)}{9b}$$

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\left(\sec^9(bx+a)\right)}{9}-\frac{2\left(\sec^7(bx+a)\right)}{7}+\frac{\left(\sec^5(bx+a)\right)}{5}}{b}$
default	$\frac{\frac{\left(\sec^9(bx+a)\right)}{9}-\frac{2\left(\sec^7(bx+a)\right)}{7}+\frac{\left(\sec^5(bx+a)\right)}{5}}{b}$
risch	$\frac{\frac{32\,e^{13i(bx+a)}}{5}-\frac{384\,e^{11i(bx+a)}}{35}+\frac{6976\,e^{9i(bx+a)}}{315}-\frac{384\,e^{7i(bx+a)}}{35}+\frac{32\,e^{5i(bx+a)}}{5}}{b\left(e^{2i(bx+a)}+1\right)^9}$
parallelrisch	$\frac{-\frac{16}{315}-\frac{32\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-16\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{112\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}-\frac{32\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}-\frac{64\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35}+\frac{16\left(\tan^2\left(\frac{bx}{2}\right.\right.}{35}}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^9}$

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/9*sec(b*x+a)^9-2/7*sec(b*x+a)^7+1/5*sec(b*x+a)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\,\cos\left(bx+a\right)^4 - 90\,\cos\left(bx+a\right)^2 + 35}{315\,b\cos\left(bx+a\right)^9}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\,\cos\left(bx+a\right)^4 - 90\,\cos\left(bx+a\right)^2 + 35}{315\,b\cos\left(bx+a\right)^9}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(40) = 80.

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.48

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx$$
$$= \frac{16\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{36(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{126(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{441(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - \frac{315(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} + \frac{210(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}\right.}{315\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^9}$$

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/315*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 36*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 126*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 441*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 315*(cos(b*x + a) - 1)^5/(cos(b*
x + a) + 1)^5 + 210*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 1)/(b*((cos
(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\cos\left(a+b\,x\right)^4-90\cos\left(a+b\,x\right)^2+35}{315\,b\cos\left(a+b\,x\right)^9}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^10,x)
```

```
[Out] (63*cos(a + b*x)^4 - 90*cos(a + b*x)^2 + 35)/(315*b*cos(a + b*x)^9)
```

3.115 $\int \sec^6(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}$$

```
[Out] 1/6*sec(b*x+a)^6/b-1/4*sec(b*x+a)^8/b+1/10*sec(b*x+a)^10/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2686, 272, 45}

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^{10}(a+bx)}{10b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^5(-1+x^2)^2\,dx, x, \sec(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int (-1+x)^2 x^2\,dx, x, \sec^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)\,dx, x, \sec^2(a+bx)\right)}{2b} \\
&= \frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}$$

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\sec^{10}(bx+a)\right)}{10}-\frac{\left(\sec^{8}(bx+a)\right)}{4}+\frac{\left(\sec^{6}(bx+a)\right)}{6}}{b}$	36
default	$\frac{\frac{\left(\sec^{10}(bx+a)\right)}{10}-\frac{\left(\sec^{8}(bx+a)\right)}{4}+\frac{\left(\sec^{6}(bx+a)\right)}{6}}{b}$	36
risch	$\frac{\frac{32\,e^{14i(bx+a)}}{3}-\frac{64\,e^{12i(bx+a)}}{3}+\frac{192\,e^{10i(bx+a)}}{5}-\frac{64\,e^{8i(bx+a)}}{3}+\frac{32\,e^{6i(bx+a)}}{3}}{b\left(e^{2i(bx+a)}+1\right)^{10}}$	75
parallelrisch	$\frac{32\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)+2\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{18\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5}+2\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{3b\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{10}}$	84

```
[In] int(sec(b*x+a)^11*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/10*sec(b*x+a)^10-1/4*sec(b*x+a)^8+1/6*sec(b*x+a)^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{10\cos(bx+a)^4-15\cos(bx+a)^2+6}{60\,b\cos(bx+a)^{10}}$$

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/60*(10*cos(b*x + a)^4 - 15*cos(b*x + a)^2 + 6)/(b*cos(b*x + a)^10)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**11*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = -\frac{10\,\sin(bx+a)^4 - 5\,\sin(bx+a)^2 + 1}{60\,\left(\sin(bx+a)^{10} - 5\,\sin(bx+a)^8 + 10\,\sin(bx+a)^6 - 10\,\sin(bx+a)^4 + 5\,\sin(bx+a)^2 - 1\right)b}$$

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/60*(10*sin(b*x + a)^4 - 5*sin(b*x + a)^2 + 1)/((sin(b*x + a)^10 - 5*sin(
b*x + a)^8 + 10*sin(b*x + a)^6 - 10*sin(b*x + a)^4 + 5*sin(b*x + a)^2 - 1)*
b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(40) = 80.

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.02

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = -\frac{32\left(\frac{5(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{10(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{18(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} - \frac{10(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6} + \frac{5(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{10}}$$

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="giac")
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 10*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 18*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 10
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 5*(cos(b*x + a) - 1)^7/(cos(b*
x + a) + 1)^7)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^10)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6\,\left(6\tan(a+b\,x)^4 + 15\tan(a+b\,x)^2 + 10\right)}{60\,b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^11,x)
[Out] (tan(a + b*x)^6*(15*tan(a + b*x)^2 + 6*tan(a + b*x)^4 + 10))/(60*b)
```

3.116 $\int \sec^7(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}$$

```
[Out] 1/7*sec(b*x+a)^7/b-2/9*sec(b*x+a)^9/b+1/11*sec(b*x+a)^11/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2686, 276}

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

```
[In] Int[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^6(-1+x^2)^2\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int (x^6-2x^8+x^{10})\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}$$

```
[In] Integrate[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\left(\sec^{11}(bx+a)\right)}{11}-\frac{2\left(\sec^{9}(bx+a)\right)}{9}+\frac{\left(\sec^{7}(bx+a)\right)}{7}}{b}$
default	$\frac{\frac{\left(\sec^{11}(bx+a)\right)}{11}-\frac{2\left(\sec^{9}(bx+a)\right)}{9}+\frac{\left(\sec^{7}(bx+a)\right)}{7}}{b}$
risch	$\frac{\frac{128\,e^{15i(bx+a)}}{7}-\frac{2560\,e^{13i(bx+a)}}{63}+\frac{47360\,e^{11i(bx+a)}}{693}-\frac{2560\,e^{9i(bx+a)}}{63}+\frac{128\,e^{7i(bx+a)}}{7}}{b\left(e^{2i(bx+a)}+1\right)^{11}}$
parallelrisch	$\frac{-\frac{16}{693}-\frac{32\left(\tan^{16}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{80\left(\tan^{14}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-\frac{176\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}-48\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{240\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{7}-\frac{48\left(\tan^{6}\right.}{}}{b\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{11}}$

```
[In] int(sec(b*x+a)^12*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/11*sec(b*x+a)^11-2/9*sec(b*x+a)^9+1/7*sec(b*x+a)^7)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\,\cos\left(bx+a\right)^4 - 154\,\cos\left(bx+a\right)^2 + 63}{693\,b\cos\left(bx+a\right)^{11}}$$

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**12*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\,\cos\left(bx+a\right)^4 - 154\,\cos\left(bx+a\right)^2 + 63}{693\,b\cos\left(bx+a\right)^{11}}$$

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(40) = 80.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.43

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx$$
$$= \frac{16\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{55\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{297\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{1485\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - \frac{2079\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} + \frac{2541\,(\cos(bx+a)\cdots}{(\cos(bx+a)+1\cdots}\right.}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{11}}$$

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 55*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1485
*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(cos
(b*x + a) + 1)^5 + 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155*(c
os(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 462*(cos(b*x + a) - 1)^8/(cos(b*x
 + a) + 1)^8 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^11)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\cos\left(a+b\,x\right)^4-154\cos\left(a+b\,x\right)^2+63}{693\,b\cos\left(a+b\,x\right)^{11}}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^12,x)
```

```
[Out] (99*cos(a + b*x)^4 - 154*cos(a + b*x)^2 + 63)/(693*b*cos(a + b*x)^11)
```

3.117 $\int \sec^8(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^8(a+bx)}{8b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^{12}(a+bx)}{12b}$$

```
[Out] 1/8*sec(b*x+a)^8/b-1/5*sec(b*x+a)^10/b+1/12*sec(b*x+a)^12/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2686, 272, 45}

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^{12}(a+bx)}{12b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^8(a+bx)}{8b}$$

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^7(-1+x^2)^2\,dx, x, \sec(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int (-1+x)^2x^3\,dx, x, \sec^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int (x^3-2x^4+x^5)\,dx, x, \sec^2(a+bx)\right)}{2b} \\
&= \frac{\sec^8(a+bx)}{8b} - \frac{\sec^{10}(a+bx)}{5b} + \frac{\sec^{12}(a+bx)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{\sec^8(a+bx)}{8b} - \frac{\sec^{10}(a+bx)}{5b} + \frac{\sec^{12}(a+bx)}{12b}$$

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\sec^{12}(bx+a)}{12}-\frac{\sec^{10}(bx+a)}{5}+\frac{\sec^{8}(bx+a)}{8}}{b}$
default	$\frac{\frac{\sec^{12}(bx+a)}{12}-\frac{\sec^{10}(bx+a)}{5}+\frac{\sec^{8}(bx+a)}{8}}{b}$
risch	$\frac{32\,e^{16i(bx+a)}-\frac{384\,e^{14i(bx+a)}}{5}+\frac{1856\,e^{12i(bx+a)}}{15}-\frac{384\,e^{10i(bx+a)}}{5}+32\,e^{8i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^{12}}$
parallelrisch	$\frac{32\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(5\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+15\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+39\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+42\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+39\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{15b\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{12}}$

```
[In] int(sec(b*x+a)^13*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/12*sec(b*x+a)^12-1/5*sec(b*x+a)^10+1/8*sec(b*x+a)^8)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{15\,\cos(bx+a)^4-24\,\cos(bx+a)^2+10}{120\,b\cos(bx+a)^{12}}$$

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/120*(15*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 10)/(b*cos(b*x + a)^12)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**13*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(40) = 80.

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx$$
$$= \frac{15 \sin\left(bx+a\right)^4 - 6 \sin\left(bx+a\right)^2 + 1}{120 \left(\sin\left(bx+a\right)^{12} - 6 \sin\left(bx+a\right)^{10} + 15 \sin\left(bx+a\right)^8 - 20 \sin\left(bx+a\right)^6 + 15 \sin\left(bx+a\right)^4 - 6 \sin(bx\right.}$$

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/120*(15*sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 1)/((sin(b*x + a)^12 - 6*sin(
b*x + a)^10 + 15*sin(b*x + a)^8 - 20*sin(b*x + a)^6 + 15*sin(b*x + a)^4 - 6
*sin(b*x + a)^2 + 1)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(40) = 80.

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx =$$
$$-\frac{32\left(\frac{5(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{15(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{39(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} - \frac{42(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6} + \frac{39(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7} - \frac{15(\cos(bx+a)-1}{(\cos(bx+a)+1)^8}\right.}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{12}}$$

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 15*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 39*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 42
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 39*(cos(b*x + a) - 1)^7/(cos(b
*x + a) + 1)^7 - 15*(cos(b*x + a) - 1)^8/(cos(b*x + a) + 1)^8 + 5*(cos(b*x
+ a) - 1)^9/(cos(b*x + a) + 1)^9)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
 + 1)^12)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{\frac{\tan(a+b\,x)^{12}}{12}+\frac{3\tan(a+b\,x)^{10}}{10}+\frac{3\tan(a+b\,x)^8}{8}+\frac{\tan(a+b\,x)^6}{6}}{b}$$

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^13,x)
```

```
[Out] (tan(a + b*x)^6/6 + (3*tan(a + b*x)^8)/8 + (3*tan(a + b*x)^10)/10 + tan(a +
 b*x)^12/12)/b
```

3.118 $\int \sin^3(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx=-\frac{5\text{arctanh}(\sin(a+bx))}{2b}+\frac{5\sin(a+bx)}{2b}+\frac{5\sin^3(a+bx)}{6b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}$$

```
[Out] -5/2*arctanh(sin(b*x+a))/b+5/2*sin(b*x+a)/b+5/6*sin(b*x+a)^3/b+1/2*sin(b*x+
a)^3*tan(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2672, 294, 308, 212}

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx=-\frac{5\text{arctanh}(\sin(a+bx))}{2b}+\frac{5\sin^3(a+bx)}{6b}+\frac{5\sin(a+bx)}{2b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}$$

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sin[a + b*x])/(2*b) + (5*Sin[a + b*x]
^3)/(6*b) + (Sin[a + b*x]^3*Tan[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\
&= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(a+bx)\right)}{2b}\\
&= \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}\\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\
&= -\frac{5\text{arctanh}(\sin(a+bx))}{2b} + \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\text{arctanh}(\sin(a+bx))}{2b}+\frac{5\sec(a+bx)\tan(a+bx)}{2b} -\frac{5\sin(a+bx)\tan^2(a+bx)}{3b}-\frac{\sin^3(a+bx)\tan^2(a+bx)}{3b}$$

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
[Out] (-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(2*b) - (5
*Sin[a + b*x]*Tan[a + b*x]^2)/(3*b) - (Sin[a + b*x]^3*Tan[a + b*x]^2)/(3*b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{\sin^7(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^5(bx+a)\right)}{2}+\frac{5\left(\sin^3(bx+a)\right)}{6}+\frac{5\sin(bx+a)}{2}-\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{\frac{\sin^7(bx+a)}{2\cos(bx+a)^2}+\frac{\left(\sin^5(bx+a)\right)}{2}+\frac{5\left(\sin^3(bx+a)\right)}{6}+\frac{5\sin(bx+a)}{2}-\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
parallelrisch	$\frac{(60\cos(2bx+2a)+60)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-60\cos(2bx+2a)-60)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+50\sin(bx+a)+25\sin(3bx+3a}{24b(1+\cos(2bx+2a))}$
risch	$\frac{ie^{3i(bx+a)}}{24b}-\frac{9ie^{i(bx+a)}}{8b}+\frac{9ie^{-i(bx+a)}}{8b}-\frac{ie^{-3i(bx+a)}}{24b}-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^2}-\frac{5\ln\left(e^{i(bx+a)}+i\right)}{2b}+\frac{5\ln(e^{i(b}}{2}$
norman	$\frac{\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{20\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{22\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{20\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{5\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}+\frac{5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}-$

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sin(b*x+a)^7/cos(b*x+a)^2+1/2*sin(b*x+a)^5+5/6*sin(b*x+a)^3+5/2*si
n(b*x+a)-5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{15\cos(bx+a)^2\log(\sin(bx+a)+1) - 15\cos(bx+a)^2\log(-\sin(bx+a)+1) + 2\left(2\cos(bx+a)^4 - 1\right.}{12\,b\cos(bx+a)^2}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^2*log(-sin
(b*x + a) + 1) + 2*(2*cos(b*x + a)^4 - 14*cos(b*x + a)^2 - 3)*sin(b*x + a))
/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{4\sin(bx+a)^3 - \frac{6\sin(bx+a)}{\sin(bx+a)^2-1} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1) + 24\sin(bx+a)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(sin(b
*x + a) + 1) + 15*log(sin(b*x + a) - 1) + 24*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{4\,\sin\left(bx+a\right)^3-\frac{6\,\sin\left(bx+a\right)}{\sin\left(bx+a\right)^2-1}-15\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+15\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)+24\,\sin\left(bx+a\right)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(abs(s
in(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)) + 24*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^9+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{3}+5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)} - \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

```
[In] int(sin(a + b*x)^6/cos(a + b*x)^3,x)
```

```
[Out] (5*tan(a/2 + (b*x)/2) + (20*tan(a/2 + (b*x)/2)^3)/3 - (22*tan(a/2 + (b*x)/2
)^5)/3 + (20*tan(a/2 + (b*x)/2)^7)/3 + 5*tan(a/2 + (b*x)/2)^9)/(b*(tan(a/2
+ (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2)^4 - 2*tan(a/2 + (b*x)/2)^6 + tan(a/2 +
(b*x)/2)^8 + tan(a/2 + (b*x)/2)^10 + 1)) - (5*atanh(tan(a/2 + (b*x)/2)))/b
```

3.119 $\int \sin(a+bx)\tan^6(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \sin(a+bx)\tan^6(a+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{3\sec(a+bx)}{b}-\frac{\sec^3(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}$$

```
[Out] cos(b*x+a)/b+3*sec(b*x+a)/b-sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 276}

$$\int \sin(a+bx)\tan^6(a+bx)\,dx=\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\
&= \frac{\cos(a+bx)}{b}+\frac{3\sec(a+bx)}{b}-\frac{\sec^3(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(a+bx)}{b}+\frac{3\sec(a+bx)}{b}-\frac{\sec^3(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}$$

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^6,x]
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result	size
norman	$\frac{-\frac{32}{5b}+\frac{128\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{32\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}$	70
risch	$\frac{5\,\mathrm{e}^{11i(bx+a)}+90\,\mathrm{e}^{9i(bx+a)}+235\,\mathrm{e}^{7i(bx+a)}+364\,\mathrm{e}^{5i(bx+a)}+235\,\mathrm{e}^{3i(bx+a)}+95\cos(bx+a)+85i\sin(bx+a)}{10b\left(\mathrm{e}^{2i(bx+a)}+1\right)^5}$	92
derivativedivides	$\frac{\frac{\sin^8(bx+a)}{5\cos(bx+a)^5}-\frac{\sin^8(bx+a)}{5\cos(bx+a)^3}+\frac{\sin^8(bx+a)}{\cos(bx+a)}+\left(\frac{16}{5}+\sin^6(bx+a)+\frac{6\left(\sin^4(bx+a)\right)}{5}+\frac{8\left(\sin^2(bx+a)\right)}{5}\right)\cos(bx+a)}{b}$	96
default	$\frac{\frac{\sin^8(bx+a)}{5\cos(bx+a)^5}-\frac{\sin^8(bx+a)}{5\cos(bx+a)^3}+\frac{\sin^8(bx+a)}{\cos(bx+a)}+\left(\frac{16}{5}+\sin^6(bx+a)+\frac{6\left(\sin^4(bx+a)\right)}{5}+\frac{8\left(\sin^2(bx+a)\right)}{5}\right)\cos(bx+a)}{b}$	96
parallelrisch	$\frac{235\cos(2bx+2a)+5\cos(6bx+6a)+160\cos(3bx+3a)+32\cos(5bx+5a)+90\cos(4bx+4a)+182+320\cos(bx+a)}{10b(\cos(5bx+5a)+5\cos(3bx+3a)+10\cos(bx+a))}$	102

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] (-32/5/b+128/5/b*tan(1/2*b*x+1/2*a)^2-32/b*tan(1/2*b*x+1/2*a)^4)/(1+tan(1/2
*b*x+1/2*a)^2)/(tan(1/2*b*x+1/2*a)^2-1)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{5\cos(bx+a)^6+15\cos(bx+a)^4-5\cos(bx+a)^2+1}{5\,b\cos(bx+a)^5}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/(b*cos(b*
x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\frac{15\cos(bx+a)^4-5\cos(bx+a)^2+1}{\cos(bx+a)^5}+5\cos(bx+a)}{5\,b}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/5*((15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/cos(b*x + a)^5 + 5*cos(b*x
+ a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(48) = 96$.

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.88

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = -\frac{2\left(\frac{5}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \frac{\frac{50\,(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{80\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{30\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{5\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + 11}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}\right)}{5\,b}$$

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="giac")
[Out] -2/5*(5/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (50*(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + 80*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 30*(c
os(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 5*(cos(b*x + a) - 1)^4/(cos(b*x +
 a) + 1)^4 + 11)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(a+b\,x)}{b} + \frac{3}{b\,\cos(a+b\,x)} - \frac{1}{b\cos(a+b\,x)^3} + \frac{1}{5\,b\cos(a+b\,x)^5}$$

```
[In] int(sin(a + b*x)^7/cos(a + b*x)^6,x)
[Out] cos(a + b*x)/b + 3/(b*cos(a + b*x)) - 1/(b*cos(a + b*x)^3) + 1/(5*b*cos(a +
 b*x)^5)
```

3.120 $\int \cos^5(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \cos^5(a+bx)\cot(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2672, 308, 212}

$$\int \cos^5(a+bx)\cot(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b) + Cos[a
+ b*x]^5/(5*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{11\cos(a+bx)}{8b}+\frac{7\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{80b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] (11*Cos[a + b*x])/(8*b) + (7*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(8
0*b) - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^5(bx+a)\right)}{5}+\frac{\left(\cos^3(bx+a)\right)}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	48
default	$\frac{\frac{\left(\cos^5(bx+a)\right)}{5}+\frac{\left(\cos^3(bx+a)\right)}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	48
parallelrisch	$\frac{240\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+368+3\cos(5bx+5a)+35\cos(3bx+3a)+330\cos(bx+a)}{240b}$	50
risch	$\frac{11\,\mathrm{e}^{i(bx+a)}}{16b}+\frac{11\,\mathrm{e}^{-i(bx+a)}}{16b}+\frac{\ln(\mathrm{e}^{i(bx+a)}-1)}{b}-\frac{\ln(\mathrm{e}^{i(bx+a)}+1)}{b}+\frac{\cos(5bx+5a)}{80b}+\frac{7\cos(3bx+3a)}{48b}$	91
norman	$\frac{\frac{6\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{46}{15b}+\frac{12\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{28\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{56\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	102

```
[In] int(cos(b*x+a)^6/sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/5*cos(b*x+a)^5+1/3*cos(b*x+a)^3+cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \cos^5(a+bx)\cot(a+bx)\,dx$$
$$= \frac{6\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+30\,\cos\left(bx+a\right)-15\,\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)+15\,\log\left(-\frac{1}{2}\,\cos\left(bx+a\right.\right.}{30\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(1/2*c
os(b*x + a) + 1/2) + 15*log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(41) = 82$.

Time = 2.06 (sec) , antiderivative size = 1085, normalized size of antiderivative = 20.47

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a),x)
```

```
[Out] Piecewise((15*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(15*b*tan(a/2 + b*
x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan
(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/
2))*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 150*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(15*b*ta
n(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 +
 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 150*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/
2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*
b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b
*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 1
5*log(tan(a/2 + b*x/2))/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 90*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75
*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)
**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 180*tan(a/2 + b*x/2)**6/(15*b*tan(
a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 1
50*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 280*tan(a/2 +
 b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*ta
n(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 +
15*b) + 140*tan(a/2 + b*x/2)**2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b) + 46/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b), Ne(b, 0)), (x*cos(a)**6/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{6\cos(bx+a)^5 + 10\cos(bx+a)^3 + 30\cos(bx+a) - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{30\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(cos(b
*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.74

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{\frac{4\left(\frac{70(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{140(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{90(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{45(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5} + 15\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 140*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 45*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 23)/((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) - 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/
b
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b} + \frac{6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8 + 12\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6 + \frac{56\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{3} + \frac{28\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{3} + \frac{46}{15}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)^5}$$

[In] int(cos(a + b*x)^6/sin(a + b*x),x)

[Out] log(tan(a/2 + (b*x)/2))/b + ((28*tan(a/2 + (b*x)/2)^2)/3 + (56*tan(a/2 + (b*x)/2)^4)/3 + 12*tan(a/2 + (b*x)/2)^6 + 6*tan(a/2 + (b*x)/2)^8 + 46/15)/(b*(tan(a/2 + (b*x)/2)^2 + 1)^5)

3.121 $\int \cos^4(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \cos^4(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}$$

```
[Out] ln(sin(b*x+a))/b-sin(b*x+a)^2/b+1/4*sin(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 272, 45}

$$\int \cos^4(a+bx)\cot(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}-\frac{\sin^2(a+bx)}{b}+\frac{\log(\sin(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x],x]
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$	33
default	$\frac{\frac{\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$	33
parallelrisch	$\frac{32\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-32\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-13+\cos(4bx+4a)+12\cos(2bx+2a)}{32b}$	54
risch	$-ix+\frac{3\,\mathrm{e}^{2i(bx+a)}}{16b}+\frac{3\,\mathrm{e}^{-2i(bx+a)}}{16b}-\frac{2ia}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}+\frac{\cos(4bx+4a)}{32b}$	71
norman	$\frac{-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	100

```
[In] int(cos(b*x+a)^5/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4*cos(b*x+a)^4+1/2*cos(b*x+a)^2+ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\cos{(bx+a)}^4+2\,\cos{(bx+a)}^2+4\,\log\left(\frac{1}{2}\,\sin{(bx+a)}\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
[Out] 1/4*(cos(b*x + a)^4 + 2*cos(b*x + a)^2 + 4*log(1/2*sin(b*x + a)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. 2(31) = 62.

Time = 1.33 (sec) , antiderivative size = 1086, normalized size of antiderivative = 27.15

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a),x)
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*ta
n(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b
*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)
```

```
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2
 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2
 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**
4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/
2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a
/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4
*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b
*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
 + 4*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**5/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\sin(bx+a)^4 - 4\sin(bx+a)^2 + 2\log\left(\sin(bx+a)^2\right)}{4b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
[Out] 1/4*(sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(38) = 76$.

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.25

$$\int \cos^4(a+bx)\cot(a+bx)\,dx =$$

$$-\frac{\frac{\frac{52(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{102(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{52(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 1\right)^4} - 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 12\log\left(\left|-\frac{\cos(bx+a}{\cos(bx+a}\right.\right.}{12b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 102*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) - 1)^4 - 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 1
2*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} + \frac{\frac{\tan(a+b\,x)^2}{2}+\frac{3}{4}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

```
[In] int(cos(a + b*x)^5/sin(a + b*x),x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2/2 + 3
/4)/(b*(2*tan(a + b*x)^2 + tan(a + b*x)^4 + 1))
```

3.122 $\int \cos^3(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos^3(a+bx)\cot(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2672, 308, 212}

$$\int \cos^3(a+bx)\cot(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = \frac{5\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x],x]
[Out] (5*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) - Log[Cos[(a + b*x)/2]]/b
+ Log[Sin[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+16+\cos(3bx+3a)+15\cos(bx+a)}{12b}$	37
derivativedivides	$\frac{\frac{\left(\cos^3(bx+a)\right)}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	38
default	$\frac{\frac{\left(\cos^3(bx+a)\right)}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	38
norman	$\frac{\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{8}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	70
risch	$\frac{5\,e^{i(bx+a)}}{8b}+\frac{5\,e^{-i(bx+a)}}{8b}+\frac{\ln(e^{i(bx+a)}-1)}{b}-\frac{\ln(e^{i(bx+a)}+1)}{b}+\frac{\cos(3bx+3a)}{12b}$	77

```
[In] int(cos(b*x+a)^4/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/12*(12*ln(tan(1/2*b*x+1/2*a))+16+cos(3*b*x+3*a)+15*cos(b*x+a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$
$$=\frac{2\,\cos(bx+a)^3+6\,\cos(bx+a)-3\,\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)+3\,\log\left(-\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)}{6\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(1/2*cos(b*x + a) + 1/2) + 3*
log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(29) = 58.

Time = 0.88 (sec) , antiderivative size = 473, normalized size of antiderivative = 12.45

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$
$$=\begin{cases}\frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b}+\frac{9\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b}+\frac{9\log(}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)} \\ \frac{x\cos^4(a)}{\sin(a)}\end{cases}$$

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a),x)
```

```
[Out] Piecewise((3*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(3*b*tan(a/2 + b*x/2
)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 +
 b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan(a/2 + b*x/2))*tan(a
/2 + b*x/2)**2/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan
(a/2 + b*x/2)**2 + 3*b) + 3*log(tan(a/2 + b*x/2))/(3*b*tan(a/2 + b*x/2)**6
+ 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 12*tan(a/2 + b
*x/2)**4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 +
 b*x/2)**2 + 3*b) + 12*tan(a/2 + b*x/2)**2/(3*b*tan(a/2 + b*x/2)**6 + 9*b*t
an(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 8/(3*b*tan(a/2 + b*x/
2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b), Ne(b, 0))
, (x*cos(a)**4/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = \frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log(\cos(bx+a)+1)+3\log(\cos(bx+a)-1)}{6b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(cos(b*x + a) + 1) + 3*log(co
s(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = \frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}+3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}+\frac{4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+\frac{8}{3}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)^3}$$

```
[In] int(cos(a + b*x)^4/sin(a + b*x),x)
[Out] log(tan(a/2 + (b*x)/2))/b + (4*tan(a/2 + (b*x)/2)^2 + 4*tan(a/2 + (b*x)/2)^
4 + 8/3)/(b*(tan(a/2 + (b*x)/2)^2 + 1)^3)
```

3.123 $\int \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cos^2(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

```
[Out] ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \cos^2(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x} - x\right)\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{2b}$$

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x],x]
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$	23
default	$\frac{\frac{\left(\cos^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$	23
parallelrisch	$\frac{4\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1+\cos(2bx+2a)}{4b}$	43
risch	$-ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	57
norman	$-\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	66

```
[In] int(cos(b*x+a)^3/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*cos(b*x+a)^2+ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\cos{(bx+a)}^2 + 2\,\log{\left(\frac{1}{2}\,\sin{(bx+a)}\right)}}{2\,b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^2 + 2*log(1/2*sin(b*x + a)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(20) = 40$.

Time = 0.61 (sec) , antiderivative size = 369, normalized size of antiderivative = 13.67

$$\int \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \begin{cases} -\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{2\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{a}{2}\right.\right.}{b\tan^4\left(\frac{a}{2}+\frac{b}{2}\right.} \\ \frac{x\cos^3{(a)}}{\sin{(a)}} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a),x)
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b
*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*ta
n(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - l
og(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/
2))/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x
/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x
*cos(a)**3/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = -\frac{\sin\left(bx+a\right)^2 - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = -\frac{\sin\left(bx+a\right)^2 - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)^2}{2} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2} + \ln\left(\tan(a+b\,x)\right)}{b}$$

```
[In] int(cos(a + b*x)^3/sin(a + b*x),x)
[Out] (log(tan(a + b*x)) - log(tan(a + b*x)^2 + 1)/2 + cos(a + b*x)^2/2)/b
```

3.124 $\int \cos(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cos(a+bx)\cot(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2672, 327, 212}

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{\cos(a+bx)}{b} - \frac{\operatorname{arctanh}(\cos(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]*Cot[a + b*x],x]
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b} \\
&= \frac{\cos(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$
\int \cos(a+bx)\cot(a+bx)\,dx = \frac{\cos(a+bx)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}
$$

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x],x]
[Out] Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1+\cos(bx+a)}{b}$	23
derivativedivides	$\frac{\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	28
default	$\frac{\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	28
norman	$-\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	47
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{\ln(e^{i(bx+a)}-1)}{b}-\frac{\ln(e^{i(bx+a)}+1)}{b}$	63

```
[In] int(cos(b*x+a)^2/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(ln(tan(1/2*b*x+1/2*a))-1+cos(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2\,\cos\left(bx+a\right)-\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(17) = 34$.

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \cos(a+bx)\cot(a+bx)\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} & \text{for } b\neq 0 \\ \frac{x\cos^2(a)}{\sin(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**2
 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**2 + b) + 2/(b*tan(a/2 +
b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**2/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2\cos(bx+a) - \log(\cos(bx+a)+1) + \log(\cos(bx+a)-1)}{2b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
[Out] 1/2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(23) = 46.

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int \cos(a+bx)\cot(a+bx)\,dx = -\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
[Out] -1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a)
 + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)} + \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

```
[In] int(cos(a + b*x)^2/sin(a + b*x),x)
[Out] 2/(b*(tan(a/2 + (b*x)/2)^2 + 1)) + log(tan(a/2 + (b*x)/2))/b
```

3.125 $\int \cot(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}$$

```
[Out] ln(sin(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3556}

$$\int \cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}$$

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral}=\frac{\log(\sin(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \cot(a+bx)\,dx = \frac{\log(\cos(a+bx))}{b} + \frac{\log(\tan(a+bx))}{b}$$

```
[In] Integrate[Cot[a + b*x],x]
[Out] Log[Cos[a + b*x]]/b + Log[Tan[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\sin(bx+a))}{b}$	12
default	$\frac{\ln(\sin(bx+a))}{b}$	12
risch	$-ix-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	29
parallelrisch	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	30
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	35

```
[In] int(cos(b*x+a)/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(sin(b*x+a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\,dx = \frac{\log\left(\frac{1}{2}\sin(bx+a)\right)}{b}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="fricas")
[Out] log(1/2*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cot(a+bx)\,dx = \begin{cases} \frac{\log(\sin(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x\cos(a)}{\sin(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x)
[Out] Piecewise((log(sin(a + b*x))/b, Ne(b, 0)), (x*cos(a)/sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(bx+a))}{b}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="maxima")
[Out] log(sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \cot(a+bx)\,dx = \frac{\log(|\sin(bx+a)|)}{b}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="giac")
[Out] log(abs(sin(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \cot(a+bx)\,dx = -\frac{\ln\left(\tan(a+b\,x)^2+1\right)-2\,\ln\left(\tan(a+b\,x)\right)}{2\,b}$$

```
[In] int(cos(a + b*x)/sin(a + b*x),x)
[Out] -(log(tan(a + b*x)^2 + 1) - 2*log(tan(a + b*x)))/(2*b)
```

3.126 $\int \csc(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\log(\tan(a+bx))}{b}$$

```
[Out] ln(tan(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2700, 29}

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] Log[Tan[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\log(\tan(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \csc(a+bx)\sec(a+bx)\,dx = 2\left(-\frac{\log(\cos(a+bx))}{2b} + \frac{\log(\sin(a+bx))}{2b}\right)$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x],x]
[Out] 2*(-1/2*Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/(2*b))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tan(bx+a))}{b}$	12
default	$\frac{\ln(\tan(bx+a))}{b}$	12
risch	$-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	35
parallelrisch	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	44
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	50

```
[In] int(sec(b*x+a)/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(tan(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \csc(a+bx)\sec(a+bx)\,dx = -\frac{\log\left(\cos\left(bx+a\right)^2\right) - \log\left(-\frac{1}{4}\cos\left(bx+a\right)^2 + \frac{1}{4}\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(log(cos(b*x + a)^2) - log(-1/4*cos(b*x + a)^2 + 1/4))/b
```

Sympy [F]

$$\int \csc(a+bx)\sec(a+bx)\,dx = \int \frac{\sec\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)/sin(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \csc(a+bx)\sec(a+bx)\,dx = -\frac{\log\left(\sin\left(bx+a\right)^2 - 1\right) - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.09

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 1\right|\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="giac")
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\ln(\tan(a+b\,x))}{b}$$

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)),x)
[Out] log(tan(a + b*x))/b
```

3.127 $\int \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \csc(a+bx)\sec^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 327, 213}

$$\int \csc(a+bx)\sec^2(a+bx)\,dx=\frac{\sec(a+bx)}{b}-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\ &= -\frac{\text{arctanh}(\cos(a+bx))}{b} + \frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\sec(a+bx)}{b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	30
default	$\frac{\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	30
norman	$-\frac{2}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	36
parallelrisch	$\frac{-2+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	56
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}$	62

```
[In] int(sec(b*x+a)^2/sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2}{2\,b\cos(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F]

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \int \frac{\sec^2(a+bx)}{\sin(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{2}{\cos(bx+a)} - \log(\cos(bx+a)+1) + \log(\cos(bx+a)-1)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
[Out] 1/2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(23) = 46.

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
[Out] 1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(abs(-cos(b*x + a)
+ 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\mathrm{atanh}(\cos(a+b\,x)) - \frac{1}{\cos(a+b\,x)}}{b}$$

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] -(atanh(cos(a + b*x)) - 1/cos(a + b*x))/b
```

3.128 $\int \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \csc(a+bx)\sec^3(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}$$

```
[Out] ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \csc(a+bx)\sec^3(a+bx)\,dx=\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b} + \frac{\log(\sin(a+bx))}{b} + \frac{\sec^2(a+bx)}{2b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^3,x]
[Out] -(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$
parallelrisch	$\frac{(-2\cos(2bx+2a)-2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-2\cos(2bx+2a)-2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(2\cos(2bx+2a)+2)\ln\left(\tan\left(\frac{bx}{2}+\right.\right.}{2b(1+\cos(2bx+2a))}$

```
[In] int(sec(b*x+a)^3/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\cos\left(bx+a\right)^2\log\left(\cos\left(bx+a\right)^2\right)-\cos\left(bx+a\right)^2\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{2\,b\cos\left(bx+a\right)^2}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
+ a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = \int \frac{\sec^3\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)**3/sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{1}{\sin\left(bx+a\right)^2-1}+\log\left(\sin\left(bx+a\right)^2-1\right)-\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.59

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = \frac{\frac{\frac{2(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2}+\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{2b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + 3)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 + log(a
bs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2}-\ln\left(\cos\left(a+b\,x\right)\right)+\frac{1}{2\cos(a+b\,x)^2}}{b}$$

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + 1/(2*cos(a + b*x)^2))/b
```

3.129 $\int \csc(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \csc(a+bx)\sec^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 308, 213}

$$\int \csc(a+bx)\sec^4(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= -\frac{\text{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^4,x]
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\frac{1}{3\cos(bx+a)^{3}}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	40
default	$\frac{\frac{1}{3\cos(bx+a)^{3}}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	40
norman	$\frac{\frac{4\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{8}{3b}-\frac{4\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	70
risch	$\frac{2\,e^{5i(bx+a)}+\frac{20\,e^{3i(bx+a)}}{3}+2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^{3}}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{b}$	87
parallelrisch	$\frac{3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{3}\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+12\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{3}}$	98

```
[In] int(sec(b*x+a)^4/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{3\,\cos{(bx+a)}^{3}\log\left(\frac{1}{2}\,\cos{(bx+a)}+\frac{1}{2}\right)-3\,\cos{(bx+a)}^{3}\log\left(-\frac{1}{2}\,\cos{(bx+a)}+\frac{1}{2}\right)-6\,\cos{(bx+a)}^{2}-}{6\,b\cos{(bx+a)}^{3}}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
[Out] -1/6*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(-
1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \int \frac{\sec^{4}(a+bx)}{\sin(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)**4/sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(3\cos(bx+a)^2+1\right)}{\cos(bx+a)^3} - 3\log\left(\cos\left(bx+a\right)+1\right) + 3\log\left(\cos\left(bx+a\right)-1\right)}{6b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
[Out] 1/6*(2*(3*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 3*log(cos(b*x + a) + 1) + 3*
log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(36) = 72.

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3} + 3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\mathrm{atanh}(\cos\left(a+b\,x\right)) - \frac{\cos(a+b\,x)^2+\frac{1}{3}}{\cos(a+b\,x)^3}}{b}$$

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)),x)
[Out] -(atanh(cos(a + b*x)) - (cos(a + b*x)^2 + 1/3)/cos(a + b*x)^3)/b
```

3.130 $\int \csc(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \csc(a+bx)\sec^5(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}$$

```
[Out] ln(tan(b*x+a))/b+tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2700, 272, 45}

$$\int \csc(a+bx)\sec^5(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}+\frac{\tan^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/b + Tan[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\ &= \frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}+\frac{\sec^4(a+bx)}{4b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^5,x]
[Out] -(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b) + Sec[a
 + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,\mathrm{e}^{6i(bx+a)}+8\,\mathrm{e}^{4i(bx+a)}+2\,\mathrm{e}^{2i(bx+a)}}{b(\mathrm{e}^{2i(bx+a)}+1)^4}-\frac{\ln(\mathrm{e}^{2i(bx+a)}+1)}{b}+\frac{\ln(\mathrm{e}^{2i(bx+a)}-1)}{b}$
norman	$\frac{\frac{2}{3b}+\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{2\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{b}$
parallelrisch	$\frac{(-16\cos(2bx+2a)-4\cos(4bx+4a)-12)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-16\cos(2bx+2a)-4\cos(4bx+4a)-12)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right.}{4b(\cos(4bx+4a)+4\cos(2bx+2a)}$

```
[In] int(sec(b*x+a)^5/sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4/cos(b*x+a)^4+1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = -\frac{2\cos(bx+a)^4\log\left(\cos(bx+a)^2\right)-2\cos(bx+a)^4\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-2\cos(bx+a)^2-1}{4\,b\cos(bx+a)^4}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
[Out] -1/4*(2*cos(b*x + a)^4*log(cos(b*x + a)^2) - 2*cos(b*x + a)^4*log(-1/4*cos(
b*x + a)^2 + 1/4) - 2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \int \frac{\sec^5(a+bx)}{\sin(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)**5/sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{2\,\sin(bx+a)^2-3}{\sin(bx+a)^4-2\,\sin(bx+a)^2+1}+2\,\log\left(\sin\left(bx+a\right)^2-1\right)-2\,\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(37) = 74.

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.36

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \frac{\frac{\frac{52\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{102\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{52\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}+6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-12\,\log\left(\left|-\frac{\cos(bx+a)-}{\cos(bx+a)+}\right.\right.}{12\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 102*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(c
os(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) + 1)^4 + 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 12
*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2}-\ln\left(\cos\left(a+b\,x\right)\right)+\frac{\frac{\cos(a+b\,x)^2}{2}+\frac{1}{4}}{\cos(a+b\,x)^4}}{b}$$

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/2 + 1/4)/cos(a
 + b*x)^4)/b
```

3.131 $\int \csc(a+bx)\sec^6(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2702, 308, 213}

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b) + Sec[a
+ b*x]^5/(5*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+bx)\right)}{b}\\
&= \frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= -\frac{\text{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{1}{5\cos(bx+a)^5}+\frac{1}{3\cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$
default	$\frac{\frac{1}{5\cos(bx+a)^5}+\frac{1}{3\cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$
norman	$\frac{-\frac{46}{15b}+\frac{12\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{6\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{28\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{56\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$
risch	$\frac{2\,e^{9i(bx+a)}+\frac{32\,e^{7i(bx+a)}}{3}+\frac{356\,e^{5i(bx+a)}}{15}+\frac{32\,e^{3i(bx+a)}}{3}+2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^5}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{b}$
parallelrisch	$\frac{15\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-90\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+180\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-280\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{15b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$

```
[In] int(sec(b*x+a)^6/sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/5/cos(b*x+a)^5+1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{15\,\cos\left(bx+a\right)^5\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-15\,\cos\left(bx+a\right)^5\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-30\,\cos\left(bx+a\right)^4}{30\,b\cos\left(bx+a\right)^5}$$

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(b*x + a)^5*log(1/2*cos(b*x + a) + 1/2) - 15*cos(b*x + a)^5*lo
g(-1/2*cos(b*x + a) + 1/2) - 30*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 6)/(b*
cos(b*x + a)^5)
```

Sympy [F]

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \int \frac{\sec^6(a+bx)}{\sin(a+bx)}\,dx$$

[In] integrate(sec(b*x+a)**6/sin(b*x+a),x)

[Out] Integral(sec(a + b*x)**6/sin(a + b*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\frac{2\left(15\cos(bx+a)^4+5\cos(bx+a)^2+3\right)}{\cos(bx+a)^5} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{30\,b}$$

[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")

[Out] 1/30*(2*(15*cos(b*x + a)^4 + 5*cos(b*x + a)^2 + 3)/cos(b*x + a)^5 - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(49) = 98.

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.74

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\frac{4\left(\frac{70(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{140(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{90(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{45(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5} + 15\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="giac")

[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 140*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 45*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 23)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\cos\left(a+b\,x\right)^4+\frac{\cos\left(a+b\,x\right)^2}{3}+\frac{1}{5}}{b\cos\left(a+b\,x\right)^5}-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{b}$$

```
[In] int(1/(cos(a + b*x)^6*sin(a + b*x)),x)
```

```
[Out] (cos(a + b*x)^2/3 + cos(a + b*x)^4 + 1/5)/(b*cos(a + b*x)^5) - atanh(cos(a
+ b*x))/b
```

3.132 $\int \csc(a+bx)\sec^7(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \csc(a+bx)\sec^7(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{3\tan^4(a+bx)}{4b}+\frac{\tan^6(a+bx)}{6b}$$

```
[Out] ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+3/4*tan(b*x+a)^4/b+1/6*tan(b*x+a)^6/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2700, 272, 45}

$$\int \csc(a+bx)\sec^7(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}+\frac{3\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

```
[Out] Log[Tan[a + b*x]]/b + (3*Tan[a + b*x]^2)/(2*b) + (3*Tan[a + b*x]^4)/(4*b) +
 Tan[a + b*x]^6/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x}+3x+x^2\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{3\tan^4(a+bx)}{4b}+\frac{\tan^6(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}+\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^7,x]
[Out] -(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b) + Sec[a
 + b*x]^4/(4*b) + Sec[a + b*x]^6/(6*b)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{1}{6\cos(bx+a)^6}+\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{6\cos(bx+a)^6}+\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{10i(bx+a)}+12\,e^{8i(bx+a)}+\frac{92\,e^{6i(bx+a)}}{3}+12\,e^{4i(bx+a)}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^6}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$
norman	$\frac{\frac{6\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{6\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{12\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{12\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{68\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-$
parallelrisch	$(-180\cos(2bx+2a)-72\cos(4bx+4a)-12\cos(6bx+6a)-120)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-180\cos(2bx+2a)-72\cos(4bx+4a$

```
[In] int(sec(b*x+a)^7/sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6/cos(b*x+a)^6+1/4/cos(b*x+a)^4+1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{6\,\cos\left(bx+a\right)^6\log\left(\cos\left(bx+a\right)^2\right)-6\,\cos\left(bx+a\right)^6\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^2+\frac{1}{4}\right)-6\,\cos\left(bx+a\right)^4-3\,\mathrm{c}}{12\,b\cos\left(bx+a\right)^6}$$

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(b*x + a)^6*log(cos(b*x + a)^2) - 6*cos(b*x + a)^6*log(-1/4*cos
(b*x + a)^2 + 1/4) - 6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 2)/(b*cos(b*x +
a)^6)
```

Sympy [F]

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \int \frac{\sec^7(a+bx)}{\sin(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**7/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)**7/sin(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{\frac{6\,\sin(bx+a)^4-15\,\sin(bx+a)^2+11}{\sin(bx+a)^6-3\,\sin(bx+a)^4+3\,\sin(bx+a)^2-1}+6\,\log\left(\sin(bx+a)^2-1\right)-6\,\log\left(\sin(bx+a)^2\right)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="maxima")
[Out] -1/12*((6*sin(b*x + a)^4 - 15*sin(b*x + a)^2 + 11)/(sin(b*x + a)^6 - 3*sin(
b*x + a)^4 + 3*sin(b*x + a)^2 - 1) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(
b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(51) = 102$.

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.75

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\frac{\frac{522\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{1485\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{1580\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{1485\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{522\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{147\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+147}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^6}+30\,\log\left(\frac{|-\cdots}{|\cos\cdots}\right.}{60\,b}$$

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="giac")
[Out] 1/60*((522*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1485*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 1580*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 522*(cos(b*x + a) - 1)^5/(c
os(b*x + a) + 1)^5 + 147*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 147)/(
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6 + 30*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) - 60*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln\left(\cos\left(a+b\,x\right)\right) + \frac{\frac{\cos(a+b\,x)^4}{2}+\frac{\cos(a+b\,x)^2}{4}+\frac{1}{6}}{\cos(a+b\,x)^6}}{b}$$

```
[In] int(1/(cos(a + b*x)^7*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/4 + cos(a + b*
x)^4/2 + 1/6)/cos(a + b*x)^6)/b
```

3.133 $\int \cos^5(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx=-\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}$$

```
[Out] -csc(b*x+a)/b-3*sin(b*x+a)/b+sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx=-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^2}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}$$

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{\cos^8(bx+a)}{\sin(bx+a)}-\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{b}$
default	$\frac{-\frac{\cos^8(bx+a)}{\sin(bx+a)}-\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{b}$
risch	$\frac{19i\mathrm{e}^{i(bx+a)}}{16b}-\frac{19i\mathrm{e}^{-i(bx+a)}}{16b}-\frac{2i\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{\sin(5bx+5a)}{80b}-\frac{3\sin(3bx+3a)}{16b}$
parallelrisch	$\frac{-5\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-90\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-235\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-364\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-235\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-5\cot\left(\frac{bx}{2}+\right.}{10b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$
norman	$\frac{-\frac{1}{2b}-\frac{9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{47\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{182\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{47\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{9\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/sin(b*x+a)*cos(b*x+a)^8-(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos
(b*x+a)^2)*sin(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^6 + 2\cos(bx+a)^4 + 8\cos(bx+a)^2 - 16}{5\,b\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^6 + 2*cos(b*x + a)^4 + 8*cos(b*x + a)^2 - 16)/(b*sin(b*x
+ a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.

Time = 0.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = \begin{cases} -\frac{16\sin^5(a+bx)}{5b} - \frac{8\sin^3(a+bx)\cos^2(a+bx)}{b} - \frac{6\sin(a+bx)\cos^4(a+bx)}{b} - \frac{\cos^6(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^7(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-16*sin(a + b*x)**5/(5*b) - 8*sin(a + b*x)**3*cos(a + b*x)**2/b
- 6*sin(a + b*x)*cos(a + b*x)**4/b - cos(a + b*x)**6/(b*sin(a + b*x)), Ne(b
, 0)), (x*cos(a)**7/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(bx+a)^5 - 5\sin(bx+a)^3 + \frac{5}{\sin(bx+a)} + 15\sin(bx+a)}{5\,b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(bx+a)^5 - 5\sin(bx+a)^3 + \frac{5}{\sin(bx+a)} + 15\sin(bx+a)}{5b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(a+bx)^6 - 5\sin(a+bx)^4 + 15\sin(a+bx)^2 + 5}{5b\sin(a+bx)}$$

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^2,x)
```

```
[Out] -(15*sin(a + b*x)^2 - 5*sin(a + b*x)^4 + sin(a + b*x)^6 + 5)/(5*b*sin(a + b
*x))
```

3.134 $\int \cos^4(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15x}{8} - \frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b}$$

```
[Out] -15/8*x-15/8*cot(b*x+a)/b+5/8*cos(b*x+a)^2*cot(b*x+a)/b+1/4*cos(b*x+a)^4*co
t(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 327, 209}

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} - \frac{15x}{8}$$

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[a + b*x])/(8*b) + (5*Cos[a + b*x]^2*Cot[a + b*x])/(8*b)
 + (Cos[a + b*x]^4*Cot[a + b*x])/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \cot(a+bx)\right)}{b}\\
&= \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{5\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{4b}\\
&= \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{15\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(a+bx)\right)}{8b}\\
&= -\frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b}\\
&\quad + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} + \frac{15\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{8b}\\
&= -\frac{15x}{8} - \frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{60a+60bx+32\cot(a+bx)+16\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] -1/32*(60*a + 60*b*x + 32*Cot[a + b*x] + 16*Sin[2*(a + b*x)] + Sin[4*(a + b
*x)])/b
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{(-120bx\sin(bx+a)-80\cos(bx+a)+15\cos(3bx+3a)+\cos(5bx+5a))\sec\left(\frac{bx}{2}+\frac{a}{2}\right)\csc\left(\frac{bx}{2}+\frac{a}{2}\right)}{128b}$
derivativedivides	$\frac{-\frac{\cos^7(bx+a)}{\sin(bx+a)}-\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)-\frac{15bx}{8}-\frac{15a}{8}}{b}$
default	$\frac{-\frac{\cos^7(bx+a)}{\sin(bx+a)}-\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)-\frac{15bx}{8}-\frac{15a}{8}}{b}$
risch	$-\frac{15x}{8}+\frac{i e^{2i(bx+a)}}{4b}-\frac{i e^{-2i(bx+a)}}{4b}-\frac{2i}{b\left(e^{2i(bx+a)}-1\right)}-\frac{\sin(4bx+4a)}{32b}$
norman	$\frac{-\frac{1}{2b}-\frac{15\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{15\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}-\frac{15x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8}-\frac{15x(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-120*b*x*sin(b*x+a)-80*cos(b*x+a)+15*cos(3*b*x+3*a)+cos(5*b*x+5*a))*
sec(1/2*b*x+1/2*a)*csc(1/2*b*x+1/2*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = \frac{2\,\cos{(bx+a)}^5+5\,\cos{(bx+a)}^3-15\,bx\sin{(bx+a)}-15\,\cos{(bx+a)}}{8\,b\sin{(bx+a)}}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(2*cos(b*x + a)^5 + 5*cos(b*x + a)^3 - 15*b*x*sin(b*x + a) - 15*cos(b*x
 + a))/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(54) = 108$.

Time = 0.65 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = \begin{cases} -\frac{15x\sin^4(a+bx)}{8}-\frac{15x\sin^2(a+bx)\cos^2(a+bx)}{4}-\frac{15x\cos^4(a+bx)}{8}-\frac{15\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{25\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{\cos^5}{b\sin} \\ \frac{x\cos^6(a)}{\sin^2(a)} \end{cases}$$

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**2,x)
[Out] Piecewise((-15*x*sin(a + b*x)**4/8 - 15*x*sin(a + b*x)**2*cos(a + b*x)**2/4
 - 15*x*cos(a + b*x)**4/8 - 15*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 25*sin(
a + b*x)*cos(a + b*x)**3/(8*b) - cos(a + b*x)**5/(b*sin(a + b*x)), Ne(b, 0)
), (x*cos(a)**6/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,bx+15\,a+\frac{15\,\tan(bx+a)^4+25\,\tan(bx+a)^2+8}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}}{8\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/8*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 25*tan(b*x + a)^2 + 8)/(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,bx + 15\,a + \frac{7\,\tan(bx+a)^3 + 9\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^2} + \frac{8}{\tan(bx+a)}}{8\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*x + 15*a + (7*tan(b*x + a)^3 + 9*tan(b*x + a))/(tan(b*x + a)^2 +
 1)^2 + 8/tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,x}{8} - \frac{\cos\left(a+b\,x\right)^4\,\left(\frac{15\,\tan(a+b\,x)^4}{8} + \frac{25\,\tan(a+b\,x)^2}{8} + 1\right)}{b\,\tan\left(a+b\,x\right)}$$

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^2,x)
```

```
[Out] - (15*x)/8 - (cos(a + b*x)^4*((25*tan(a + b*x)^2)/8 + (15*tan(a + b*x)^4)/8
 + 1))/(b*tan(a + b*x))
```

3.135 $\int \cos^3(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx=-\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}$$

```
[Out] -csc(b*x+a)/b-2*sin(b*x+a)/b+1/3*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}$$

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{-\frac{\cos^6(bx+a)}{\sin(bx+a)}-\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{b}$	52
default	$\frac{-\frac{\cos^6(bx+a)}{\sin(bx+a)}-\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{b}$	52
risch	$\frac{7i e^{i(bx+a)}}{8b}-\frac{7i e^{-i(bx+a)}}{8b}-\frac{2i e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{\sin(3bx+3a)}{12b}$	74
parallelrisch	$\frac{-3\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-36\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-50\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-36\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{6b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	83
norman	$\frac{-\frac{1}{2b}-\frac{6\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{25\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{6\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	98

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-cos(b*x+a)^6/sin(b*x+a)-(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^4+4\cos(bx+a)^2-8}{3\,b\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*(cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 8)/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \begin{cases} -\frac{8\sin^3(a+bx)}{3b} - \frac{4\sin(a+bx)\cos^2(a+bx)}{b} - \frac{\cos^4(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^5(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**2,x)
[Out] Piecewise((-8*sin(a + b*x)**3/(3*b) - 4*sin(a + b*x)*cos(a + b*x)**2/b - co
s(a + b*x)**4/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**5/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\sin(bx+a)^3 - \frac{3}{\sin(bx+a)} - 6\sin(bx+a)}{3\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\sin(bx+a)^3 - \frac{3}{\sin(bx+a)} - 6\sin(bx+a)}{3b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = -\frac{-\sin(a+bx)^4 + 6\sin(a+bx)^2 + 3}{3b\sin(a+bx)}$$

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^2,x)
[Out] -(6*sin(a + b*x)^2 - sin(a + b*x)^4 + 3)/(3*b*sin(a + b*x))
```

3.136 $\int \cos^2(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=-\frac{3x}{2}-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}$$

```
[Out] -3/2*x-3/2*cot(b*x+a)/b+1/2*cos(b*x+a)^2*cot(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 327, 209}

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}-\frac{3x}{2}$$

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[a + b*x])/(2*b) + (Cos[a + b*x]^2*Cot[a + b*x])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b} \\
&= \frac{\cos^2(a+bx)\cot(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b} \\
&= -\frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b} \\
&= -\frac{3x}{2} - \frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{6(a+bx)+4\cot(a+bx)+\sin(2(a+bx))}{4b}$$

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
[Out] -1/4*(6*(a + b*x) + 4*Cot[a + b*x] + Sin[2*(a + b*x)])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

method	result	si
risch	$-\frac{3x}{2}+\frac{i\mathrm{e}^{2i(bx+a)}}{8b}-\frac{i\mathrm{e}^{-2i(bx+a)}}{8b}-\frac{2i}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}$	54
derivativedivides	$\frac{-\frac{\cos^5(bx+a)}{\sin(bx+a)}-\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	56
default	$\frac{-\frac{\cos^5(bx+a)}{\sin(bx+a)}-\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	56
parallelrisch	$\frac{(-2\cos(bx+a)+\cos(2bx+2a)-3)\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-6bx+2\sec\left(\frac{bx}{2}+\frac{a}{2}\right)\csc\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}$	60
norman	$\frac{-\frac{1}{2b}-\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}-\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2}-3x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{3x\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	12

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -3/2*x+1/8*I/b*exp(2*I*(b*x+a))-1/8*I/b*exp(-2*I*(b*x+a))-2*I/b/(exp(2*I*(b
*x+a))-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=\frac{\cos\left(bx+a\right)^3-3\,bx\sin\left(bx+a\right)-3\,\cos\left(bx+a\right)}{2\,b\sin\left(bx+a\right)}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^3 - 3*b*x*sin(b*x + a) - 3*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{3x\sin^2(a+bx)}{2}-\frac{3x\cos^2(a+bx)}{2}-\frac{3\sin(a+bx)\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)}{b\sin(a+bx)} & \text{for } b\neq 0\\ \frac{x\cos^4(a)}{\sin^2(a)} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2/2 - 3*x*cos(a + b*x)**2/2 - 3*sin(a + b*x)*
cos(a + b*x)/(2*b) - cos(a + b*x)**3/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)
**4/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{9\,\cos\left(a+b\,x\right)-\cos\left(3\,a+3\,b\,x\right)+12\,b\,x\,\sin\left(a+b\,x\right)}{8\,b\,\sin\left(a+b\,x\right)}$$

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^2,x)
```

```
[Out] -(9*cos(a + b*x) - cos(3*a + 3*b*x) + 12*b*x*sin(a + b*x))/(8*b*sin(a + b*x
))
```

3.137 $\int \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b} - \frac{\sin(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b-sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2670, 14}

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(a+bx)}{b} - \frac{\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\csc(a+bx)}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b} - \frac{\sin(a+bx)}{b}$$

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
parallelrisch	$\frac{(-3+\cos(2bx+2a))\sec\left(\frac{bx}{2}+\frac{a}{2}\right)\csc\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}$	35
derivativedivides	$\frac{-\frac{\cos^4(bx+a)}{\sin(bx+a)}-\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b}$	42
default	$\frac{-\frac{\cos^4(bx+a)}{\sin(bx+a)}-\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b}$	42
risch	$\frac{i\left(e^{3i(bx+a)}-5\cos(bx+a)-7i\sin(bx+a)\right)}{2b\left(e^{2i(bx+a)}-1\right)}$	47
norman	$\frac{-\frac{1}{2b}-\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	66

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4/b*(-3+cos(2*b*x+2*a))*sec(1/2*b*x+1/2*a)*csc(1/2*b*x+1/2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^2-2}{b\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
[Out] (cos(b*x + a)^2 - 2)/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = \begin{cases} -\frac{2\sin(a+bx)}{b} - \frac{\cos^2(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**2,x)
[Out] Piecewise((-2*sin(a + b*x)/b - cos(a + b*x)**2/(b*sin(a + b*x)), Ne(b, 0)),
 (x*cos(a)**3/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)}+\sin(bx+a)}{b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)}+\sin(bx+a)}{b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(a+b\,x)^2+1}{b\,\sin(a+b\,x)}$$

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^2,x)
[Out] -(sin(a + b*x)^2 + 1)/(b*sin(a + b*x))
```

3.138 $\int \cot^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \cot^2(a+bx)\,dx = -x-\frac{\cot(a+bx)}{b}$$

```
[Out] -x-cot(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \cot^2(a+bx)\,dx = -\frac{\cot(a+bx)}{b}-x$$

```
[In] Int[Cot[a + b*x]^2,x]
```

```
[Out] -x - Cot[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\cot(a+bx)}{b} - \int 1\,dx$$
$$= -x - \frac{\cot(a+bx)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cot^2(a+bx)\,dx = -\frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+bx)\right)}{b}$$

```
[In] Integrate[Cot[a + b*x]^2,x]
[Out] -((Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{-\cot(bx+a)-bx-a}{b}$	21
default	$\frac{-\cot(bx+a)-bx-a}{b}$	21
risch	$-x-\frac{2i}{b\left(e^{2i(bx+a)}-1\right)}$	24
parallelrisch	$\frac{-2bx-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	31
norman	$\frac{-\frac{1}{2b}+\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}-x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	47

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-cot(b*x+a)-b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cot^2(a+bx)\,dx = -\frac{bx\sin(bx+a)+\cos(bx+a)}{b\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*x*sin(b*x + a) + cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(10) = 20.

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cot^2(a+bx)\,dx = \begin{cases} -x - \frac{\cos(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^2(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-x - cos(a + b*x)/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**2/sin(
a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \cot^2(a+bx)\,dx = -\frac{bx+a+\frac{1}{\tan(bx+a)}}{b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a + 1/tan(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \cot^2(a+bx)\,dx = -\frac{2\,bx + 2\,a + \frac{1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)} - \tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(2*b*x + 2*a + 1/tan(1/2*b*x + 1/2*a) - tan(1/2*b*x + 1/2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^2(a+bx)\,dx = -x - \frac{\cot(a+b\,x)}{b}$$

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^2,x)
[Out] - x - cot(a + b*x)/b
```

3.139 $\int \cot(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2686, 8}

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{\csc(a+bx)}{b}$$

```
[In] Int[Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}(\int 1\,dx, x, \csc(a+bx))}{b}$$
$$= -\frac{\csc(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{\csc(a+bx)}{b}$$

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x],x]
[Out] -(Csc[a + b*x]/b)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{1}{\sin(bx+a)b}$	14
default	$-\frac{1}{\sin(bx+a)b}$	14
parallelrisch	$-\frac{\sec\left(\frac{bx}{2}+\frac{a}{2}\right)\csc\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	24
risch	$-\frac{2i\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}$	29
norman	$\frac{-\frac{1}{2b}-\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	35

```
[In] int(cos(b*x+a)/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/sin(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx=-\frac{1}{b\sin\left(bx+a\right)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \cot(a+bx)\csc(a+bx)\,dx=\begin{cases}-\frac{1}{b\sin\left(a+bx\right)} & \text{for } b\neq 0\\ \frac{x\cos\left(a\right)}{\sin^{2}\left(a\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**2,x)
[Out] Piecewise((-1/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)/sin(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx=-\frac{1}{b\sin\left(bx+a\right)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/(b*sin(b*x + a))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/(b*sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\,\sin(a+b\,x)}$$

```
[In] int(cos(a + b*x)/sin(a + b*x)^2,x)
[Out] -1/(b*sin(a + b*x))
```

3.140 $\int \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+a))/b-csc(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2701, 327, 213}

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	30
default	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	30
parallelrisch	$\frac{-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	57
risch	$-\frac{2i\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}$	65
norman	$\frac{-\frac{1}{2b}-\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	69

```
[In] int(sec(b*x+a)/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)\sin(bx+a)-\log(-\sin(bx+a)+1)\sin(bx+a)-2}{2\,b\sin(bx+a)}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \int \frac{\sec(a+bx)}{\sin^2(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**2,x)
[Out] Integral(sec(a + b*x)/sin(a + b*x)**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2}{\sin(bx+a)} - \log(|\sin(bx+a)+1|) + \log(|\sin(bx+a)-1|)}{2\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2/sin(b*x + a) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) -
1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x)) - \frac{1}{\sin(a+b\,x)}}{b}$$

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] (atanh(sin(a + b*x)) - 1/sin(a + b*x))/b
```

3.141 $\int \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)}{b}$$

```
[Out] -cot(b*x+a)/b+tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 14}

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] -(Cot[a + b*x]/b) + Tan[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\cot(a+bx)}{b} + \frac{\tan(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cot(2(a+bx))}{b}$$

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] (-2*Cot[2*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{1}{\sin(bx+a)\cos(bx+a)}-2\cot(bx+a)}{b}$	31
default	$\frac{\frac{1}{\sin(bx+a)\cos(bx+a)}-2\cot(bx+a)}{b}$	31
risch	$-\frac{4i}{b\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a)}-1\right)}$	33
parallelrisch	$\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)-6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2b}$	54
norman	$\frac{\frac{1}{2b}-\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	66

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/sin(b*x+a)/cos(b*x+a)-2*cot(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cos(bx+a)^2-1}{b\cos(bx+a)\sin(bx+a)}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = \int \frac{\sec^2(a+bx)}{\sin^2(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**2,x)
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{1}{\tan(bx+a)}-\tan(bx+a)}{b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -(1/tan(b*x + a) - tan(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2}{b\tan(2bx+2a)}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*tan(2*b*x + 2*a))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cot(2\,a+2\,b\,x)}{b}$$

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] -(2*cot(2*a + 2*b*x))/b
```

3.142 $\int \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{2b} - \frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(sin(b*x+a))/b-3/2*csc(b*x+a)/b+1/2*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2701, 294, 327, 213}

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{2b} - \frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*Csc[a + b*x])/(2*b) + (Csc[a + b*x]*Se
c[a + b*x]^2)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{b}\\
&= \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= -\frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= \frac{3\text{arctanh}(\sin(a+bx))}{2b} - \frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 2, \frac{1}{2}, \sin^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{1}{2\cos(bx+a)^2\sin(bx+a)}-\frac{3}{2\sin(bx+a)}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{\frac{1}{2\cos(bx+a)^2\sin(bx+a)}-\frac{3}{2\sin(bx+a)}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
risch	$-\frac{i(3\,\mathrm{e}^{5i(bx+a)}+2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)})}{b(\mathrm{e}^{2i(bx+a)}+1)^2(\mathrm{e}^{2i(bx+a)}-1)}+\frac{3\ln(\mathrm{e}^{i(bx+a)}+i)}{2b}-\frac{3\ln(\mathrm{e}^{i(bx+a)}-i)}{2b}$
parallelrisch	$\frac{(-3\cos(2bx+2a)-3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(3\cos(2bx+2a)+3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(-6\cos(bx+a)+6)\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-2}{2b(1+\cos(2bx+2a))}$
norman	$\frac{-\frac{1}{2b}+\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/cos(b*x+a)^2/sin(b*x+a)-3/2/sin(b*x+a)+3/2*ln(sec(b*x+a)+tan(b*x+a
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$=\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a)-6}{4\,b\cos(bx+a)^2\sin(bx+a)}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^2
*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x + a
)^2*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \int \frac{\sec^3(a+bx)}{\sin^2(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**2,x)
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log(\sin(bx+a)+1) + 3\log(\sin(bx+a)-1)}{4b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(
b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log(|\sin(bx+a)+1|) + 3\log(|\sin(bx+a)-1|)}{4b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(abs(
sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx=\frac{3\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{2\,b}+\frac{\frac{3\,{\sin\left(a+b\,x\right)}^2}{2}-1}{b\,\left(\sin\left(a+b\,x\right)-{\sin\left(a+b\,x\right)}^3\right)}$$

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] (3*atanh(sin(a + b*x)))/(2*b) + ((3*sin(a + b*x)^2)/2 - 1)/(b*(sin(a + b*x)
 - sin(a + b*x)^3))
```

3.143 $\int \csc^2(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=-\frac{\cot(a+bx)}{b}+\frac{2\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] -cot(b*x+a)/b+2*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 276}

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=\frac{\tan^3(a+bx)}{3b}+\frac{2\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]/b) + (2*Tan[a + b*x])/b + Tan[a + b*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{\cot(a+bx)}{b}+\frac{2\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = -\frac{\cot(a+bx)}{b}+\frac{5\tan(a+bx)}{3b}+\frac{\sec^2(a+bx)\tan(a+bx)}{3b}$$

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
[Out] -(Cot[a + b*x]/b) + (5*Tan[a + b*x])/(3*b) + (Sec[a + b*x]^2*Tan[a + b*x])/
(3*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{16i\left(2\,e^{2i(bx+a)}+1\right)}{3b\left(e^{2i(bx+a)}+1\right)^3\left(e^{2i(bx+a)}-1\right)}$	46
derivativedivides	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
default	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
parallelrisch	$\frac{3\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-36\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+50\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-36\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+3\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{6b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	94
norman	$\frac{\frac{1}{2b}-\frac{6\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{25\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{6\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}$	98

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -16/3*I*(2*exp(2*I*(b*x+a))+1)/b/(exp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-1
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = -\frac{8\,\cos\left(bx+a\right)^4 - 4\,\cos\left(bx+a\right)^2 - 1}{3\,b\cos\left(bx+a\right)^3\sin\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \int \frac{\sec^4\left(a+bx\right)}{\sin^2\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3 - \frac{3}{\tan(bx+a)} + 6\,\tan\left(bx+a\right)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \frac{\tan(bx+a)^3 - \frac{3}{\tan(bx+a)} + 6\,\tan(bx+a)}{3\,b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \frac{\tan(a+b\,x)^4 + 6\tan(a+b\,x)^2 - 3}{3\,b\tan(a+b\,x)}$$

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^2),x)
[Out] (6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(3*b*tan(a + b*x))
```

3.144 $\int \csc^2(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\text{arctanh}(\sin(a+bx))}{8b} - \frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b}$$

```
[Out] 15/8*arctanh(sin(b*x+a))/b-15/8*csc(b*x+a)/b+5/8*csc(b*x+a)*sec(b*x+a)^2/b+
1/4*csc(b*x+a)*sec(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2701, 294, 327, 213}

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\text{arctanh}(\sin(a+bx))}{8b} - \frac{15\csc(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b}$$

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(8*b) - (15*Csc[a + b*x])/(8*b) + (5*Csc[a + b*x
]*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{b}\\
&= \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{4b}\\
&= \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{8b}\\
&= -\frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b}\\
&\quad + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{8b}\\
&= \frac{15\operatorname{arctanh}(\sin(a+bx))}{8b} - \frac{15\csc(a+bx)}{8b}\\
&\quad + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)}+\frac{5}{8\cos(bx+a)^2\sin(bx+a)}-\frac{15}{8\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)}+\frac{5}{8\cos(bx+a)^2\sin(bx+a)}-\frac{15}{8\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(15\,e^{9i(bx+a)}+40\,e^{7i(bx+a)}+18\,e^{5i(bx+a)}+40\,e^{3i(bx+a)}+15\,e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4\left(e^{2i(bx+a)}-1\right)}+\frac{15\ln\left(e^{i(bx+a)}+i\right)}{8b}-\frac{15\ln\left(e^{i(bx+a)}-i\right)}{8b}$
norman	$\frac{-\frac{1}{2b}+\frac{15\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{5\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{15\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}-\frac{15\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-\right.}{8b}$
parallelrisch	$\frac{(-60\cos(2bx+2a)-15\cos(4bx+4a)-45)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(60\cos(2bx+2a)+15\cos(4bx+4a)+45)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right.}{8b(\cos(4bx+4a)+4\cos(2bx}$

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4/cos(b*x+a)^4/sin(b*x+a)+5/8/cos(b*x+a)^2/sin(b*x+a)-15/8/sin(b*x+a
)+15/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{15\,\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a)-15\,\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a)-}{16\,b\cos(bx+a)^4\sin(bx+a)}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x + a
)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \int \frac{\sec^5(a+bx)}{\sin^2(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**2,x)
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{2\left(15\sin(bx+a)^4-25\sin(bx+a)^2+8\right)}{\sin(bx+a)^5-2\sin(bx+a)^3+\sin(bx+a)} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1)}{16\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/16*(2*(15*sin(b*x + a)^4 - 25*sin(b*x + a)^2 + 8)/(sin(b*x + a)^5 - 2*si
n(b*x + a)^3 + sin(b*x + a)) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x +
a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{2\left(7\sin(bx+a)^3-9\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2} + \frac{16}{\sin(bx+a)} - 15\log(|\sin(bx+a)+1|) + 15\log(|\sin(bx+a)-1|)}{16\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/16*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/si
n(b*x + a) - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))
/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\,\mathrm{atanh}(\sin(a+b\,x))}{8\,b} - \frac{\frac{15\sin(a+b\,x)^4}{8} - \frac{25\sin(a+b\,x)^2}{8} + 1}{b\,\left(\sin(a+b\,x)^5 - 2\sin(a+b\,x)^3 + \sin(a+b\,x)\right)}$$

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^2),x)
[Out] (15*atanh(sin(a + b*x)))/(8*b) - ((15*sin(a + b*x)^4)/8 - (25*sin(a + b*x)^
2)/8 + 1)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

3.145 $\int \cos^4(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\sin^4(a+bx)}{4b}$$

```
[Out] -1/2*csc(b*x+a)^2/b-3*ln(sin(b*x+a))/b+3/2*sin(b*x+a)^2/b-1/4*sin(b*x+a)^4/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2670, 272, 45}

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\sin^4(a+bx)}{4b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*Csc[a + b*x]^2/b - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
- Sin[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^3}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^2}\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}-\frac{3}{x}-x\right)\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= -\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\sin^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\sin^4(a+bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
[Out] -1/2*Csc[a + b*x]^2/b - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
- Sin[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{\cos^{8}(bx+a)}{2\sin(bx+a)^{2}}-\frac{\left(\cos^{6}(bx+a)\right)}{2}-\frac{3\left(\cos^{4}(bx+a)\right)}{4}-\frac{3\left(\cos^{2}(bx+a)\right)}{2}-3\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\cos^{8}(bx+a)}{2\sin(bx+a)^{2}}-\frac{\left(\cos^{6}(bx+a)\right)}{2}-\frac{3\left(\cos^{4}(bx+a)\right)}{4}-\frac{3\left(\cos^{2}(bx+a)\right)}{2}-3\ln(\sin(bx+a))}{b}$
risch	$3ix-\frac{5\,e^{2i(bx+a)}}{16b}-\frac{5\,e^{-2i(bx+a)}}{16b}+\frac{6ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^{2}}-\frac{3\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\cos(4bx+4a)}{32b}$
parallelrisch	$\frac{96\ln\left(\sec^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-96\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(-92\cos(bx+a)+28\cos(2bx+2a)-4\cos(3bx+3a)+\cos(4bx+4a)+43)\left(\cot^{2}\right.}{32b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{10\left(\tan^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{57\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{57\left(\tan^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{4}\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{3\ln\left(1+\tan^{2}\right.}{b}$

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^8-1/2*cos(b*x+a)^6-3/4*cos(b*x+a)^4-3/2*c
os(b*x+a)^2-3*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{8\,\cos\left(bx+a\right)^{6}+24\,\cos\left(bx+a\right)^{4}-51\,\cos\left(bx+a\right)^{2}+96\,\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)+3}{32\,\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*cos(b*x + a)^6 + 24*cos(b*x + a)^4 - 51*cos(b*x + a)^2 + 96*(cos(b
*x + a)^2 - 1)*log(1/2*sin(b*x + a)) + 3)/(b*cos(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1484 vs. $2(48) = 96$.

Time = 3.46 (sec) , antiderivative size = 1484, normalized size of antiderivative = 25.59

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**3,x)
[Out] Piecewise((24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(8*b*tan(a/
2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b
*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)**
2 + 1)*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2
)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 +
b*x/2)**2) + 144*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(
a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x
/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2
+ b*x/2)**2) + 24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan
(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 3
2*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**10/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) - 96*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2
)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2
+ b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 144*log(tan(a/2 + b*x/2))*tan(a/2
+ b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan
(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 96
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b
*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)
**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2
)**12/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 +
 b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 57*tan(a
/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*
tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
 80*tan(a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) + 57*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 +
 b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(
a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 +
 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)
**2), Ne(b, 0)), (x*cos(a)**7/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\sin(bx+a)^4 - 6\,\sin(bx+a)^2 + \frac{2}{\sin(bx+a)^2} + 6\,\log\left(\sin(bx+a)^2\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 2/sin(b*x + a)^2 + 6*log(sin(b*x
+ a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(52) = 104.

Time = 0.37 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.98

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} + \frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{2\left(\frac{76(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{118(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{76(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}}{8\,b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2*(76*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 +
 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^4 - 12*
log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \frac{3\ln\left(\tan(a+bx)^2+1\right)}{2b} - \frac{3\ln\left(\tan(a+bx)\right)}{b} - \frac{\frac{3\tan(a+bx)^4}{2}+\frac{9\tan(a+bx)^2}{4}+\frac{1}{2}}{b\left(\tan\left(a+bx\right)^6+2\tan\left(a+bx\right)^4+\tan\left(a+bx\right)^2\right)}$$

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^3,x)
```

```
[Out] (3*log(tan(a + b*x)^2 + 1))/(2*b) - (3*log(tan(a + b*x)))/b - ((9*tan(a + b
*x)^2)/4 + (3*tan(a + b*x)^4)/2 + 1/2)/(b*(tan(a + b*x)^2 + 2*tan(a + b*x)^
4 + tan(a + b*x)^6))
```

3.146 $\int \cos^3(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{5\text{arctanh}(\cos(a+bx))}{2b} - \frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}$$

```
[Out] 5/2*arctanh(cos(b*x+a))/b-5/2*cos(b*x+a)/b-5/6*cos(b*x+a)^3/b-1/2*cos(b*x+a
)^3*cot(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2672, 294, 308, 212}

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{5\text{arctanh}(\cos(a+bx))}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{5\cos(a+bx)}{2b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}$$

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Cos[a + b*x]])/(2*b) - (5*Cos[a + b*x])/(2*b) - (5*Cos[a + b*x]^
3)/(6*b) - (Cos[a + b*x]^3*Cot[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\
&= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cos(a+bx)\right)}{2b}\\
&= -\frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\
&= \frac{5\text{arctanh}(\cos(a+bx))}{2b} - \frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{9\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{12b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{5\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{5\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (-9*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) - Csc[(a + b*x)/2]^2/(8*b
) + (5*Log[Cos[(a + b*x)/2]])/(2*b) - (5*Log[Sin[(a + b*x)/2]])/(2*b) + Sec
[(a + b*x)/2]^2/(8*b)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{\cos^7(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^5(bx+a)\right)}{2}-\frac{5\left(\cos^3(bx+a)\right)}{6}-\frac{5\cos(bx+a)}{2}-\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
default	$\frac{-\frac{\cos^7(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^5(bx+a)\right)}{2}-\frac{5\left(\cos^3(bx+a)\right)}{6}-\frac{5\cos(bx+a)}{2}-\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
parallelrisch	$\frac{\left(60\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos(2bx+2a)-50\cos(bx+a)+65\cos(2bx+2a)+25\cos(3bx+3a)+\cos(5bx+5a)-60\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{192b}$
norman	$\frac{-\frac{1}{8b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{75\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{65\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}-\frac{55\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$
risch	$-\frac{e^{3i(bx+a)}}{24b}-\frac{9\,e^{i(bx+a)}}{8b}-\frac{9\,e^{-i(bx+a)}}{8b}-\frac{e^{-3i(bx+a)}}{24b}+\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{5\ln\left(e^{i(bx+a)}-1\right)}{2b}+\frac{5\ln\left(e^{i(bx+\right.}}{2b}$

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)^7/sin(b*x+a)^2-1/2*cos(b*x+a)^5-5/6*cos(b*x+a)^3-5/2*c
os(b*x+a)-5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{4\cos(bx+a)^5+20\cos(bx+a)^3-15\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\left(\cos(bx+a)^2-\right.}{12\left(b\cos(bx+a)^2-b\right)}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^5 + 20*cos(b*x + a)^3 - 15*(cos(b*x + a)^2 - 1)*log(1
/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/
2) - 30*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(58) = 116.

Time = 2.25 (sec) , antiderivative size = 719, normalized size of antiderivative = 10.89

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(24*b*tan(a/
2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*
tan(a/2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(24*b*
tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 +
 24*b*tan(a/2 + b*x/2)**2) - 60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)
**4 + 24*b*tan(a/2 + b*x/2)**2) + 3*tan(a/2 + b*x/2)**10/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 165*tan(a/2 + b*x/2)**6/(24*b*tan(a/2 + b*x/2)**8 + 72*b*t
an(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) -
 225*tan(a/2 + b*x/2)**4/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)*
*6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 130*tan(a/2 + b
*x/2)**2/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/
2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 3/(24*b*tan(a/2 + b*x/2)**8 + 7
2*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)*
*2), Ne(b, 0)), (x*cos(a)**6/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{4\cos(bx+a)^3 - \frac{6\cos(bx+a)}{\cos(bx+a)^2-1} + 24\cos(bx+a) - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*cos(b*x + a)^3 - 6*cos(b*x + a)/(cos(b*x + a)^2 - 1) + 24*cos(b*x
+ a) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(58) = 116.

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.47

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} - \frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3} - 30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(
cos(b*x + a) - 1) - 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(b
*x + a) - 1)/(cos(b*x + a) + 1) - 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 - 30*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.95

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} - \frac{5\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b} - \frac{\frac{49\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{8} + \frac{67\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{8} + \frac{121\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{24} + \frac{1}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8 + 3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6 + 3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4 + \tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - (5*log(tan(a/2 + (b*x)/2)))/(2*b) - ((121*tan(
a/2 + (b*x)/2)^2)/24 + (67*tan(a/2 + (b*x)/2)^4)/8 + (49*tan(a/2 + (b*x)/2)
^6)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + 3*tan(a/2 + (b*x)/2)^4 + 3*tan(a/2
+ (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8))
```

3.147 $\int \cos^2(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b} - \frac{2\log(\sin(a+bx))}{b} + \frac{\sin^2(a+bx)}{2b}$$

```
[Out] -1/2*csc(b*x+a)^2/b-2*ln(sin(b*x+a))/b+1/2*sin(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2670, 272, 45}

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{\sin^2(a+bx)}{2b} - \frac{\csc^2(a+bx)}{2b} - \frac{2\log(\sin(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*Csc[a + b*x]^2/b - (2*Log[Sin[a + b*x]])/b + Sin[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \sin^2(a+bx)\right)}{2b} \\
&= -\frac{\csc^2(a+bx)}{2b} - \frac{2\log(\sin(a+bx))}{b} + \frac{\sin^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)+4\log(\sin(a+bx))-\sin^2(a+bx)}{2b}$$

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
[Out] -1/2*(Csc[a + b*x]^2 + 4*Log[Sin[a + b*x]] - Sin[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{\cos^6(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^4(bx+a)\right)}{2}-\left(\cos^2(bx+a)\right)-2\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\cos^6(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^4(bx+a)\right)}{2}-\left(\cos^2(bx+a)\right)-2\ln(\sin(bx+a))}{b}$
risch	$2ix-\frac{e^{2i(bx+a)}}{8b}-\frac{e^{-2i(bx+a)}}{8b}+\frac{4ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{2\ln\left(e^{2i(bx+a)}-1\right)}{b}$
parallelrisch	$\frac{16\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-16\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(-8\cos(bx+a)+2\cos(2bx+2a)+2)\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(-\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\right.}{8b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{9\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)^6/sin(b*x+a)^2-1/2*cos(b*x+a)^4-cos(b*x+a)^2-2*ln(sin(
b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{2\,\cos(bx+a)^4-3\,\cos(bx+a)^2+8\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\,\sin(bx+a)\right)-1}{4\left(b\cos(bx+a)^2-b\right)}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 8*(cos(b*x + a)^2 - 1)*log(1/2*
sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(34) = 68.

Time = 1.33 (sec) , antiderivative size = 614, normalized size of antiderivative = 14.28

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \begin{cases} \frac{16\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{32\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{16\log\left(\tan^2\left(\frac{a}{2}\right.\right.}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\ldots} \\ \frac{x\cos^5(a)}{\sin^3(a)} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(a/2
 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 32*log
(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/
2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*
x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/
2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b
*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*
x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 18*tan(a/2
+ b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a
/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8
*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**5/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx=\frac{\sin{(bx+a)}^2-\frac{1}{\sin{(bx+a)}^2}-2\,\log{\left(\sin{(bx+a)}^2\right)}}{2\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sin(b*x + a)^2 - 1/sin(b*x + a)^2 - 2*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(39) = 78$.

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.35

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx$$

$$=\frac{\frac{\left(\frac{8\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{8\left(\frac{4\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^2}-8\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+16\,\log}{8\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 8*(4*(cos(b*x + a)
```

```
- 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 3)/
((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^2 - 8*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{b} - \frac{2\,\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\tan(a+b\,x)^2+\frac{1}{2}}{b\,\left(\tan\left(a+b\,x\right)^4+\tan\left(a+b\,x\right)^2\right)}$$

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^3,x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/b - (2*log(tan(a + b*x)))/b - (tan(a + b*x)^2 + 1/2
)/(b*(tan(a + b*x)^2 + tan(a + b*x)^4))
```

3.148 $\int \cos(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \cos(a+bx)\cot^3(a+bx)\,dx=\frac{3\text{arctanh}(\cos(a+bx))}{2b}-\frac{3\cos(a+bx)}{2b}-\frac{\cos(a+bx)\cot^2(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(cos(b*x+a))/b-3/2*cos(b*x+a)/b-1/2*cos(b*x+a)*cot(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2672, 294, 327, 212}

$$\int \cos(a+bx)\cot^3(a+bx)\,dx=\frac{3\text{arctanh}(\cos(a+bx))}{2b}-\frac{3\cos(a+bx)}{2b}-\frac{\cos(a+bx)\cot^2(a+bx)}{2b}$$

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[a + b*x]])/(2*b) - (3*Cos[a + b*x])/(2*b) - (Cos[a + b*x]*Co
t[a + b*x]^2)/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\
&= -\frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\
&= \frac{3\text{arctanh}(\cos(a+bx))}{2b} - \frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{\cos(a+bx)}{b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^3,x]

[Out] -(Cos[a + b*x]/b) - Csc[(a + b*x)/2]^2/(8*b) + (3*Log[Cos[(a + b*x)/2]])/(2*b) - (3*Log[Sin[(a + b*x)/2]])/(2*b) + Sec[(a + b*x)/2]^2/(8*b)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-\frac{\cos^5(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^3(bx+a)\right)}{2}-\frac{3\cos(bx+a)}{2}-\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	60
default	$\frac{-\frac{\cos^5(bx+a)}{2\sin(bx+a)^2}-\frac{\left(\cos^3(bx+a)\right)}{2}-\frac{3\cos(bx+a)}{2}-\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	60
parallelrisch	$\frac{-12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(8\cos(bx+a)-25)\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)+15\right)\left(\csc^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	66
norman	$\frac{-\frac{1}{8b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$	82
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}+\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{3\ln\left(e^{i(bx+a)}-1\right)}{2b}+\frac{3\ln\left(e^{i(bx+a)}+1\right)}{2b}$	100

[In] int(cos(b*x+a)^4/sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/b*(-1/2*cos(b*x+a)^5/sin(b*x+a)^2-1/2*cos(b*x+a)^3-3/2*cos(b*x+a)-3/2*ln(csc(b*x+a)-cot(b*x+a)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{4\cos(bx+a)^3-3\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)}{4\left(b\cos(bx+a)^2-b\right)}$$

[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")

[Out] -1/4*(4*cos(b*x + a)^3 - 3*(cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) + 3*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b*cos(b*x + a)^2 - b)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(42) = 84$.

Time = 0.87 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \begin{cases} -\frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{18\tan^2}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} \\ \frac{x\cos^4(a)}{\sin^3(a)} \end{cases}$$

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 18*tan(a/2 + b*x
/2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2
 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**4/sin(a)**3,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} - 4\cos(bx+a) + 3\log(\cos(bx+a)+1) - 3\log(\cos(bx+a)-1)}{4b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - 4*cos(b*x + a) + 3*log(cos(b*x +
 a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(43) = 86.

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.86

$$\int \cos(a+bx)\cot^3(a+bx)\,dx$$
$$= -\frac{\frac{\frac{14\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x +
 a) - 1)^2/(cos(b*x + a) + 1)^2) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}-\frac{3\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b} - \frac{\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8}+\frac{1}{8}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - (3*log(tan(a/2 + (b*x)/2)))/(2*b) - ((17*tan(a
/2 + (b*x)/2)^2)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b*x)/2)^4))
```

3.149 $\int \cot^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \cot^3(a+bx)\,dx = -\frac{\cot^2(a+bx)}{2b} - \frac{\log(\sin(a+bx))}{b}$$

```
[Out] -1/2*cot(b*x+a)^2/b-ln(sin(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \cot^3(a+bx)\,dx = -\frac{\cot^2(a+bx)}{2b} - \frac{\log(\sin(a+bx))}{b}$$

```
[In] Int[Cot[a + b*x]^3,x]
```

```
[Out] -1/2*Cot[a + b*x]^2/b - Log[Sin[a + b*x]]/b
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\cot^2(a+bx)}{2b} - \int \cot(a+bx)\,dx$$
$$= -\frac{\cot^2(a+bx)}{2b} - \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \cot^3(a+bx)\,dx = -\frac{\cot^2(a+bx) + 2\log(\cos(a+bx)) + 2\log(\tan(a+bx))}{2b}$$

```
[In] Integrate[Cot[a + b*x]^3,x]
[Out] -1/2*(Cot[a + b*x]^2 + 2*Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\left(\cot^2(bx+a)\right)}{2}-\ln(\sin(bx+a))}{b}$	25
default	$\frac{-\frac{\left(\cot^2(bx+a)\right)}{2}-\ln(\sin(bx+a))}{b}$	25
risch	$ix+\frac{2ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	57
parallelrisch	$\frac{-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	59
norman	$\frac{-\frac{1}{8b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	69

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2*cot(b*x+a)^2-ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \cot^3(a+bx)\,dx = -\frac{2\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)-1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(2*(cos(b*x + a)^2 - 1)*log(1/2*sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 -
 b)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \cot^3(a+bx)\,dx = \begin{cases} -\frac{\log\left(\sin\left(a+bx\right)\right)}{b} - \frac{\cos^2\left(a+bx\right)}{2b\sin^2\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos^3\left(a\right)}{\sin^3\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**3,x)
[Out] Piecewise((-log(sin(a + b*x))/b - cos(a + b*x)**2/(2*b*sin(a + b*x)**2), Ne
(b, 0)), (x*cos(a)**3/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \cot^3(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \cot^3(a+bx)\,dx = \frac{\frac{\sin(bx+a)^2-1}{\sin(bx+a)^2} - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*((sin(b*x + a)^2 - 1)/sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \cot^3(a+bx)\,dx = -\frac{\cot(a+b\,x)^2 - \ln\left(\tan(a+b\,x)^2+1\right) + 2\,\ln\left(\tan(a+b\,x)\right)}{2\,b}$$

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^3,x)
[Out] -(2*log(tan(a + b*x)) - log(tan(a + b*x)^2 + 1) + cot(a + b*x)^2)/(2*b)
```

3.150 $\int \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \cot^2(a+bx)\csc(a+bx)\,dx=\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(cos(b*x+a))/b-1/2*cot(b*x+a)*csc(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \cot^2(a+bx)\csc(a+bx)\,dx=\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(2*b) - (Cot[a + b*x]*Csc[a + b*x])/(2*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot(a+bx)\csc(a+bx)}{2b} - \frac{1}{2}\int \csc(a+bx)\,dx \\ &= \frac{\operatorname{arctanh}(\cos(a+bx))}{2b} - \frac{\cot(a+bx)\csc(a+bx)}{2b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = -\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x],x]

[Out] -1/8*Csc[(a + b*x)/2]^2/b + Log[Cos[(a + b*x)/2]]/(2*b) - Log[Sin[(a + b*x)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b)

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

method	result	size
parallelrisch	$\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	43
derivativedivides	$\frac{-\frac{\cos^3(bx+a)}{2\sin(bx+a)^2}-\frac{\cos(bx+a)}{2}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	50
default	$\frac{-\frac{\cos^3(bx+a)}{2\sin(bx+a)^2}-\frac{\cos(bx+a)}{2}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	50
norman	$\frac{-\frac{1}{8b}+\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$	51
risch	$\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(e^{i(bx+a)}-1\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}+1\right)}{2b}$	72

[In] int(cos(b*x+a)^2/sin(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/8*(tan(1/2*b*x+1/2*a)^2-cot(1/2*b*x+1/2*a)^2-4*ln(tan(1/2*b*x+1/2*a)))/b

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)+2\cos\left(bx+\right.}{4\left(b\cos\left(bx+a\right)^2-b\right)}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 - 1
)*log(-1/2*cos(b*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{2b}+\frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}-\frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0 \\ \frac{x\cos^2\left(a\right)}{\sin^3\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(2*b) + tan(a/2 + b*x/2)**2/(8*b) - 1/(8*
b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**2/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1}+\log\left(\cos\left(bx+a\right)+1\right)-\log\left(\cos\left(bx+a\right)-1\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(
b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(30) = 60$.

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.74

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\frac{\left(\frac{2(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 2\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*log(abs(-cos(b*x
+ a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} - \frac{1}{8\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b}$$

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - 1/(8*b*tan(a/2 + (b*x)/2)^2) - log(tan(a/2 + (
b*x)/2))/(2*b)
```

3.151 $\int \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^2(a+bx)\,dx=-\frac{\csc^2(a+bx)}{2b}$$

```
[Out] -1/2*csc(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2686, 30}

$$\int \cot(a+bx)\csc^2(a+bx)\,dx=-\frac{\csc^2(a+bx)}{2b}$$

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*Csc[a + b*x]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int x\,dx, x, \csc(a+bx)\right)}{b} \\ &= -\frac{\csc^2(a+bx)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}$$

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] -1/2*Csc[a + b*x]^2/b
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{1}{2\sin(bx+a)^2 b}$	14
default	$-\frac{1}{2\sin(bx+a)^2 b}$	14
risch	$\frac{2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$	28
parallelrisch	$\frac{-1-\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$	32
norman	$\frac{-\frac{1}{8b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$	35

```
[In] int(cos(b*x+a)/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/sin(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = \frac{1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2/(b*cos(b*x + a)^2 - b)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = \begin{cases} -\frac{1}{2b\sin^2\left(a+bx\right)} & \text{for}\ b\neq 0 \\ \frac{x\cos\left(a\right)}{\sin^3\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**3,x)
[Out] Piecewise((-1/(2*b*sin(a + b*x)**2), Ne(b, 0)), (x*cos(a)/sin(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{2\,b\sin\left(bx+a\right)^2}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2/(b*sin(b*x + a)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{2\,b\sin\left(bx+a\right)^2}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/2/(b*sin(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{2\,b\sin\left(a+b\,x\right)^2}$$

```
[In] int(cos(a + b*x)/sin(a + b*x)^3,x)
[Out] -1/(2*b*sin(a + b*x)^2)
```

3.152 $\int \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \csc^3(a+bx)\sec(a+bx)\,dx=-\frac{\cot^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+ln(tan(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2700, 14}

$$\int \csc^3(a+bx)\sec(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}$$

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*Cot[a + b*x]^2/b + Log[Tan[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{\cot^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}$$

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x],x]
[Out] -1/2*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$	23
default	$\frac{-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$	23
risch	$\frac{2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}$	62
parallelrisch	$\frac{8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	73
norman	$\frac{-\frac{1}{8b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	84

```
[In] int(sec(b*x+a)/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\cos\left(bx+a\right)^2\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = \int \frac{\sec\left(a+bx\right)}{\sin^3\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)/sin(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin\left(bx+a\right)^2-1\right)-\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.41

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\frac{\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 8\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{8\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\ln\left(\cos\left(a+b\,x\right)\right) - \frac{\ln\left(\sin(a+b\,x)^2\right)}{2} + \frac{1}{2\sin(a+b\,x)^2}}{b}$$

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] -(log(cos(a + b*x)) - log(sin(a + b*x)^2)/2 + 1/(2*sin(a + b*x)^2))/b
```

3.153 $\int \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

```
[Out] -3/2*arctanh(cos(b*x+a))/b+3/2*sec(b*x+a)/b-1/2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2702, 294, 327, 213}

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*Sec[a + b*x])/(2*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= -\frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}\\
&= \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}\\
&= -\frac{3\text{arctanh}(\cos(a+bx))}{2b} + \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.92

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$= \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a\right.}{2b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\text{s}\right.}$$

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(2*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{-\frac{1}{2\cos(bx+a)\sin(bx+a)^2}+\frac{3}{2\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	52
default	$\frac{-\frac{1}{2\cos(bx+a)\sin(bx+a)^2}+\frac{3}{2\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	52
parallelrisch	$\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)+12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)-12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-18}{8b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8b}$	81
norman	$\frac{\frac{1}{8b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$	82
risch	$\frac{3\,e^{5i(bx+a)}-2\,e^{3i(bx+a)}+3\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2\left(e^{2i(bx+a)}+1\right)}-\frac{3\ln\left(e^{i(bx+a)}+1\right)}{2b}+\frac{3\ln\left(e^{i(bx+a)}-1\right)}{2b}$	100

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/cos(b*x+a)/sin(b*x+a)^2+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b*x+
a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.96

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{6\cos(bx+a)^2-3\left(\cos(bx+a)^3-\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^3-\cos(bx+a)\right)}{4\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x +
 a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2)
 - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{\sec^2(a+bx)}{\sin^3(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{2\left(3\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)} - 3\log(\cos(bx+a)+1) + 3\log(\cos(bx+a)-1)}{4b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b
*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(43) = 86.

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.86

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{\frac{14(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(co
s(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos(a+b\,x))}{2\,b} - \frac{\frac{3\cos(a+b\,x)^2}{2}-1}{b\,\left(\cos(a+b\,x)-\cos(a+b\,x)^3\right)}$$

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] - (3*atanh(cos(a + b*x)))/(2*b) - ((3*cos(a + b*x)^2)/2 - 1)/(b*(cos(a + b*
x) - cos(a + b*x)^3))
```

3.154 $\int \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx=-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+2*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx=\frac{\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*Cot[a + b*x]^2/b + (2*Log[Tan[a + b*x]])/b + Tan[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3}\,dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, \tan^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)\,dx, x, \tan^2(a+bx)\right)}{2b} \\
&= -\frac{\cot^2(a+bx)}{2b} + \frac{2\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\left(-\frac{\csc^2(a+bx)}{16b} - \frac{\log(\cos(a+bx))}{4b} + \frac{\log(\sin(a+bx))}{4b} + \frac{\sec^2(a+bx)}{16b}\right)$$

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] 8*(-1/16*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/(4*b) + Log[Sin[a + b*x]]/(4*
b) + Sec[a + b*x]^2/(16*b))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{1}{2\cos(bx+a)^2\sin(bx+a)^2}-\frac{1}{\sin(bx+a)^2}+2\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{2\cos(bx+a)^2\sin(bx+a)^2}-\frac{1}{\sin(bx+a)^2}+2\ln(\tan(bx+a))}{b}$
risch	$\frac{4\,\mathrm{e}^{6i(bx+a)}+4\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{2\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}+\frac{2\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{9\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$
parallelrisch	$\frac{(-8\cos(2bx+2a)-8)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-8\cos(2bx+2a)-8)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(8\cos(2bx+2a)+8)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{4b(1+\cos(2bx+2a))}$

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2/cos(b*x+a)^2/sin(b*x+a)^2-1/sin(b*x+a)^2+2*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{2\cos(bx+a)^2 - 2\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 2\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\text{lo}}{2\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(cos(b*x + a
)^2) + 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) -
 1)/(b*cos(b*x + a)^4 - b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = \int \frac{\sec^3(a+bx)}{\sin^3(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)^2-1}{\sin(bx+a)^4-\sin(bx+a)^2} + 2\log\left(\sin(bx+a)^2-1\right) - 2\log\left(\sin(bx+a)^2\right)}{2\,b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(
b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(39) = 78.

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.37

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} - \frac{\cos(bx+a)-1}{\cos(bx+a)+1} - \frac{8\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2} - 8\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 16}{8\,b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 8*(4*(cos(b*x + a)
 - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 3)
/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 - 8*log(abs(-cos(b*x + a) +
1)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) +
1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = \frac{\tan(a+b\,x)^2}{2\,b} - \frac{1}{2\,b\tan\left(a+b\,x\right)^2} + \frac{2\,\ln\left(\tan(a+b\,x)\right)}{b}$$

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] tan(a + b*x)^2/(2*b) - 1/(2*b*tan(a + b*x)^2) + (2*log(tan(a + b*x)))/b
```

3.155 $\int \csc^3(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = -\frac{5\text{arctanh}(\cos(a+bx))}{2b}+\frac{5\sec(a+bx)}{2b}+\frac{5\sec^3(a+bx)}{6b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

```
[Out] -5/2*arctanh(cos(b*x+a))/b+5/2*sec(b*x+a)/b+5/6*sec(b*x+a)^3/b-1/2*csc(b*x+
a)^2*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2702, 294, 308, 213}

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = -\frac{5\text{arctanh}(\cos(a+bx))}{2b}+\frac{5\sec^3(a+bx)}{6b}+\frac{5\sec(a+bx)}{2b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(2*b) + (5*Sec[a + b*x])/(2*b) + (5*Sec[a + b*x]
^3)/(6*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{b}\\
&= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}\\
&= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+bx)\right)}{2b}\\
&= \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}\\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}\\
&= -\frac{5\text{arctanh}(\cos(a+bx))}{2b} + \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(66) = 132.

Time = 0.38 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.11

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{2\csc^8(a+bx)\left(22-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\,\text{co}\right.}{}$$

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[
4*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[S
in[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]
*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(3*b*(Csc[(a
 + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)^2}-\frac{5}{6\cos(bx+a)\sin(bx+a)^2}+\frac{5}{2\cos(bx+a)}+\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
default	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)^2}-\frac{5}{6\cos(bx+a)\sin(bx+a)^2}+\frac{5}{2\cos(bx+a)}+\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
norman	$\frac{\frac{1}{8b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{75\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{65\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}-\frac{55\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$
risch	$\frac{15\,e^{9i(bx+a)}+20\,e^{7i(bx+a)}-22\,e^{5i(bx+a)}+20\,e^{3i(bx+a)}+15\,e^{i(bx+a)}}{3b\left(e^{2i(bx+a)}+1\right)^3\left(e^{2i(bx+a)}-1\right)^2}+\frac{5\ln\left(e^{i(bx+a)}-1\right)}{2b}-\frac{5\ln\left(e^{i(bx+a)}+1\right)}{2b}$
parallelrisch	$\frac{60\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-165\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+}{24b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/3/cos(b*x+a)^3/sin(b*x+a)^2-5/6/cos(b*x+a)/sin(b*x+a)^2+5/2/cos(b*x+
a)+5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{30\cos(bx+a)^4 - 20\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) - 4}{12\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/12*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
+ a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)*
log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \int \frac{\sec^4(a+bx)}{\sin^3(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(15\cos(bx+a)^4 - 10\cos(bx+a)^2 - 2\right)}{\cos(bx+a)^5 - \cos(bx+a)^3} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/12*(2*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 2)/(cos(b*x + a)^5 - cos(b
*x + a)^3) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.47

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = -\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}-30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")

[Out] -1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{-\frac{5\cos(a+b\,x)^4}{2}+\frac{5\cos(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{2\,b}$$

[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^3),x)

[Out] ((5*cos(a + b*x)^2)/3 - (5*cos(a + b*x)^4)/2 + 1/3)/(b*(cos(a + b*x)^3 - cos(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(2*b)

3.156 $\int \csc^3(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx=-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+3*ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

```
[Out] -1/2*Cot[a + b*x]^2/b + (3*Log[Tan[a + b*x]])/b + (3*Tan[a + b*x]^2)/(2*b)
+ Tan[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= -\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\cos(a+bx))}{b}+\frac{3\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{b}+\frac{\sec^4(a+bx)}{4b}$$

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
[Out] -1/2*Csc[a + b*x]^2/b - (3*Log[Cos[a + b*x]])/b + (3*Log[Sin[a + b*x]])/b +
 Sec[a + b*x]^2/b + Sec[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)^2}+\frac{3}{4\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)^2}+\frac{3}{4\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
risch	$\frac{6\,\mathrm{e}^{10i(bx+a)}+12\,\mathrm{e}^{8i(bx+a)}-4\,\mathrm{e}^{6i(bx+a)}+12\,\mathrm{e}^{4i(bx+a)}+6\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^4\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{3\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}+\frac{3\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{10\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{57\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{57\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{b}$
parallelrisch	$\frac{(-24\cos(2bx+2a)-6\cos(4bx+4a)-18)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-24\cos(2bx+2a)-6\cos(4bx+4a)-18)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+\right.}{}$

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/cos(b*x+a)^4/sin(b*x+a)^2+3/4/cos(b*x+a)^2/sin(b*x+a)^2-3/2/sin(b*
x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.93

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx$$
$$=\frac{6\,\cos\left(bx+a\right)^4-3\,\cos\left(bx+a\right)^2-6\,\left(\cos\left(bx+a\right)^6-\cos\left(bx+a\right)^4\right)\log\left(\cos\left(bx+a\right)^2\right)+6\,\left(\cos\left(bx+a\right)^6\right.}{4\,\left(b\cos\left(bx+a\right)^6-b\cos\left(bx+a\right)^4\right)}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x + a)
^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(
b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \int \frac{\sec^5{(a+bx)}}{\sin^3{(a+bx)}}\,dx$$

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{6\sin(bx+a)^4-9\sin(bx+a)^2+2}{\sin(bx+a)^6-2\sin(bx+a)^4+\sin(bx+a)^2}+6\log\left(\sin\left(bx+a\right)^2-1\right)-6\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/4*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/(sin(b*x + a)^6 - 2*sin(b*x
 + a)^4 + sin(b*x + a)^2) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^
2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(52) = 104$.

Time = 0.36 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.00

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{2\left(\frac{76\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{118\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{76\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}}{8\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*(76*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2
+ 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(cos(b*x + a) - 1)^4/(c
os(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^4 - 12
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{2\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{b}+\frac{-\frac{3\cos\left(a+b\,x\right)^4}{2}+\frac{3\cos\left(a+b\,x\right)^2}{4}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^4-\cos\left(a+b\,x\right)^6\right)}$$

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^3),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)^
2)/4 - (3*cos(a + b*x)^4)/2 + 1/4)/(b*(cos(a + b*x)^4 - cos(a + b*x)^6))
```

3.157 $\int \cos^5(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx=\frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] 4*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+6*sin(b*x+a)/b-4/3*sin(b*x+a)^3/b+1/5*sin
(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx=\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{\cos^{10}(bx+a)}{3\sin(bx+a)^3}+\frac{7\left(\cos^{10}(bx+a)\right)}{3\sin(bx+a)}+\frac{7\left(\frac{128}{35}+\cos^8(bx+a)+\frac{8\left(\cos^6(bx+a)\right)}{7}+\frac{48\left(\cos^4(bx+a)\right)}{35}+\frac{64\left(\cos^2(bx+a)\right)}{35}\right)\sin(bx+a)}{3}}{b}$
default	$\frac{-\frac{\cos^{10}(bx+a)}{3\sin(bx+a)^3}+\frac{7\left(\cos^{10}(bx+a)\right)}{3\sin(bx+a)}+\frac{7\left(\frac{128}{35}+\cos^8(bx+a)+\frac{8\left(\cos^6(bx+a)\right)}{7}+\frac{48\left(\cos^4(bx+a)\right)}{35}+\frac{64\left(\cos^2(bx+a)\right)}{35}\right)\sin(bx+a)}{3}}{b}$
risch	$-\frac{i\left(3\,e^{11i(bx+a)}+56\,e^{9i(bx+a)}+1044\,e^{7i(bx+a)}-7524\cos(bx+a)-9612i\sin(bx+a)-8565\cos(5bx+5a)-8571i\sin(5bx+5a)\right.}{480b\left(e^{2i(bx+a)}-1\right)^3}$
parallelrisch	$\frac{-5\left(\tan^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+200\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2740\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+7800\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+11298\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+7800\left(\right.}{120b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5}$
norman	$\frac{-\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{137\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6b}+\frac{65\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{1883\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{20b}+\frac{65\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{137\left(\tan^{12}\left(\frac{bx}{2}\right.\right.}{6b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^10+7/3/sin(b*x+a)*cos(b*x+a)^10+7/3*(128/
35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin
(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = -\frac{3\,\cos(bx+a)^8+8\,\cos(bx+a)^6+48\,\cos(bx+a)^4-192\,\cos(bx+a)^2+128}{15\,\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 48*cos(b*x + a)^4 - 192*cos(b*
x + a)^2 + 128)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx$$
$$=\begin{cases} \frac{128\sin^5(a+bx)}{15b}+\frac{64\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{16\sin(a+bx)\cos^4(a+bx)}{b}+\frac{8\cos^6(a+bx)}{3b\sin(a+bx)}-\frac{\cos^8(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^9(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((128*sin(a + b*x)**5/(15*b) + 64*sin(a + b*x)**3*cos(a + b*x)**2/
(3*b) + 16*sin(a + b*x)*cos(a + b*x)**4/b + 8*cos(a + b*x)**6/(3*b*sin(a +
b*x)) - cos(a + b*x)**8/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**9/sin(
a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx$$
$$=\frac{3\sin(bx+a)^5-20\sin(bx+a)^3+\frac{5\left(12\sin(bx+a)^2-1\right)}{\sin(bx+a)^3}+90\sin(bx+a)}{15b}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx$$
$$=\frac{3\sin(bx+a)^5-20\sin(bx+a)^3+\frac{5\left(12\sin(bx+a)^2-1\right)}{\sin(bx+a)^3}+90\sin(bx+a)}{15b}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{3\sin(a+b\,x)^8-20\sin(a+b\,x)^6+90\sin(a+b\,x)^4+60\sin(a+b\,x)^2-5}{15\,b\sin(a+b\,x)^3}$$

```
[In] int(cos(a + b*x)^9/sin(a + b*x)^4,x)
[Out] (60*sin(a + b*x)^2 + 90*sin(a + b*x)^4 - 20*sin(a + b*x)^6 + 3*sin(a + b*x)
^8 - 5)/(15*b*sin(a + b*x)^3)
```

3.158 $\int \cos^4(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx=\frac{35x}{8}+\frac{35\cot(a+bx)}{8b}-\frac{35\cot^3(a+bx)}{24b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}$$

```
[Out] 35/8*x+35/8*cot(b*x+a)/b-35/24*cot(b*x+a)^3/b+7/8*cos(b*x+a)^2*cot(b*x+a)^3
/b+1/4*cos(b*x+a)^4*cot(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 308, 209}

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx=-\frac{35\cot^3(a+bx)}{24b}+\frac{35\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{35x}{8}$$

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

```
[Out] (35*x)/8 + (35*Cot[a + b*x])/(8*b) - (35*Cot[a + b*x]^3)/(24*b) + (7*Cos[a
+ b*x]^2*Cot[a + b*x]^3)/(8*b) + (Cos[a + b*x]^4*Cot[a + b*x]^3)/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^3}\,dx, x, \cot(a+bx)\right)}{b}\\
&= \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{7\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{4b}\\
&= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \cot(a+bx)\right)}{8b}\\
&= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}\\
&\quad - \frac{35\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \cot(a+bx)\right)}{8b}\\
&= \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}\\
&\quad + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{8b}
\end{aligned}
$$

$$= \frac{35x}{8} + \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{420(a+bx) - 32\cot(a+bx)\left(-10+\csc^2(a+bx)\right) + 72\sin(2(a+bx)) + 3\sin(4(a+bx))}{96b}$$

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
[Out] (420*(a + b*x) - 32*Cot[a + b*x]*(-10 + Csc[a + b*x]^2) + 72*Sin[2*(a + b*x
)] + 3*Sin[4*(a + b*x)])/(96*b)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{(2520bx\sin(bx+a)-840bx\sin(3bx+3a)+525\cos(bx+a)+63\cos(5bx+5a)-847\cos(3bx+3a)+3\cos(7bx+7a))\left(\sec^3\left(\frac{bx}{2}+\right.\right.}{6144b}$
derivativedivides	$\frac{-\frac{\cos^9(bx+a)}{3\sin(bx+a)^3}+\frac{2\left(\cos^9(bx+a)\right)}{\sin(bx+a)}+2\left(\cos^7(bx+a)+\frac{7\left(\cos^5(bx+a)\right)}{6}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)+\frac{35bx}{8}+\frac{35a}{8}}{b}$
default	$\frac{-\frac{\cos^9(bx+a)}{3\sin(bx+a)^3}+\frac{2\left(\cos^9(bx+a)\right)}{\sin(bx+a)}+2\left(\cos^7(bx+a)+\frac{7\left(\cos^5(bx+a)\right)}{6}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)+\frac{35bx}{8}+\frac{35a}{8}}{b}$
risch	$\frac{35x}{8}-\frac{i e^{4i(bx+a)}}{64b}-\frac{3i e^{2i(bx+a)}}{8b}+\frac{3i e^{-2i(bx+a)}}{8b}+\frac{i e^{-4i(bx+a)}}{64b}+\frac{4i\left(6\,e^{4i(bx+a)}-9\,e^{2i(bx+a)}+5\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}$
norman	$\frac{-\frac{1}{24b}+\frac{35\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{63\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{35\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{35\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{63\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{35\left(\tan^{12}\left(\frac{bx}{2}+\right.\right.}{24b}}{\left(1+\tan^2\left(\frac{bx}{2}+\right.\right.}$

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/6144*(2520*b*x*sin(b*x+a)-840*b*x*sin(3*b*x+3*a)+525*cos(b*x+a)+63*cos(5*
b*x+5*a)-847*cos(3*b*x+3*a)+3*cos(7*b*x+7*a))*sec(1/2*b*x+1/2*a)^3*csc(1/2*
b*x+1/2*a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = -\frac{6\cos(bx+a)^7+21\cos(bx+a)^5-140\cos(bx+a)^3-105\left(bx\cos(bx+a)^2-bx\right)\sin(bx+a)+105\cos(bx+a)}{24\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="fricas")
[Out] -1/24*(6*cos(b*x + a)^7 + 21*cos(b*x + a)^5 - 140*cos(b*x + a)^3 - 105*(b*x
*cos(b*x + a)^2 - b*x)*sin(b*x + a) + 105*cos(b*x + a))/((b*cos(b*x + a)^2
- b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \begin{cases} \frac{35x\sin^4(a+bx)}{8}+\frac{35x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{35x\cos^4(a+bx)}{8}+\frac{35\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{175\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{7\cos^5(a+bx)}{3b\sin(a+bx)}-\frac{\cos^7(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^8(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**4,x)
[Out] Piecewise((35*x*sin(a + b*x)**4/8 + 35*x*sin(a + b*x)**2*cos(a + b*x)**2/4
+ 35*x*cos(a + b*x)**4/8 + 35*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 175*sin(
a + b*x)*cos(a + b*x)**3/(24*b) + 7*cos(a + b*x)**5/(3*b*sin(a + b*x)) - co
s(a + b*x)**7/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**8/sin(a)**4, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{105\,bx+105\,a+\frac{105\,\tan(bx+a)^6+175\,\tan(bx+a)^4+56\,\tan(bx+a)^2-8}{\tan(bx+a)^7+2\,\tan(bx+a)^5+\tan(bx+a)^3}}{24\,b}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/24*(105*b*x + 105*a + (105*tan(b*x + a)^6 + 175*tan(b*x + a)^4 + 56*tan(b
*x + a)^2 - 8)/(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3))/b
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{105\,bx+105\,a+\frac{3\left(11\,\tan(bx+a)^3+13\,\tan(bx+a)\right)}{\left(\tan(bx+a)^2+1\right)^2}+\frac{8\left(9\,\tan(bx+a)^2-1\right)}{\tan(bx+a)^3}}{24\,b}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*b*x + 105*a + 3*(11*tan(b*x + a)^3 + 13*tan(b*x + a))/(tan(b*x +
a)^2 + 1)^2 + 8*(9*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{35\,x}{8}+\frac{\cos\left(a+b\,x\right)^4\,\left(\frac{35\,\tan\left(a+b\,x\right)^6}{8}+\frac{175\,\tan\left(a+b\,x\right)^4}{24}+\frac{7\,\tan\left(a+b\,x\right)^2}{3}-\frac{1}{3}\right)}{b\,\tan\left(a+b\,x\right)^3}$$

```
[In] int(cos(a + b*x)^8/sin(a + b*x)^4,x)
```

```
[Out] (35*x)/8 + (cos(a + b*x)^4*((7*tan(a + b*x)^2)/3 + (175*tan(a + b*x)^4)/24
+ (35*tan(a + b*x)^6)/8 - 1/3))/(b*tan(a + b*x)^3)
```

3.159 $\int \cos^3(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx=\frac{3\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] 3*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+3*sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2670, 276}

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx=-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{3\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx=\frac{3\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(\cos(6bx+6a)-273\cos(2bx+2a)+30\cos(4bx+4a)+210)}{768b}$
derivativedivides	$\frac{-\frac{\cos^8(bx+a)}{3\sin(bx+a)^3}+\frac{5\left(\cos^8(bx+a)\right)}{3\sin(bx+a)}+\frac{5\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{3}}{b}$
default	$\frac{-\frac{\cos^8(bx+a)}{3\sin(bx+a)^3}+\frac{5\left(\cos^8(bx+a)\right)}{3\sin(bx+a)}+\frac{5\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{3}}{b}$
risch	$-\frac{i\left(e^{9i(bx+a)}+30\,e^{7i(bx+a)}-273\,e^{5i(bx+a)}-243\cos(bx+a)-303i\sin(bx+a)+421\cos(3bx+3a)+419i\sin(3bx+3a)\right)}{24b\left(e^{2i(bx+a)}-1\right)^3}$
norman	$\frac{-\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{91\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{35\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{91\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{5\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

[Out] 1/768*sec(1/2*b*x+1/2*a)^3*csc(1/2*b*x+1/2*a)^3*(cos(6*b*x+6*a)-273*cos(2*b*x+2*a)+30*cos(4*b*x+4*a)+210)/b

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\cos(bx+a)^6 + 6\cos(bx+a)^4 - 24\cos(bx+a)^2 + 16}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="fricas")

[Out] -1/3*(cos(b*x + a)^6 + 6*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 16)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \begin{cases} \frac{16\sin^3(a+bx)}{3b} + \frac{8\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2\cos^4(a+bx)}{b\sin(a+bx)} - \frac{\cos^6(a+bx)}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^7(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

[In] integrate(cos(b*x+a)**7/sin(b*x+a)**4,x)

[Out] Piecewise((16*sin(a + b*x)**3/(3*b) + 8*sin(a + b*x)*cos(a + b*x)**2/b + 2*cos(a + b*x)**4/(b*sin(a + b*x)) - cos(a + b*x)**6/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**7/sin(a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\sin(bx+a)^3 - \frac{9\sin(bx+a)^2 - 1}{\sin(bx+a)^3} - 9\sin(bx+a)}{3b}$$

[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="maxima")

[Out] -1/3*(sin(b*x + a)^3 - (9*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 - 9*sin(b*x + a))/b

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx=-\frac{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^3-\frac{12}{\sin(bx+a)}-12\sin(bx+a)}{3b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/3*((1/sin(b*x + a) + sin(b*x + a))^3 - 12/sin(b*x + a) - 12*sin(b*x + a)
)/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx=\frac{-\sin(a+b\,x)^6+9\sin(a+b\,x)^4+9\sin(a+b\,x)^2-1}{3\,b\sin(a+b\,x)^3}$$

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^4,x)
[Out] (9*sin(a + b*x)^2 + 9*sin(a + b*x)^4 - sin(a + b*x)^6 - 1)/(3*b*sin(a + b*x
)^3)
```

3.160 $\int \cos^2(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx=\frac{5x}{2}+\frac{5\cot(a+bx)}{2b}-\frac{5\cot^3(a+bx)}{6b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}$$

```
[Out] 5/2*x+5/2*cot(b*x+a)/b-5/6*cot(b*x+a)^3/b+1/2*cos(b*x+a)^2*cot(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2671, 294, 308, 209}

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx=-\frac{5\cot^3(a+bx)}{6b}+\frac{5\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}+\frac{5x}{2}$$

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

```
[Out] (5*x)/2 + (5*Cot[a + b*x])/(2*b) - (5*Cot[a + b*x]^3)/(6*b) + (Cos[a + b*x]
^2*Cot[a + b*x]^3)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b} \\
&= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b} \\
&= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx, x, \cot(a+bx)\right)}{2b} \\
&= \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b} \\
&= \frac{5x}{2} + \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{30(a+bx)-4\cot(a+bx)\left(-7+\csc^2(a+bx)\right)+3\sin(2(a+bx))}{12b}$$

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
[Out] (30*(a + b*x) - 4*Cot[a + b*x]*(-7 + Csc[a + b*x]^2) + 3*Sin[2*(a + b*x)])/
(12*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

method	result
risch	$\frac{5x}{2}-\frac{i e^{2i(bx+a)}}{8b}+\frac{i e^{-2i(bx+a)}}{8b}+\frac{2i\left(9\,e^{4i(bx+a)}-12\,e^{2i(bx+a)}+7\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}$
parallelrisch	$\frac{(180bx\sin(bx+a)-60bx\sin(3bx+3a)+30\cos(bx+a)-65\cos(3bx+3a)+3\cos(5bx+5a))\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{768b}$
derivativedivides	$\frac{-\frac{\cos^7(bx+a)}{3\sin(bx+a)^3}+\frac{4\left(\cos^7(bx+a)\right)}{3\sin(bx+a)}+\frac{4\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{3}+\frac{5bx}{2}+\frac{5a}{2}}{b}$
default	$\frac{-\frac{\cos^7(bx+a)}{3\sin(bx+a)^3}+\frac{4\left(\cos^7(bx+a)\right)}{3\sin(bx+a)}+\frac{4\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{3}+\frac{5bx}{2}+\frac{5a}{2}}{b}$
norman	$\frac{-\frac{1}{24b}+\frac{25\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{25\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}-\frac{25\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}-\frac{25\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}+\frac{5x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+5}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 5/2*x-1/8*I/b*exp(2*I*(b*x+a))+1/8*I/b*exp(-2*I*(b*x+a))+2/3*I*(9*exp(4*I*(
b*x+a))-12*exp(2*I*(b*x+a))+7)/b/(exp(2*I*(b*x+a))-1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = -\frac{3\cos(bx+a)^5-20\cos(bx+a)^3-15\left(bx\cos(bx+a)^2-bx\right)\sin(bx+a)+15\cos(bx+a)}{6\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(b*x + a)^5 - 20*cos(b*x + a)^3 - 15*(b*x*cos(b*x + a)^2 - b*x)*
sin(b*x + a) + 15*cos(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \begin{cases} \frac{5x\sin^2(a+bx)}{2}+\frac{5x\cos^2(a+bx)}{2}+\frac{5\sin(a+bx)\cos(a+bx)}{2b}+\frac{5\cos^3(a+bx)}{3b\sin(a+bx)}-\frac{\cos^5(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^6(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**2/2 + 5*x*cos(a + b*x)**2/2 + 5*sin(a + b*x)*c
os(a + b*x)/(2*b) + 5*cos(a + b*x)**3/(3*b*sin(a + b*x)) - cos(a + b*x)**5/
(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**6/sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{15\,bx+15\,a+\frac{15\,\tan(bx+a)^4+10\,\tan(bx+a)^2-2}{\tan(bx+a)^5+\tan(bx+a)^3}}{6\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 10*tan(b*x + a)^2 - 2)/(tan(b*x +
 a)^5 + tan(b*x + a)^3))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{15\,bx + 15\,a + \frac{3\,\tan(bx+a)}{\tan(bx+a)^2+1} + \frac{2\left(6\,\tan(bx+a)^2-1\right)}{\tan(bx+a)^3}}{6\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*b*x + 15*a + 3*tan(b*x + a)/(tan(b*x + a)^2 + 1) + 2*(6*tan(b*x + a
)^2 - 1)/tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{5\,x}{2} + \frac{\cos\left(a+b\,x\right)^2\,\left(\frac{5\tan(a+b\,x)^4}{2}+\frac{5\tan(a+b\,x)^2}{3}-\frac{1}{3}\right)}{b\tan\left(a+b\,x\right)^3}$$

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^4,x)
```

```
[Out] (5*x)/2 + (cos(a + b*x)^2*((5*tan(a + b*x)^2)/3 + (5*tan(a + b*x)^4)/2 - 1/
3))/(b*tan(a + b*x)^3)
```

3.161 $\int \cos(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{2\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{\sin(a+bx)}{b}$$

```
[Out] 2*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+sin(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2670, 276}

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\sin(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{2\csc(a+bx)}{b}$$

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -\sin(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx, x, -\sin(a+bx)\right)}{b} \\
&= \frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^4,x]
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

method	result	size
parallelrisch	$-\frac{(-25-3\cos(4bx+4a)+36\cos(2bx+2a))\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}$	52
derivativedivides	$\frac{-\frac{\cos^6(bx+a)}{3\sin(bx+a)^3}+\frac{\cos^6(bx+a)}{\sin(bx+a)}+\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{b}$	68
default	$\frac{-\frac{\cos^6(bx+a)}{3\sin(bx+a)^3}+\frac{\cos^6(bx+a)}{\sin(bx+a)}+\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{b}$	68
risch	$-\frac{i\left(3\,e^{7i(bx+a)}-36\,e^{5i(bx+a)}+50\,e^{3i(bx+a)}-33\cos(bx+a)-39i\sin(bx+a)\right)}{6b\left(e^{2i(bx+a)}-1\right)^3}$	71
norman	$\frac{-\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6b}+\frac{15\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	98

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/192*(-25-3*cos(4*b*x+4*a)+36*cos(2*b*x+2*a))*sec(1/2*b*x+1/2*a)^3*csc(1/
2*b*x+1/2*a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^4 - 12\,\cos\left(bx+a\right)^2 + 8}{3\,\left(b\cos\left(bx+a\right)^2 - b\right)\sin\left(bx+a\right)}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
[Out] -1/3*(3*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 8)/((b*cos(b*x + a)^2 - b)*sin
(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \begin{cases} \frac{8\sin\left(a+bx\right)}{3b} + \frac{4\cos^2\left(a+bx\right)}{3b\sin\left(a+bx\right)} - \frac{\cos^4\left(a+bx\right)}{3b\sin^3\left(a+bx\right)} & \text{for}\ b\neq 0 \\ \frac{x\cos^5\left(a\right)}{\sin^4\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**4,x)
[Out] Piecewise((8*sin(a + b*x)/(3*b) + 4*cos(a + b*x)**2/(3*b*sin(a + b*x)) - co
s(a + b*x)**4/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**5/sin(a)**4, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\frac{6\,\sin\left(bx+a\right)^2-1}{\sin\left(bx+a\right)^3} + 3\,\sin\left(bx+a\right)}{3\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\frac{6\,\sin(bx+a)^2-1}{\sin(bx+a)^3}+3\,\sin\left(bx+a\right)}{3\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^4+2\sin\left(a+b\,x\right)^2-\frac{1}{3}}{b\sin\left(a+b\,x\right)^3}$$

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^4,x)
[Out] (2*sin(a + b*x)^2 + sin(a + b*x)^4 - 1/3)/(b*sin(a + b*x)^3)
```

3.162 $\int \cot^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \cot^4(a+bx)\,dx = x+\frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}$$

```
[Out] x+cot(b*x+a)/b-1/3*cot(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \cot^4(a+bx)\,dx = -\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

```
[In] Int[Cot[a + b*x]^4,x]
```

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^3(a+bx)}{3b} - \int \cot^2(a+bx)\,dx \\ &= \frac{\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} + \int 1\,dx \\ &= x + \frac{\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \cot^4(a+bx)\,dx = -\frac{\cot^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Cot[a + b*x]^4,x]
[Out] -1/3*(Cot[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{\left(\cot^3(bx+a)\right)}{3}+\cot(bx+a)+bx+a}{b}$	26
default	$\frac{-\frac{\left(\cot^3(bx+a)\right)}{3}+\cot(bx+a)+bx+a}{b}$	26
risch	$x+\frac{4i\left(3\,e^{4i(bx+a)}-3\,e^{2i(bx+a)}+2\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}$	46
parallelrisch	$\frac{-\left(\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)+24bx+15\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}$	57
norman	$\frac{x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$	80

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3*cot(b*x+a)^3+cot(b*x+a)+b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \cot^4(a+bx)\,dx = \frac{4\cos(bx+a)^3 + 3\left(bx\cos(bx+a)^2 - bx\right)\sin(bx+a) - 3\cos(bx+a)}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(4*cos(b*x + a)^3 + 3*(b*x*cos(b*x + a)^2 - b*x)*sin(b*x + a) - 3*cos(b
*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \cot^4(a+bx)\,dx = \begin{cases} x + \frac{\cos(a+bx)}{b\sin(a+bx)} - \frac{\cos^3(a+bx)}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^4(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**4,x)
[Out] Piecewise((x + cos(a + b*x)/(b*sin(a + b*x)) - cos(a + b*x)**3/(3*b*sin(a +
 b*x)**3), Ne(b, 0)), (x*cos(a)**4/sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \cot^4(a+bx)\,dx = \frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^2 - 1}{\tan(bx+a)^3}}{3\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \cot^4(a+bx)\,dx = \frac{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3+24\,bx+24\,a+\frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3}-15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{24\,b}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(tan(1/2*b*x + 1/2*a)^3 + 24*b*x + 24*a + (15*tan(1/2*b*x + 1/2*a)^2 -
 1)/tan(1/2*b*x + 1/2*a)^3 - 15*tan(1/2*b*x + 1/2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cot^4(a+bx)\,dx = x+\frac{\tan(a+b\,x)^2-\frac{1}{3}}{b\tan\left(a+b\,x\right)^3}$$

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^4,x)
```

```
[Out] x + (tan(a + b*x)^2 - 1/3)/(b*tan(a + b*x)^3)
```

3.163 $\int \cot^3(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}$$

```
[Out] csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2686\}$

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}$$

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x],x]
```

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(a+bx)\right)}{b} \\ &= \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}$$

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x],x]
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

method	result	size
risch	$\frac{2i\left(3\,\mathrm{e}^{5i(bx+a)}-2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	54
parallelrisch	$\frac{-\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+9\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+9\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}$	55
derivativedivides	$\frac{-\frac{\cos^4(bx+a)}{3\sin(bx+a)^3}+\frac{\cos^4(bx+a)}{3\sin(bx+a)}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}}{b}$	60
default	$\frac{-\frac{\cos^4(bx+a)}{3\sin(bx+a)^3}+\frac{\cos^4(bx+a)}{3\sin(bx+a)}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}}{b}$	60
norman	$\frac{-\frac{1}{24b}+\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$	67

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 2/3*I/b/(exp(2*I*(b*x+a))-1)^3*(3*exp(5*I*(b*x+a))-2*exp(3*I*(b*x+a))+3*exp
(I*(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\,\cos(bx+a)^2-2}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(3*cos(b*x + a)^2 - 2)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \begin{cases} \frac{2}{3b\sin(a+bx)} - \frac{\cos^2(a+bx)}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**4,x)
[Out] Piecewise((2/(3*b*sin(a + b*x)) - cos(a + b*x)**2/(3*b*sin(a + b*x)**3), Ne
(b, 0)), (x*cos(a)**3/sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\sin(bx+a)^2 - 1}{3\,b\sin(bx+a)^3}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\sin(bx+a)^2 - 1}{3\,b\sin(bx+a)^3}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\sin(a+b\,x)^2-\frac{1}{3}}{b\sin(a+b\,x)^3}$$

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^4,x)
[Out] (sin(a + b*x)^2 - 1/3)/(b*sin(a + b*x)^3)
```

3.164 $\int \cot^2(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^3(a+bx)}{3b}$$

```
[Out] -1/3*cot(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^3(a+bx)}{3b}$$

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

```
[Out] -1/3*Cot[a + b*x]^3/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(a+bx)\right)}{b}$$
$$= -\frac{\cot^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot^3(a+bx)}{3b}$$

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
[Out] -1/3*Cot[a + b*x]^3/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$-\frac{\cos^3(bx+a)}{3\sin(bx+a)^3 b}$	22
default	$-\frac{\cos^3(bx+a)}{3\sin(bx+a)^3 b}$	22
risch	$\frac{2i\left(3\,\mathrm{e}^{4i(bx+a)}+1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	33
parallelrisch	$-\frac{(\cos(3bx+3a)+3\cos(bx+a))\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{96b}$	46
norman	$\frac{-\frac{1}{24b}+\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$	67

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*cos(b*x+a)^3/sin(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = \frac{\cos(bx+a)^3}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*cos(b*x + a)^3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(12) = 24$.

Time = 0.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.73

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = \begin{cases} \frac{\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)}{24b} - \frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{1}{8b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{1}{24b\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b \neq 0 \\ \frac{x\cos^2(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**4,x)
[Out] Piecewise((tan(a/2 + b*x/2)**3/(24*b) - tan(a/2 + b*x/2)/(8*b) + 1/(8*b*tan
(a/2 + b*x/2)) - 1/(24*b*tan(a/2 + b*x/2)**3), Ne(b, 0)), (x*cos(a)**2/sin(
a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{3\,b\tan(bx+a)^3}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/3/(b*tan(b*x + a)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{3\,b\tan{(bx+a)}^3}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/3/(b*tan(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot(a+b\,x)^3}{3\,b}$$

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^4,x)
[Out] -cot(a + b*x)^3/(3*b)
```

3.165 $\int \cot(a+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^3(a+bx)\,dx=-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] -1/3*csc(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \cot(a+bx)\csc^3(a+bx)\,dx=-\frac{\csc^3(a+bx)}{3b}$$

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

```
[Out] -1/3*Csc[a + b*x]^3/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^2\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{\csc^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{\csc^3(a+bx)}{3b}$$

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^3,x]
[Out] -1/3*Csc[a + b*x]^3/b
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{1}{3\sin(bx+a)^3 b}$	14
default	$-\frac{1}{3\sin(bx+a)^3 b}$	14
parallelrisch	$-\frac{\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}$	28
risch	$\frac{8i\,\mathrm{e}^{3i(bx+a)}}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	29
norman	$\frac{-\frac{1}{24b}-\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$	67

```
[In] int(cos(b*x+a)/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3/sin(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = \frac{1}{3\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = \begin{cases} -\frac{1}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**4,x)
[Out] Piecewise((-1/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)/sin(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(bx+a\right)^3}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
[Out] -1/3/(b*sin(b*x + a)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(bx+a\right)^3}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/3/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(a+b\,x\right)^3}$$

```
[In] int(cos(a + b*x)/sin(a + b*x)^4,x)
[Out] -1/(3*b*sin(a + b*x)^3)
```

3.166 $\int \csc^4(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \csc^4(a+bx)\sec(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] arctanh(sin(b*x+a))/b-csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2701, 308, 213}

$$\int \csc^4(a+bx)\sec(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\csc^3(a+bx)}{3b}-\frac{\csc(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx)\right)}{b} \\
&= -\frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b} \\
&= \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x],x]
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{3\sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$
parallelrisch	$\frac{-\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-15\cot\left(\frac{bx}{2}+\frac{a}{2}\right)+24\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-24\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1}{24b}$
risch	$-\frac{2i\left(3\,e^{5i(bx+a)}-10\,e^{3i(bx+a)}+3\,e^{i(bx+a)}\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}$
norman	$\frac{-\frac{1}{24b}-\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$

```
[In] int(sec(b*x+a)/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= \frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin}{6\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/6*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*x
 + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/((
b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = \int \frac{\sec\left(a+bx\right)}{\sin^4\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**4,x)
[Out] Integral(sec(a + b*x)/sin(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{6\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3
*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{6\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)
) + 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = \frac{\mathrm{atanh}(\sin\left(a+b\,x\right)) - \frac{\sin(a+b\,x)^2+\frac{1}{3}}{\sin(a+b\,x)^3}}{b}$$

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^4),x)
```

```
[Out] (atanh(sin(a + b*x)) - (sin(a + b*x)^2 + 1/3)/sin(a + b*x)^3)/b
```

3.167 $\int \csc^4(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} + \frac{\tan(a+bx)}{b}$$

```
[Out] -2*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+tan(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2700, 276}

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \frac{\tan(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} - \frac{2\cot(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

```
[Out] (-2*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + Tan[a + b*x]/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{2\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{5\cot(a+bx)}{3b}-\frac{\cot(a+bx)\csc^2(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
[Out] (-5*Cot[a + b*x])/(3*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(3*b) + Tan[a + b*x
]/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{16i\left(2\,e^{2i(bx+a)}-1\right)}{3b\left(e^{2i(bx+a)}-1\right)^3\left(e^{2i(bx+a)}+1\right)}$	46
derivativedivides	$\frac{-\frac{1}{3\cos(bx+a)\sin(bx+a)^3}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
default	$\frac{-\frac{1}{3\cos(bx+a)\sin(bx+a)^3}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
parallelrisch	$\frac{\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)+20\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right)-90\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+20\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-24b}$	80
norman	$\frac{\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6b}-\frac{15\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6b}+\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$	98

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

[Out] 16/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{8\,\cos\left(bx+a\right)^4 - 12\,\cos\left(bx+a\right)^2 + 3}{3\,\left(b\cos\left(bx+a\right)^3 - b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")

[Out] -1/3*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))

Sympy [F]

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \int \frac{\sec^2\left(a+bx\right)}{\sin^4\left(a+bx\right)}\,dx$$

[In] integrate(sec(b*x+a)**2/sin(b*x+a)**4,x)

[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{6\,\tan\left(bx+a\right)^2+1}{\tan\left(bx+a\right)^3} - 3\,\tan\left(bx+a\right)}{3\,b}$$

[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")

[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{6\,\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\,\tan{(bx+a)}}{3\,b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{b} - \frac{2\tan(a+b\,x)^2+\frac{1}{3}}{b\tan{(a+b\,x)}^3}$$

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^4),x)
[Out] tan(a + b*x)/b - (2*tan(a + b*x)^2 + 1/3)/(b*tan(a + b*x)^3)
```

3.168 $\int \csc^4(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \frac{5\text{arctanh}(\sin(a+bx))}{2b} - \frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] 5/2*arctanh(sin(b*x+a))/b-5/2*csc(b*x+a)/b-5/6*csc(b*x+a)^3/b+1/2*csc(b*x+a
)^3*sec(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2701, 294, 308, 213}

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \frac{5\text{arctanh}(\sin(a+bx))}{2b} - \frac{5\csc^3(a+bx)}{6b} - \frac{5\csc(a+bx)}{2b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(2*b) - (5*Csc[a + b*x])/(2*b) - (5*Csc[a + b*x]^
3)/(6*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b} \\
&= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx)\right)}{2b} \\
&= -\frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b} \\
&= \frac{5\operatorname{arctanh}(\sin(a+bx))}{2b} - \frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.47

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{1}{3\cos(bx+a)^2\sin(bx+a)^3}+\frac{5}{6\cos(bx+a)^2\sin(bx+a)}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{-\frac{1}{3\cos(bx+a)^2\sin(bx+a)^3}+\frac{5}{6\cos(bx+a)^2\sin(bx+a)}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
risch	$-\frac{i\left(15\,e^{9i(bx+a)}-20\,e^{7i(bx+a)}-22\,e^{5i(bx+a)}-20\,e^{3i(bx+a)}+15\,e^{i(bx+a)}\right)}{3b\left(e^{2i(bx+a)}-1\right)^3\left(e^{2i(bx+a)}+1\right)^2}+\frac{5\ln\left(e^{i(bx+a)}+i\right)}{2b}-\frac{5\ln\left(e^{i(bx+a)}-i\right)}{2b}$
norman	$\frac{-\frac{1}{24b}-\frac{25\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{25\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}+\frac{25\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{12b}-\frac{25\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}-\frac{5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{2b}$
parallelrisch	$\frac{30(-1-\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+30(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+60(-3+\cos(bx+a))\left(\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{12b(1+\cos(2bx+2a))}$

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/3/cos(b*x+a)^2/sin(b*x+a)^3+5/6/cos(b*x+a)^2/sin(b*x+a)-5/2/sin(b*x
+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx =$$

$$-\frac{30\cos(bx+a)^4-15\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\left(\cos(bx+\right.}{12\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \int \frac{\sec^3(a+bx)}{\sin^4(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\left(15\sin(bx+a)^4-10\sin(bx+a)^2-2\right)}{\sin(bx+a)^5-\sin(bx+a)^3} - 15\log\left(\sin\left(bx+a\right)+1\right) + 15\log\left(\sin\left(bx+a\right)-1\right)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(2*(15*sin(b*x + a)^4 - 10*sin(b*x + a)^2 - 2)/(sin(b*x + a)^5 - sin(
b*x + a)^3) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{6\sin(bx+a)}{\sin(bx+a)^2-1} + \frac{4\left(6\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 15\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 15\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{12\,b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(b
*x + a)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))/
b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{2\,b} - \frac{-\frac{5\,{\sin\left(a+b\,x\right)}^4}{2}+\frac{5\,{\sin\left(a+b\,x\right)}^2}{3}+\frac{1}{3}}{b\,\left({\sin\left(a+b\,x\right)}^3-{\sin\left(a+b\,x\right)}^5\right)}$$

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^4),x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(2*b) - ((5*sin(a + b*x)^2)/3 - (5*sin(a + b*x)^4)/
2 + 1/3)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

3.169 $\int \csc^4(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] -3*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+3*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2700, 276}

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{3\cot(a+bx)}{b}$$

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + (3*Tan[a + b*x])/b + Tan[a + b
*x]^3/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = 16\left(-\frac{\cot(2(a+bx))}{3b}-\frac{\cot(2(a+bx))\csc^2(2(a+bx))}{6b}\right)$$

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
[Out] 16*(-1/3*Cot[2*(a + b*x)]/b - (Cot[2*(a + b*x)]*Csc[2*(a + b*x)]^2)/(6*b))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result
risch	$\frac{32i\left(3\,e^{4i(bx+a)}-1\right)}{3b\left(e^{2i(bx+a)}-1\right)^3\left(e^{2i(bx+a)}+1\right)^3}$
derivativedivides	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)^3}-\frac{2}{3\cos(bx+a)\sin(bx+a)^3}+\frac{8}{3\sin(bx+a)\cos(bx+a)}-\frac{16\cot(bx+a)}{3}}{b}$
default	$\frac{\frac{1}{3\cos(bx+a)^3\sin(bx+a)^3}-\frac{2}{3\cos(bx+a)\sin(bx+a)^3}+\frac{8}{3\sin(bx+a)\cos(bx+a)}-\frac{16\cot(bx+a)}{3}}{b}$
parallelrisch	$\frac{\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)+30\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-273\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\cot^3\left(\frac{bx}{2}+\frac{a}{2}\right)+420\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+30\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-273\tan}{24b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$
norman	$\frac{\frac{1}{24b}+\frac{5\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{91\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{35\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}-\frac{91\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{5\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$

[In] int(sec(b*x+a)^4/sin(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] 32/3*I*(3*exp(4*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1)^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{16\,\cos\left(bx+a\right)^6 - 24\,\cos\left(bx+a\right)^4 + 6\,\cos\left(bx+a\right)^2 + 1}{3\left(b\cos\left(bx+a\right)^5 - b\cos\left(bx+a\right)^3\right)\sin\left(bx+a\right)}$$

[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")

[Out] -1/3*(16*cos(b*x + a)^6 - 24*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 1)/((b*cos(b*x + a)^5 - b*cos(b*x + a)^3)*sin(b*x + a))

Sympy [F]

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \int \frac{\sec^4\left(a+bx\right)}{\sin^4\left(a+bx\right)}\,dx$$

[In] integrate(sec(b*x+a)**4/sin(b*x+a)**4,x)

[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3 - \frac{9\,\tan(bx+a)^2+1}{\tan(bx+a)^3} + 9\,\tan\left(bx+a\right)}{3\,b}$$

[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")

[Out] 1/3*(tan(b*x + a)^3 - (9*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 + 9*tan(b*x + a))/b

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{8\left(3\,\tan\left(2\,bx+2\,a\right)^2+1\right)}{3\,b\tan\left(2\,bx+2\,a\right)^3}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
[Out] -8/3*(3*tan(2*b*x + 2*a)^2 + 1)/(b*tan(2*b*x + 2*a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{-\tan(a+b\,x)^6-9\tan(a+b\,x)^4+9\tan(a+b\,x)^2+1}{3\,b\tan\left(a+b\,x\right)^3}$$

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^4),x)
[Out] -(9*tan(a + b*x)^2 - 9*tan(a + b*x)^4 - tan(a + b*x)^6 + 1)/(3*b*tan(a + b*
x)^3)
```

3.170 $\int \csc^4(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\text{arctanh}(\sin(a+bx))}{8b} - \frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}$$

```
[Out] 35/8*arctanh(sin(b*x+a))/b-35/8*csc(b*x+a)/b-35/24*csc(b*x+a)^3/b+7/8*csc(b
*x+a)^3*sec(b*x+a)^2/b+1/4*csc(b*x+a)^3*sec(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2701, 294, 308, 213}

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\text{arctanh}(\sin(a+bx))}{8b} - \frac{35\csc^3(a+bx)}{24b} - \frac{35\csc(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b}$$

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(8*b) - (35*Csc[a + b*x])/(8*b) - (35*Csc[a + b*
x]^3)/(24*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]^3*Se
c[a + b*x]^4)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{b}\\
&= \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{4b}\\
&= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}\\
&\quad - \frac{35\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{8b}\\
&= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}\\
&\quad - \frac{35\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx)\right)}{8b}\\
&= -\frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b}\\
&\quad + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{8b}
\end{aligned}$$

$$= \frac{35\text{arctanh}(\sin(a+bx))}{8b} - \frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)^3}-\frac{7}{12\cos(bx+a)^2\sin(bx+a)^3}+\frac{35}{24\cos(bx+a)^2\sin(bx+a)}-\frac{35}{8\sin(bx+a)}+\frac{35\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{1}{4\cos(bx+a)^4\sin(bx+a)^3}-\frac{7}{12\cos(bx+a)^2\sin(bx+a)^3}+\frac{35}{24\cos(bx+a)^2\sin(bx+a)}-\frac{35}{8\sin(bx+a)}+\frac{35\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(105\,e^{13i(bx+a)}+70\,e^{11i(bx+a)}-329\,e^{9i(bx+a)}-204\,e^{7i(bx+a)}-329\,e^{5i(bx+a)}+70\,e^{3i(bx+a)}+105\,e^{i(bx+a)}\right)}{12b\left(e^{2i(bx+a)}+1\right)^4\left(e^{2i(bx+a)}-1\right)^3}-\frac{35\ln(e^{i(}}{8}$
norman	$\frac{-\frac{1}{24b}-\frac{35\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{63\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{35\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{35\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{63\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{35\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{24b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}$
parallelrisch	$\frac{35\left(\csc^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\sec^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\left(\sin(7bx+7a)-3\sin(bx+a)-3\sin(3bx+3a)+\sin(5bx+5a)\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-\mathrm{s}\right.}{512b(\cos(4bx+}$

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/4/cos(b*x+a)^4/sin(b*x+a)^3-7/12/cos(b*x+a)^2/sin(b*x+a)^3+35/24/cos
(b*x+a)^2/sin(b*x+a)-35/8/sin(b*x+a)+35/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx =$$
$$-\frac{210\cos(bx+a)^6 - 280\cos(bx+a)^4 - 105\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(b}{48\left(b\cos(bx+a)^6 - b\,c\right.}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)/(
(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \int \frac{\sec^5(a+bx)}{\sin^4(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx =$$
$$-\frac{\frac{2\left(105\sin(bx+a)^6 - 175\sin(bx+a)^4 + 56\sin(bx+a)^2 + 8\right)}{\sin(bx+a)^7 - 2\sin(bx+a)^5 + \sin(bx+a)^3} - 105\log(\sin(bx+a)+1) + 105\log(\sin(bx+a)-1)}{48\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(105*sin(b*x + a)^6 - 175*sin(b*x + a)^4 + 56*sin(b*x + a)^2 + 8)/
(sin(b*x + a)^7 - 2*sin(b*x + a)^5 + sin(b*x + a)^3) - 105*log(sin(b*x + a)
 + 1) + 105*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{6\left(11\sin(bx+a)^3-13\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16\left(9\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-105\log\left(|\sin\left(bx+a\right)+1|\right)+105\log\left(|\sin\left(bx+a\right)-1|\right)}{48\,b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/48*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16*
(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(abs(sin(b*x + a) + 1)) + 10
5*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{8\,b} - \frac{\frac{35\sin(a+b\,x)^6}{8}-\frac{175\sin(a+b\,x)^4}{24}+\frac{7\sin(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\sin\left(a+b\,x\right)^7-2\sin\left(a+b\,x\right)^5+\sin\left(a+b\,x\right)^3\right)}$$

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^4),x)
```

```
[Out] (35*atanh(sin(a + b*x)))/(8*b) - ((7*sin(a + b*x)^2)/3 - (175*sin(a + b*x)^
4)/24 + (35*sin(a + b*x)^6)/8 + 1/3)/(b*(sin(a + b*x)^3 - 2*sin(a + b*x)^5
+ sin(a + b*x)^7))
```

3.171 $\int \cos^4(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{2\csc^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{6\log(\sin(a+bx))}{b} - \frac{2\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}$$

```
[Out] 2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+6*ln(sin(b*x+a))/b-2*sin(b*x+a)^2/b+1/4
*sin(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2670, 272, 45}

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{\sin^4(a+bx)}{4b} - \frac{2\sin^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{2\csc^2(a+bx)}{b} + \frac{6\log(\sin(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

```
[Out] (2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2*
Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^5}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, \sin^2(a+bx)\right)}{2b}\\
&= \frac{2\csc^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{6\log(\sin(a+bx))}{b}-\frac{2\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{2\csc^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{6\log(\sin(a+bx))}{b}-\frac{2\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}$$

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
[Out] (2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2*
Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{\cos^{10}(bx+a)}{4\sin(bx+a)^4}+\frac{3\left(\cos^{10}(bx+a)\right)}{4\sin(bx+a)^2}+\frac{3\left(\cos^8(bx+a)\right)}{4}+\cos^6(bx+a)+\frac{3\left(\cos^4(bx+a)\right)}{2}+3\left(\cos^2(bx+a)\right)+6\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\cos^{10}(bx+a)}{4\sin(bx+a)^4}+\frac{3\left(\cos^{10}(bx+a)\right)}{4\sin(bx+a)^2}+\frac{3\left(\cos^8(bx+a)\right)}{4}+\cos^6(bx+a)+\frac{3\left(\cos^4(bx+a)\right)}{2}+3\left(\cos^2(bx+a)\right)+6\ln(\sin(bx+a))}{b}$
risch	$-6ix+\frac{e^{4i(bx+a)}}{64b}+\frac{7\,e^{2i(bx+a)}}{16b}+\frac{7\,e^{-2i(bx+a)}}{16b}+\frac{e^{-4i(bx+a)}}{64b}-\frac{12ia}{b}-\frac{4\left(2\,e^{6i(bx+a)}-3\,e^{4i(bx+a)}+2\,e^{2i(bx+}\right.}{b\left(e^{2i(bx+a)}-1\right)^4}$
parallelrisch	$-\frac{3\left(\left(\frac{3}{4}-\cos(2bx+2a)+\frac{\cos(4bx+4a)}{4}\right)\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(\cos(2bx+2a)-\frac{\cos(4bx+4a)}{4}-\frac{3}{4}\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\frac{71\cos(2b}{192}\right.}{16b}$
norman	$\frac{-\frac{1}{64b}+\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+\frac{3\left(\tan^{14}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{\tan^{16}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{93\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{93\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}-\frac{591\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^10+3/4/sin(b*x+a)^2*cos(b*x+a)^10+3/4*cos
(b*x+a)^8+cos(b*x+a)^6+3/2*cos(b*x+a)^4+3*cos(b*x+a)^2+6*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{8\,\cos(bx+a)^8+32\,\cos(bx+a)^6-115\,\cos(bx+a)^4+38\,\cos(bx+a)^2+192\left(\cos(bx+a)^4-2\,\cos(bx\right.}{32\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/32*(8*cos(b*x + a)^8 + 32*cos(b*x + a)^6 - 115*cos(b*x + a)^4 + 38*cos(b*
x + a)^2 + 192*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)
) + 29)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1664 vs. 2(58) = 116.

Time = 7.68 (sec) , antiderivative size = 1664, normalized size of antiderivative = 24.12

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-384*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**12/(64*b*tan
(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8
+ 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1536*log(tan(a/2
+ b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*ta
n(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6
+ 64*b*tan(a/2 + b*x/2)**4) - 2304*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b
*x/2)**8/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*ta
n(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) -
 1536*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/
2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan
(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 384*log(tan(a/2 + b*x/2)**2
+ 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2
)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/
2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**12/(64*b*tan(a
/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 +
256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/2 +
b*x/2))*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b
*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*ta
n(a/2 + b*x/2)**4) + 2304*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(64*b*t
an(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**
8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2
+ b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b
*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b
*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)
**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*x
/2)**16/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
24*tan(a/2 + b*x/2)**14/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)
**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2
 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b
*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)*
*6 + 64*b*tan(a/2 + b*x/2)**4) - 1182*tan(a/2 + b*x/2)**8/(64*b*tan(a/2 + b
*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*
tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**6/(
```

```
64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*
x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 24*tan(a/
2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384
*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)*
*4) - 1/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), N
e(b, 0)), (x*cos(a)**9/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{\sin(bx+a)^4 - 8\,\sin(bx+a)^2 + \frac{8\,\sin(bx+a)^2-1}{\sin(bx+a)^4} + 12\,\log\left(\sin(bx+a)^2\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(b*x + a)^4 - 8*sin(b*x + a)^2 + (8*sin(b*x + a)^2 - 1)/sin(b*x + a
)^4 + 12*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(65) = 130$.

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 4.01

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = -\frac{\frac{\left(\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{288(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{32\left(\frac{84(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{126(\cos(bx\cdots}{(\cos(bx+\cdots}\cdots}{\cdots}}{64\,b}$$

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 288*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
 1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 126*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
25*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos
(b*x + a) + 1) - 1)^4 - 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1
)) + 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{6\ \ln\left(\tan(a+b\,x)\right)}{b} - \frac{3\ \ln\left(\tan(a+b\,x)^2+1\right)}{b} + \frac{3\tan(a+b\,x)^6+\frac{9\tan(a+b\,x)^4}{2}+\tan(a+b\,x)^2-\frac{1}{4}}{b\,\left(\tan\left(a+b\,x\right)^8+2\tan\left(a+b\,x\right)^6+\tan\left(a+b\,x\right)^4\right)}$$

```
[In] int(cos(a + b*x)^9/sin(a + b*x)^5,x)
```

```
[Out] (6*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/b + (tan(a + b*x)^2 +
 (9*tan(a + b*x)^4)/2 + 3*tan(a + b*x)^6 - 1/4)/(b*(tan(a + b*x)^4 + 2*tan(
a + b*x)^6 + tan(a + b*x)^8))
```

3.172 $\int \cos^3(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{8b}+\frac{35\cos(a+bx)}{8b}+\frac{35\cos^3(a+bx)}{24b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}$$

```
[Out] -35/8*arctanh(cos(b*x+a))/b+35/8*cos(b*x+a)/b+35/24*cos(b*x+a)^3/b+7/8*cos(
b*x+a)^3*cot(b*x+a)^2/b-1/4*cos(b*x+a)^3*cot(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2672, 294, 308, 212}

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{8b}+\frac{35\cos^3(a+bx)}{24b}+\frac{35\cos(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}$$

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Cos[a + b*x])/(8*b) + (35*Cos[a + b
*x]^3)/(24*b) + (7*Cos[a + b*x]^3*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]^3*C
ot[a + b*x]^4)/(4*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)^3}\,dx, x, \cos(a+bx)\right)}{b}$$

$$= -\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} + \frac{7\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{4b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{8b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cos(a+bx)\right)}{8b}$$

$$= \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{8b}$$

$$= -\frac{35\operatorname{arctanh}(\cos(a+bx))}{8b} + \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \frac{13\cos(a+bx)}{4b} + \frac{\cos(3(a+bx))}{12b} + \frac{13\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{35\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{35\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{13\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (13*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) + (13*Csc[(a + b*x)/2]^2)
/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (35*Log[Cos[(a + b*x)/2]])/(8*b) + (3
5*Log[Sin[(a + b*x)/2]])/(8*b) - (13*Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a +
b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{\cos^9(bx+a)}{4\sin(bx+a)^4}+\frac{5\left(\cos^9(bx+a)\right)}{8\sin(bx+a)^2}+\frac{5\left(\cos^7(bx+a)\right)}{8}+\frac{7\left(\cos^5(bx+a)\right)}{8}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{8}+\frac{35\ln(\csc(bx+a)-\cot(bx+a}{8}}{b}$
default	$\frac{-\frac{\cos^9(bx+a)}{4\sin(bx+a)^4}+\frac{5\left(\cos^9(bx+a)\right)}{8\sin(bx+a)^2}+\frac{5\left(\cos^7(bx+a)\right)}{8}+\frac{7\left(\cos^5(bx+a)\right)}{8}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{8}+\frac{35\ln(\csc(bx+a)-\cot(bx+a}{8}}{b}$
parallelrisch	$\frac{\left(\csc^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\sec^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(-3360\left(\cos(2bx+2a)-\frac{\cos(4bx+4a)}{4}-\frac{3}{4}\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+840\cos(bx+a)-3388\cos(2\right.}{24576b}$
norman	$\frac{-\frac{1}{64b}+\frac{21\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}-\frac{21\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{\tan^{14}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{21\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{511\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}+\frac{847\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{96b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$
risch	$\frac{e^{3i(bx+a)}}{24b}+\frac{13\,e^{i(bx+a)}}{8b}+\frac{13\,e^{-i(bx+a)}}{8b}+\frac{e^{-3i(bx+a)}}{24b}-\frac{13\,e^{7i(bx+a)}-5\,e^{5i(bx+a)}-5\,e^{3i(bx+a)}+13\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}-$

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*cos(b*x+a)^9/sin(b*x+a)^4+5/8/sin(b*x+a)^2*cos(b*x+a)^9+5/8*cos(b
*x+a)^7+7/8*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/8*cos(b*x+a)+35/8*ln(csc(b*x
+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.48

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \frac{16\,\cos(bx+a)^7 + 112\,\cos(bx+a)^5 - 350\,\cos(bx+a)^3 - 105\,\left(\cos(bx+a)^4 - 2\,\cos(bx+a)^2 + 1\right)\log\left(\frac{1}{2}\right.}{48\,\left(b\cos(bx+a)^4 - \right.}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(16*cos(b*x + a)^7 + 112*cos(b*x + a)^5 - 350*cos(b*x + a)^3 - 105*(co
s(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 105*(cos
(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 210*cos(
b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(80) = 160$.

Time = 4.65 (sec) , antiderivative size = 869, normalized size of antiderivative = 9.76

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((840*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(192*b*tan(a/2 +
b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*
tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x
/2)**6/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(
a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 840*log(tan(a/2 + b*x/2))*ta
n(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 +
 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 3*tan(a/2 + b*x/2
)**14/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a
/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 63*tan(a/2 + b*x/2)**12/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2016*tan(a/2 + b*x/2)**8/(192*b*tan(a/2
```

```
+ b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*
b*tan(a/2 + b*x/2)**4) + 3066*tan(a/2 + b*x/2)**6/(192*b*tan(a/2 + b*x/2)**
10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2
+ b*x/2)**4) + 1694*tan(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b
*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**
4) + 63*tan(a/2 + b*x/2)**2/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b
*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 3/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**8/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \frac{16\cos(bx+a)^3 - \frac{6\left(13\cos(bx+a)^3-11\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} + 144\cos(bx+a) - 105\log(\cos(bx+a)+1) + 105\log(\cos(bx+a)-1)}{48\,b}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*cos(b*x + a)^3 - 6*(13*cos(b*x + a)^3 - 11*cos(b*x + a))/(cos(b*x
+ a)^4 - 2*cos(b*x + a)^2 + 1) + 144*cos(b*x + a) - 105*log(cos(b*x + a) +
1) + 105*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(79) = 158.

Time = 0.35 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.35

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3} - 420\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{192\,b}$$

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x +
 a) + 1)^2 - 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) -
1)^3 - 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.76

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b}-\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}+\frac{35\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$
$$+\frac{\frac{67\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8}{8}+\frac{839\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{64}+\frac{1487\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{192}+\frac{21\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{64}-\frac{1}{64}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\right)}$$

```
[In] int(cos(a + b*x)^8/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - (3*tan(a/2 + (b*x)/2)^2)/(8*b) + (35*log(tan(
a/2 + (b*x)/2)))/(8*b) + ((21*tan(a/2 + (b*x)/2)^2)/64 + (1487*tan(a/2 + (b
*x)/2)^4)/192 + (839*tan(a/2 + (b*x)/2)^6)/64 + (67*tan(a/2 + (b*x)/2)^8)/8
 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + 3*tan(a/2 + (b*x)/2)^6 + 3*tan(a/2 + (b
*x)/2)^8 + tan(a/2 + (b*x)/2)^10))
```

3.173 $\int \cos^2(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{3\csc^2(a+bx)}{2b} - \frac{\csc^4(a+bx)}{4b} + \frac{3\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{2b}$$

```
[Out] 3/2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+3*ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2670, 272, 45}

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = -\frac{\sin^2(a+bx)}{2b} - \frac{\csc^4(a+bx)}{4b} + \frac{3\csc^2(a+bx)}{2b} + \frac{3\log(\sin(a+bx))}{b}$$

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

```
[Out] (3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b -
 Sin[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, \sin^2(a+bx)\right)}{2b}\\ &= \frac{3\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{3\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
[Out] (3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b -
 Sin[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{\cos^8(bx+a)}{4\sin(bx+a)^4}+\frac{\cos^8(bx+a)}{2\sin(bx+a)^2}+\frac{\left(\cos^6(bx+a)\right)}{2}+\frac{3\left(\cos^4(bx+a)\right)}{4}+\frac{3\left(\cos^2(bx+a)\right)}{2}+3\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\cos^8(bx+a)}{4\sin(bx+a)^4}+\frac{\cos^8(bx+a)}{2\sin(bx+a)^2}+\frac{\left(\cos^6(bx+a)\right)}{2}+\frac{3\left(\cos^4(bx+a)\right)}{4}+\frac{3\left(\cos^2(bx+a)\right)}{2}+3\ln(\sin(bx+a))}{b}$
risch	$-3ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}-\frac{6ia}{b}-\frac{2\left(3\,e^{6i(bx+a)}-4\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}-1\right)^4}+\frac{3\ln\left(e^{2i(bx+a)}-1\right)}{b}$
parallelrisch	$\frac{-192\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+192\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(-128\cos(bx+a)+16\cos(2bx+2a)+159)\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+34\left(\cot^2\left(\frac{bx}{2}\right.\right.}{64b}$
norman	$\frac{-\frac{1}{64b}+\frac{9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}+\frac{9\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{83\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\ln\left(1+\tan^2\right.}{b}$

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*cos(b*x+a)^8/sin(b*x+a)^4+1/2/sin(b*x+a)^2*cos(b*x+a)^8+1/2*cos(b
*x+a)^6+3/4*cos(b*x+a)^4+3/2*cos(b*x+a)^2+3*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx$$

$$=\frac{2\cos(bx+a)^6-5\cos(bx+a)^4-2\cos(bx+a)^2+12\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\sin(b\right.}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^6 - 5*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 12*(cos(b*x +
 a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)) + 4)/(b*cos(b*x + a)^4
- 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(48) = 96$.

Time = 3.22 (sec) , antiderivative size = 733, normalized size of antiderivative = 12.64

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-192*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(64*b*tan(
a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 3
84*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)*
*8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 192*log(tan(a/
2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**8 + 128*b*ta
n(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 192*log(tan(a/2 + b*x/2))*t
an(a/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 +
64*b*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(
64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2
)**4) + 192*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2
)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*
x/2)**12/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a
/2 + b*x/2)**4) + 18*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**8 + 128*b
*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 166*tan(a/2 + b*x/2)**6/
(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/
2)**4) + 18*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 +
 b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1/(64*b*tan(a/2 + b*x/2)**8 + 128*
b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**7/
sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = -\frac{2\sin(bx+a)^2 - \frac{6\sin(bx+a)^2-1}{\sin(bx+a)^4} - 6\log\left(\sin(bx+a)^2\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)^2 - (6*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 - 6*log(sin(
b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(52) = 104$.

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.00

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx =$$

$$-\frac{\frac{20\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{\frac{18\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{111\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{72\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2}-96\,\log\left(\frac{|-c}{|c}\right.}{64\,b}$$

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 111*(cos(b*x
 + a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
 1)^3 - 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/((cos(b*x + a) -
1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 - 96*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 192*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{3\,\ln\left(\tan(a+b\,x)\right)}{b}-\frac{3\,\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}+\frac{\frac{3\tan(a+b\,x)^4}{2}+\frac{3\tan(a+b\,x)^2}{4}-\frac{1}{4}}{b\,\left(\tan\left(a+b\,x\right)^6+\tan\left(a+b\,x\right)^4\right)}$$

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^5,x)
```

```
[Out] (3*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/(2*b) + ((3*tan(a + b
*x)^2)/4 + (3*tan(a + b*x)^4)/2 - 1/4)/(b*(tan(a + b*x)^4 + tan(a + b*x)^6)
)
```

3.174 $\int \cos(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{8b}+\frac{15\cos(a+bx)}{8b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}$$

```
[Out] -15/8*arctanh(cos(b*x+a))/b+15/8*cos(b*x+a)/b+5/8*cos(b*x+a)*cot(b*x+a)^2/b
-1/4*cos(b*x+a)*cot(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2672, 294, 327, 212}

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{8b}+\frac{15\cos(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}$$

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Cos[a + b*x])/(8*b) + (5*Cos[a + b*
x]*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]*Cot[a + b*x]^4)/(4*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\cos(a+bx)\cot^4(a+bx)}{4b} + \frac{5\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{4b}\\
&= \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{8b}\\
&= \frac{15\cos(a+bx)}{8b} + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b}\\
&\quad - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{8b}\\
&= -\frac{15\text{arctanh}(\cos(a+bx))}{8b} + \frac{15\cos(a+bx)}{8b}\\
&\quad + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \cos(a+bx)\cot^5(a+bx)\,dx &= \frac{\cos(a+bx)}{b}+\frac{9\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} \\ &\quad -\frac{15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b} \\ &\quad -\frac{9\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}\end{aligned}$$

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

```
[Out] Cos[a + b*x]/b + (9*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b)
- (15*Log[Cos[(a + b*x)/2]])/(8*b) + (15*Log[Sin[(a + b*x)/2]])/(8*b) - (9*
Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{\cos^7(bx+a)}{4\sin(bx+a)^4}+\frac{3\left(\cos^7(bx+a)\right)}{8\sin(bx+a)^2}+\frac{3\left(\cos^5(bx+a)\right)}{8}+\frac{5\left(\cos^3(bx+a)\right)}{8}+\frac{15\cos(bx+a)}{8}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{\cos^7(bx+a)}{4\sin(bx+a)^4}+\frac{3\left(\cos^7(bx+a)\right)}{8\sin(bx+a)^2}+\frac{3\left(\cos^5(bx+a)\right)}{8}+\frac{5\left(\cos^3(bx+a)\right)}{8}+\frac{15\cos(bx+a)}{8}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
parallelrisch	$\frac{120\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-15\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+15\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-160}{64b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
norman	$\frac{-\frac{1}{64b}+\frac{15\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}-\frac{15\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{15\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}-\frac{9\,e^{7i(bx+a)}-e^{5i(bx+a)}-e^{3i(bx+a)}+9\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}-\frac{15\ln\left(e^{i(bx+a)}+1\right)}{8b}+\frac{15\ln\left(e^{i(bx+a)}-1\right)}{8b}$

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*cos(b*x+a)^7/sin(b*x+a)^4+3/8*cos(b*x+a)^7/sin(b*x+a)^2+3/8*cos(b
*x+a)^5+5/8*cos(b*x+a)^3+15/8*cos(b*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \frac{16\cos(bx+a)^5 - 50\cos(bx+a)^3 - 15\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \cdots}{16\left(b\cos(bx+a)^4 - 2\,b\cos(bx \cdots\right.}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*cos(b*x + a)^5 - 50*cos(b*x + a)^3 - 15*(cos(b*x + a)^4 - 2*cos(b*
x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^4 - 2*cos(b*x
+ a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 30*cos(b*x + a))/(b*cos(b*x + a)
^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(63) = 126$.

Time = 1.71 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.71

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \begin{cases} \frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{\tan^{10}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{15\tan \cdots}{64b\tan^6\left(\frac{a}{2}+\cdots\right.} \\ \frac{x\cos^6(a)}{\sin^5(a)} \end{cases}$$

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((120*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*
x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 120*log(tan(a/2 + b*x/2))*tan(a/2 + b
*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + tan(a/2 +
b*x/2)**10/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 15*tan(a
/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 160*
tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
 15*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**
4) - 1/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x
*cos(a)**6/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{\frac{2\left(9\cos(bx+a)^3-7\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} - 16\cos(bx+a) + 15\log(\cos(bx+a)+1) - 15\log(\cos(bx+a)-1)}{16\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(9*cos(b*x + a)^3 - 7*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
a)^2 + 1) - 16*cos(b*x + a) + 15*log(cos(b*x + a) + 1) - 15*log(cos(b*x + a
) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.34

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - 60\log\left(\frac{|-\mathrm{co}}{|\mathrm{cos}}\right.}{64\,b}$$

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 90*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - 60*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{4\,b} + \frac{15\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b} + \frac{\frac{9\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{4} + \frac{15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{64} - \frac{1}{64}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6 + \tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\right)}$$

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(4*b) + (15*log(tan(a/2
+ (b*x)/2)))/(8*b) + ((15*tan(a/2 + (b*x)/2)^2)/64 + (9*tan(a/2 + (b*x)/2)^
4)/4 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + tan(a/2 + (b*x)/2)^6))
```

3.175 $\int \cot^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \cot^5(a+bx)\,dx = \frac{\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\sin(a+bx))}{b}$$

```
[Out] 1/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(sin(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 3556}

$$\int \cot^5(a+bx)\,dx = -\frac{\cot^4(a+bx)}{4b} + \frac{\cot^2(a+bx)}{2b} + \frac{\log(\sin(a+bx))}{b}$$

```
[In] Int[Cot[a + b*x]^5,x]
```

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cot^4(a+bx)}{4b} - \int \cot^3(a+bx)\,dx \\
&= \frac{\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \int \cot(a+bx)\,dx \\
&= \frac{\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\sin(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \cot^5(a+bx)\,dx = \frac{\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\cos(a+bx))}{b} + \frac{\log(\tan(a+bx))}{b}$$

```
[In] Integrate[Cot[a + b*x]^5,x]
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Cos[a + b*x]]/b + Log[Tan
[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{\left(\cot^4(bx+a)\right)}{4}+\frac{\left(\cot^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\left(\cot^4(bx+a)\right)}{4}+\frac{\left(\cot^2(bx+a)\right)}{2}+\ln(\sin(bx+a))}{b}$
risch	$-ix-\frac{2ia}{b}-\frac{4\left(e^{6i(bx+a)}-e^{4i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$
parallelrisch	$\frac{-\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+12\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+12\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+64\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-64\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{64b}$
norman	$\frac{-\frac{1}{64b}+\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b}+\frac{3\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/4*cot(b*x+a)^4+1/2*cot(b*x+a)^2+ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \cot^5(a+bx)\,dx = -\frac{4\,\cos\left(bx+a\right)^2-4\,\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)-3}{4\,\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^2 - 4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*
sin(b*x + a)) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \cot^5(a+bx)\,dx = \begin{cases} \frac{\log\left(\sin\left(a+bx\right)\right)}{b}+\frac{\cos^2\left(a+bx\right)}{2b\sin^2\left(a+bx\right)}-\frac{\cos^4\left(a+bx\right)}{4b\sin^4\left(a+bx\right)} & \text{for } b\neq 0 \\ \frac{x\cos^5\left(a\right)}{\sin^5\left(a\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((log(sin(a + b*x))/b + cos(a + b*x)**2/(2*b*sin(a + b*x)**2) - co
s(a + b*x)**4/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)**5/sin(a)**5, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \cot^5(a+bx)\,dx = \frac{\frac{4\,\sin\left(bx+a\right)^2-1}{\sin\left(bx+a\right)^4}+2\,\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(38) = 76.

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.90

$$\int \cot^5(a+bx)\,dx = -\frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{48\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-32\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+}{64\,b}$$

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 48*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 - 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 64*log(abs(-(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \cot^5(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)\right)}{b}-\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}+\frac{\frac{\tan(a+b\,x)^2}{2}-\frac{1}{4}}{b\tan\left(a+b\,x\right)^4}$$

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^5,x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2/2 - 1
/4)/(b*tan(a + b*x)^4)
```

3.176 $\int \cot^4(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \cot^4(a+bx)\csc(a+bx)\,dx=-\frac{3\text{arctanh}(\cos(a+bx))}{8b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

```
[Out] -3/8*arctanh(cos(b*x+a))/b+3/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)^3*csc
(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2691, 3855}

$$\int \cot^4(a+bx)\csc(a+bx)\,dx=-\frac{3\text{arctanh}(\cos(a+bx))}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}$$

```
[In] Int[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Cot[a + b*x]*Csc[a + b*x])/(8*b) - (C
ot[a + b*x]^3*Csc[a + b*x])/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot^3(a+bx)\csc(a+bx)}{4b} - \frac{3}{4}\int \cot^2(a+bx)\csc(a+bx)\,dx\\ &= \frac{3\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot^3(a+bx)\csc(a+bx)}{4b} + \frac{3}{8}\int \csc(a+bx)\,dx\\ &= -\frac{3\text{arctanh}(\cos(a+bx))}{8b} + \frac{3\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot^3(a+bx)\csc(a+bx)}{4b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \cot^4(a+bx)\csc(a+bx)\,dx = & \frac{5\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}\\ & - \frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b} + \frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}\\ & - \frac{5\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}\end{aligned}$$

```
[In] Integrate[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (5*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (3*Log[Cos[(a +
 b*x)/2]])/(8*b) + (3*Log[Sin[(a + b*x)/2]])/(8*b) - (5*Sec[(a + b*x)/2]^2)
/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result	size
parallelrisch	$\frac{\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+24\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}$	69
derivativedivides	$\frac{-\frac{\cos^5(bx+a)}{4\sin(bx+a)^4}+\frac{\cos^5(bx+a)}{8\sin(bx+a)^2}+\frac{\left(\cos^3(bx+a)\right)}{8}+\frac{3\cos(bx+a)}{8}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	78
default	$\frac{-\frac{\cos^5(bx+a)}{4\sin(bx+a)^4}+\frac{\cos^5(bx+a)}{8\sin(bx+a)^2}+\frac{\left(\cos^3(bx+a)\right)}{8}+\frac{3\cos(bx+a)}{8}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	78
norman	$\frac{-\frac{1}{64b}+\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	83
risch	$-\frac{5\,e^{7i(bx+a)}+3\,e^{5i(bx+a)}+3\,e^{3i(bx+a)}+5\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}-\frac{3\ln\left(e^{i(bx+a)}+1\right)}{8b}+\frac{3\ln\left(e^{i(bx+a)}-1\right)}{8b}$	99

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(tan(1/2*b*x+1/2*a)^4-cot(1/2*b*x+1/2*a)^4-8*tan(1/2*b*x+1/2*a)^2+8*co
t(1/2*b*x+1/2*a)^2+24*ln(tan(1/2*b*x+1/2*a)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{10\,\cos(bx+a)^3+3\left(\cos(bx+a)^4-2\,\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)-3\left(\cos(bx+a)^4-2\right.}{16\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(b*x + a)^3 + 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/
2*cos(b*x + a) + 1/2) - 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*
cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^
2 + b)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \begin{cases} \frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b} + \frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b} - \frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b \neq 0 \\ \frac{x\cos^4(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

[In] integrate(cos(b*x+a)**4/sin(b*x+a)**5,x)

[Out] Piecewise((3*log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - tan (a/2 + b*x/2)**2/(8*b) + 1/(8*b*tan(a/2 + b*x/2)**2) - 1/(64*b*tan(a/2 + b* x/2)**4), Ne(b, 0)), (x*cos(a)**4/sin(a)**5, True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{\frac{2\left(5\cos(bx+a)^3-3\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} + 3\log\left(\cos\left(bx+a\right)+1\right) - 3\log\left(\cos\left(bx+a\right)-1\right)}{16b}$$

[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")

[Out] -1/16*(2*(5*cos(b*x + a)^3 - 3*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1) + 3*log(cos(b*x + a) + 1) - 3*log(cos(b*x + a) - 1))/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{18(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 12\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64b}$$

[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")

```
[Out] -1/64*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 18*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 8*(cos
(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 12*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} + \frac{3\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b} + \frac{\cot\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\left(\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8}-\frac{1}{64}\right)}{b}$$

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(8*b) + (3*log(tan(a/2 +
 (b*x)/2)))/(8*b) + (cot(a/2 + (b*x)/2)^4*(tan(a/2 + (b*x)/2)^2/8 - 1/64))/
b
```

3.177 $\int \cot^3(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^4(a+bx)}{4b}$$

```
[Out] -1/4*cot(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2687, 30}

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^4(a+bx)}{4b}$$

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

```
[Out] -1/4*Cot[a + b*x]^4/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^3\,dx, x, -\cot(a+bx)\right)}{b}\\ &= -\frac{\cot^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot^4(a+bx)}{4b}$$

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
[Out] -1/4*Cot[a + b*x]^4/b
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$-\frac{\cos^4(bx+a)}{4\sin(bx+a)^4 b}$	22
default	$-\frac{\cos^4(bx+a)}{4\sin(bx+a)^4 b}$	22
risch	$-\frac{2\left(e^{6i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}-1\right)^4}$	38
parallelrisch	$\frac{-\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+4\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}$	59
norman	$\frac{-\frac{1}{64b}+\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b}+\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$	67

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/4*cos(b*x+a)^4/sin(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{2\cos(bx+a)^2-1}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \begin{cases} \frac{1}{4b\sin^2(a+bx)} - \frac{\cos^2(a+bx)}{4b\sin^4(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**5,x)
[Out] Piecewise((1/(4*b*sin(a + b*x)**2) - cos(a + b*x)**2/(4*b*sin(a + b*x)**4),
 Ne(b, 0)), (x*cos(a)**3/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \frac{2\sin(bx+a)^2-1}{4\,b\sin(bx+a)^4}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \frac{2\,\sin\left(bx+a\right)^2-1}{4\,b\sin\left(bx+a\right)^4}$$

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\left(\sin\left(a+b\,x\right)^2-1\right)^2}{4\,b\sin\left(a+b\,x\right)^4}$$

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^5,x)
[Out] -(sin(a + b*x)^2 - 1)^2/(4*b*sin(a + b*x)^4)
```

3.178 $\int \cot^2(a+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cos(a+bx))}{8b} + \frac{\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

```
[Out] 1/8*arctanh(cos(b*x+a))/b+1/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)*csc(b*
x+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2691, 3853, 3855}

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cos(a+bx))}{8b} - \frac{\cot(a+bx)\csc^3(a+bx)}{4b} + \frac{\cot(a+bx)\csc(a+bx)}{8b}$$

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(8*b) + (Cot[a + b*x]*Csc[a + b*x])/(8*b) - (Cot[a +
b*x]*Csc[a + b*x]^3)/(4*b)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cot(a+bx)\csc^3(a+bx)}{4b}-\frac{1}{4}\int\csc^3(a+bx)\,dx\\ &= \frac{\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}-\frac{1}{8}\int\csc(a+bx)\,dx\\ &= \frac{\operatorname{arctanh}(\cos(a+bx))}{8b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\begin{aligned}\int\cot^2(a+bx)\csc^3(a+bx)\,dx = {} &\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}\\ &+\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}\\ &-\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}\end{aligned}$$

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
[Out] Csc[(a + b*x)/2]^2/(32*b) - Csc[(a + b*x)/2]^4/(64*b) + Log[Cos[(a + b*x)/2
]]/(8*b) - Log[Sin[(a + b*x)/2]]/(8*b) - Sec[(a + b*x)/2]^2/(32*b) + Sec[(a
 + b*x)/2]^4/(64*b)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

method	result	size
norman	$\frac{-\frac{1}{64b}+\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$	51
parallelrisch	$\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}{64b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$	53
derivativedivides	$\frac{-\frac{\cos^3(bx+a)}{4\sin(bx+a)^4}-\frac{\cos^3(bx+a)}{8\sin(bx+a)^2}-\frac{\cos(bx+a)}{8}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	68
default	$\frac{-\frac{\cos^3(bx+a)}{4\sin(bx+a)^4}-\frac{\cos^3(bx+a)}{8\sin(bx+a)^2}-\frac{\cos(bx+a)}{8}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	68
risch	$-\frac{e^{7i(bx+a)}+7e^{5i(bx+a)}+7e^{3i(bx+a)}+e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{i(bx+a)}+1\right)}{8b}-\frac{\ln\left(e^{i(bx+a)}-1\right)}{8b}$	95

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] (-1/64/b+1/64/b*tan(1/2*b*x+1/2*a)^8)/tan(1/2*b*x+1/2*a)^4-1/8/b*ln(tan(1/2
*b*x+1/2*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(49) = 98.

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = -\frac{2\cos(bx+a)^3-\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\left(\cos(bx+a)^4-2\cos\right.}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/16*(2*cos(b*x + a)^3 - (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*c
os(b*x + a) + 1/2) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b
*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b
)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b} + \frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b} - \frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b \neq 0 \\ \frac{x\cos^2(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - 1/(6
4*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**2/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = -\frac{\frac{2\left(\cos(bx+a)^3+\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} - \log(\cos(bx+a)+1) + \log(\cos(bx+a)-1)}{16\,b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(cos(b*x + a)^3 + cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2
 + 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\frac{\left(\frac{2(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((2*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^
2/(cos(b*x + a) - 1)^2 + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 4*log(
abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{1}{64\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - 1/(64*b*tan(a/2 + (b*x)/2)^4) - log(tan(a/2 +
 (b*x)/2))/(8*b)
```

3.179 $\int \cot(a+bx)\csc^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{\csc^4(a+bx)}{4b}$$

```
[Out] -1/4*csc(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2686, 30}

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{\csc^4(a+bx)}{4b}$$

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^4,x]
```

```
[Out] -1/4*Csc[a + b*x]^4/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^3\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{\csc^4(a+bx)}{4b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{\csc^4(a+bx)}{4b}$$

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^4,x]
[Out] -1/4*Csc[a + b*x]^4/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{1}{4\sin(bx+a)^4 b}$	14
default	$-\frac{1}{4\sin(bx+a)^4 b}$	14
risch	$-\frac{4\,e^{4i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^4}$	28
parallelrisch	$\frac{-\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}$	59
norman	$\frac{-\frac{1}{64b}-\frac{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b}-\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)}{16b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$	67

```
[In] int(cos(b*x+a)/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/4/sin(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\left(b\cos\left(bx+a\right)^4 - 2\,b\cos\left(bx+a\right)^2 + b\right)}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = \begin{cases} -\frac{1}{4b\sin^4{(a+bx)}} & \text{for } b \neq 0 \\ \frac{x\cos{(a)}}{\sin^5{(a)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**5,x)
[Out] Piecewise((-1/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)/sin(a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\,b\sin\left(bx+a\right)^4}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/4/(b*sin(b*x + a)^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\,b\sin\left(bx+a\right)^4}$$

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
[Out] -1/4/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{\cot(a+b\,x)^2\,\left(\cot(a+b\,x)^2+2\right)}{4\,b}$$

```
[In] int(cos(a + b*x)/sin(a + b*x)^5,x)
[Out] -(cot(a + b*x)^2*(cot(a + b*x)^2 + 2))/(4*b)
```

3.180 $\int \csc^5(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\cot^2(a+bx)}{b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\tan(a+bx))}{b}$$

```
[Out] -cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(tan(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2700, 272, 45}

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\cot^4(a+bx)}{4b} - \frac{\cot^2(a+bx)}{b} + \frac{\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -(Cot[a + b*x]^2/b) - Cot[a + b*x]^4/(4*b) + Log[Tan[a + b*x]]/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx, x, \tan^2(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)\,dx, x, \tan^2(a+bx)\right)}{2b} \\
&= -\frac{\cot^2(a+bx)}{b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\tan(a+bx))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b} - \frac{\csc^4(a+bx)}{4b} - \frac{\log(\cos(a+bx))}{b} + \frac{\log(\sin(a+bx))}{b}$$

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -1/2*Csc[a + b*x]^2/b - Csc[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b + Log[Si
n[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(bx+a)^4}-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{4\sin(bx+a)^4}-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{6i(bx+a)}-8\,e^{4i(bx+a)}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
parallelrisch	$\frac{-\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+64\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-64\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-\right.}{64b}$
norman	$\frac{-\frac{1}{64b}-\frac{3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b}-\frac{3\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16b}-\frac{\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\right.\right.}{b}$

```
[In] int(sec(b*x+a)/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/sin(b*x+a)^4-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(38) = 76.

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \csc^5(a+bx)\sec(a+bx)\,dx$$
$$=\frac{2\,\cos\left(bx+a\right)^2-2\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\log\left(\cos\left(bx+a\right)^2\right)+2\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+\right.\right.}{4\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(cos(b
*x + a)^2) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/4*cos(b*x + a
)^2 + 1/4) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F]

$$\int \csc^5(a+bx)\sec(a+bx)\,dx=\int \frac{\sec\left(a+bx\right)}{\sin^5\left(a+bx\right)}\,dx$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2\,\sin(bx+a)^2+1}{\sin(bx+a)^4}+2\,\log\left(\sin\left(bx+a\right)^2-1\right)-2\,\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 + 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2 - 1) - 2
*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 4.12

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \frac{\frac{\left(\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{48\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+32\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-64}{64\,b}$$

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 48*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 64*log(abs(-(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \frac{\ln\left(\frac{\cos(2\,a+2\,b\,x)}{2}-\frac{1}{2}\right)}{2\,b}-\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{b}-\frac{\frac{\cos(2\,a+2\,b\,x)}{4}-\frac{1}{2}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)-{\left(\frac{\cos(2\,a+2\,b\,x)}{2}+\frac{1}{2}\right)}^2\right)}$$

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^5),x)
```

```
[Out] log(cos(2*a + 2*b*x)/2 - 1/2)/(2*b) - log(cos(a + b*x))/b - (cos(2*a + 2*b*
x)/4 - 1/2)/(b*(cos(2*a + 2*b*x) - (cos(2*a + 2*b*x)/2 + 1/2)^2))
```

3.181 $\int \csc^5(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{8b}+\frac{15\sec(a+bx)}{8b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}$$

```
[Out] -15/8*arctanh(cos(b*x+a))/b+15/8*sec(b*x+a)/b-5/8*csc(b*x+a)^2*sec(b*x+a)/b
-1/4*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2702, 294, 327, 213}

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{8b}+\frac{15\sec(a+bx)}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Sec[a + b*x])/(8*b) - (5*Csc[a + b*
x]^2*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{b} \\
&= -\frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b} \\
&= -\frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} \\
&\quad + \frac{15\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b} \\
&= \frac{15\sec(a+bx)}{8b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} \\
&\quad - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{15\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b} \\
&= -\frac{15\text{arctanh}(\cos(a+bx))}{8b} + \frac{15\sec(a+bx)}{8b} \\
&\quad - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = -\frac{14\csc^2\left(\frac{1}{2}(a+bx)\right)+\csc^4\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(78+\cos(a+bx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right.\right.\right.}{-1+\tan^2\left(\frac{1}{2}(a+bx)\right)}}{64b}$$

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] -1/64*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(78
 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2]
]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^2
))/b
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{1}{4\cos(bx+a)\sin(bx+a)^4}-\frac{5}{8\cos(bx+a)\sin(bx+a)^2}+\frac{15}{8\cos(bx+a)}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{1}{4\cos(bx+a)\sin(bx+a)^4}-\frac{5}{8\cos(bx+a)\sin(bx+a)^2}+\frac{15}{8\cos(bx+a)}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
parallelrisch	$\frac{\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)+\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right)+15\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+120\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+15\left(\cot^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\ln(}{64b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$
norman	$\frac{\frac{1}{64b}+\frac{15\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{15\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{15\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$
risch	$\frac{15\,e^{9i(bx+a)}-40\,e^{7i(bx+a)}+18\,e^{5i(bx+a)}-40\,e^{3i(bx+a)}+15\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)}-\frac{15\ln\left(e^{i(bx+a)}+1\right)}{8b}+\frac{15\ln\left(e^{i(bx+a)}-1\right)}{8b}$

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/cos(b*x+a)/sin(b*x+a)^4-5/8/cos(b*x+a)/sin(b*x+a)^2+15/8/cos(b*x+
a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.89

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{30\cos(bx+a)^4 - 50\cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a\right.}{16\left(b\cos(bx+a)^5 - 2b\cos(bx+a\right.}$$

[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")

[Out] 1/16*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))

Sympy [F]

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \int \frac{\sec^2(a+bx)}{\sin^5(a+bx)}\,dx$$

[In] integrate(sec(b*x+a)**2/sin(b*x+a)**5,x)

[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\frac{2\left(15\cos(bx+a)^4 - 25\cos(bx+a)^2 + 8\right)}{\cos(bx+a)^5 - 2\cos(bx+a)^3 + \cos(bx+a)} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{16b}$$

[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")

[Out] 1/16*(2*(15*cos(b*x + a)^4 - 25*cos(b*x + a)^2 + 8)/(cos(b*x + a)^5 - 2*cos(b*x + a)^3 + cos(b*x + a)) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.33

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$=\frac{\frac{\left(\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\,\log\left(\frac{|-\cos}{|\cos(}\right.}{64\,b}$$

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx=\frac{\frac{15\cos(a+b\,x)^4}{8}-\frac{25\cos(a+b\,x)^2}{8}+1}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{8\,b}$$

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^5),x)
```

```
[Out] ((15*cos(a + b*x)^4)/8 - (25*cos(a + b*x)^2)/8 + 1)/(b*(cos(a + b*x) - 2*co
s(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(8*b)
```

3.182 $\int \csc^5(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{3\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{3\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

```
[Out] -3/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+3*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/
b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2700, 272, 45}

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \frac{\tan^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} - \frac{3\cot^2(a+bx)}{2b} + \frac{3\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(2*b) - Cot[a + b*x]^4/(4*b) + (3*Log[Tan[a + b*x]])/b
+ Tan[a + b*x]^2/(2*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= -\frac{3\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\frac{3\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{4b}$$

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
[Out] -1/4*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Sin
[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{4\cos(bx+a)^2\sin(bx+a)^4}+\frac{3}{4\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{4\cos(bx+a)^2\sin(bx+a)^4}+\frac{3}{4\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
risch	$\frac{6\,e^{10i(bx+a)}-12\,e^{8i(bx+a)}-4\,e^{6i(bx+a)}-12\,e^{4i(bx+a)}+6\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)^2}+\frac{3\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{3\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{-\frac{1}{64b}-\frac{9\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}-\frac{9\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}-\frac{\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{83\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\right.\right.}{b}$
parallelrisch	$\left(-192\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+384\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-192\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\left(-192\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+384\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\right.$

[In] int(sec(b*x+a)^3/sin(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] 1/b*(-1/4/cos(b*x+a)^2/sin(b*x+a)^4+3/4/cos(b*x+a)^2/sin(b*x+a)^2-3/2/sin(b*x+a)^2+3*ln(tan(b*x+a)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(52) = 104.

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$=\frac{6\cos(bx+a)^4-9\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right)+}{4\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4+b\right.}$$

[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")

[Out] 1/4*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)

Sympy [F]

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \int \frac{\sec^3(a+bx)}{\sin^5(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**5,x)
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{6\sin(bx+a)^4-3\sin(bx+a)^2-1}{\sin(bx+a)^6-\sin(bx+a)^4}+6\log\left(\sin(bx+a)^2-1\right)-6\log\left(\sin(bx+a)^2\right)}{4b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/4*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/(sin(b*x + a)^6 - sin(b*x +
 a)^4) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(52) = 104$.

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.00

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2}+96\log\left(\frac{|-\cos(}{|\cos(b}\right.}{64b}$$

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
[Out] 1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(
b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 111*(cos(b*x
+ a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
1)^3 + 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 1)/((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 + 96*lo
g(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 192*log(abs(-(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \frac{3\ \ln\left(\sin\left(a+b\,x\right)^2\right)}{2\,b}-\frac{3\ \ln\left(\cos\left(a+b\,x\right)\right)}{b}+\frac{\frac{3\cos\left(a+b\,x\right)^4}{2}-\frac{9\cos\left(a+b\,x\right)^2}{4}+\frac{1}{2}}{b\ \left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^5),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)^
4)/2 - (9*cos(a + b*x)^2)/4 + 1/2)/(b*(cos(a + b*x)^2 - 2*cos(a + b*x)^4 +
cos(a + b*x)^6))
```

3.183 $\int \csc^5(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx=-\frac{35\text{arctanh}(\cos(a+bx))}{8b}+\frac{35\sec(a+bx)}{8b}+\frac{35\sec^3(a+bx)}{24b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}$$

```
[Out] -35/8*arctanh(cos(b*x+a))/b+35/8*sec(b*x+a)/b+35/24*sec(b*x+a)^3/b-7/8*csc(
b*x+a)^2*sec(b*x+a)^3/b-1/4*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2702, 294, 308, 213}

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx=-\frac{35\text{arctanh}(\cos(a+bx))}{8b}+\frac{35\sec^3(a+bx)}{24b}+\frac{35\sec(a+bx)}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}$$

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Sec[a + b*x])/(8*b) + (35*Sec[a + b
*x]^3)/(24*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(8*b) - (Csc[a + b*x]^4*S
ec[a + b*x]^3)/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{b}\\
&= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b}\\
&= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}\\
&\quad + \frac{35\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b}\\
&= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}\\
&\quad + \frac{35\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+bx)\right)}{8b}\\
&= \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}\\
&\quad - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{35\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b}
\end{aligned}
$$

$$= -\frac{35\text{arctanh}(\cos(a+bx))}{8b} + \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(89) = 178$.

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.01

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = -\frac{\csc^{10}(a+bx)\,(-204 + 658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx$$

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] -1/24*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
+ 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*Co
s[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
+ 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2]
]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log[
Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(a
 + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{1}{4\cos(bx+a)^3\sin(bx+a)^4}+\frac{7}{12\cos(bx+a)^3\sin(bx+a)^2}-\frac{35}{24\cos(bx+a)\sin(bx+a)^2}+\frac{35}{8\cos(bx+a)}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{1}{4\cos(bx+a)^3\sin(bx+a)^4}+\frac{7}{12\cos(bx+a)^3\sin(bx+a)^2}-\frac{35}{24\cos(bx+a)\sin(bx+a)^2}+\frac{35}{8\cos(bx+a)}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
risch	$\frac{105\,e^{13i(bx+a)}-70\,e^{11i(bx+a)}-329\,e^{9i(bx+a)}+204\,e^{7i(bx+a)}-329\,e^{5i(bx+a)}-70\,e^{3i(bx+a)}+105\,e^{i(bx+a)}}{12b(e^{2i(bx+a)}-1)^4(e^{2i(bx+a)}+1)^3}-\frac{35\ln(e^{i(bx+}}{8b}$
norman	$\frac{\frac{1}{64b}+\frac{21\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{21\left(\tan^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{64b}+\frac{\tan^{14}\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{21\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}+\frac{511\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b}-\frac{847\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{96b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}$
parallelrisch	$\frac{840\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+63\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\left(\cot^4\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{192b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4/cos(b*x+a)^3/sin(b*x+a)^4+7/12/cos(b*x+a)^3/sin(b*x+a)^2-35/24/co
s(b*x+a)/sin(b*x+a)^2+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{210\cos(bx+a)^6 - 350\cos(bx+a)^4 + 112\cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx\right.}{48\left(b\cos(bx+a)^7 - 2\right.}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/48*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(c
os(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1
/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*cos
(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x +
 a)^3)
```

Sympy [F]

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \int \frac{\sec^4(a+bx)}{\sin^5(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**5,x)
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(105\cos(bx+a)^6 - 175\cos(bx+a)^4 + 56\cos(bx+a)^2 + 8\right)}{\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3} - 105\log(\cos(bx+a)+1) + 105\log(\cos(bx+a)-1)}{48\,b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")
[Out] 1/48*(2*(105*cos(b*x + a)^6 - 175*cos(b*x + a)^4 + 56*cos(b*x + a)^2 + 8)/(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3) - 105*log(cos(b*x + a)
+ 1) + 105*log(cos(b*x + a) - 1))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(79) = 158$.

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.35

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{6(\cos(bx+a}{(\cos(bx+a)}\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}}{192\,b}$$

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 72
*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1
)^3 + 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{8}-\frac{175\cos(a+b\,x)^4}{24}+\frac{7\cos(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{8\,b}$$

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^5),x)
```

```
[Out] ((7*cos(a + b*x)^2)/3 - (175*cos(a + b*x)^4)/24 + (35*cos(a + b*x)^6)/8 + 1
/3)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*atanh(co
s(a + b*x)))/(8*b)
```

3.184 $\int \csc^5(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = -\frac{2\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{6\log(\tan(a+bx))}{b}+\frac{2\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}$$

```
[Out] -2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+6*ln(tan(b*x+a))/b+2*tan(b*x+a)^2/b+1/
4*tan(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2700, 272, 45}

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{\tan^4(a+bx)}{4b}+\frac{2\tan^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}-\frac{2\cot^2(a+bx)}{b}+\frac{6\log(\tan(a+bx))}{b}$$

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] (-2*Cot[a + b*x]^2)/b - Cot[a + b*x]^4/(4*b) + (6*Log[Tan[a + b*x]])/b + (2
*Tan[a + b*x]^2)/b + Tan[a + b*x]^4/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^5}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{2b}\\
&= -\frac{2\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{6\log(\tan(a+bx))}{b}+\frac{2\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = 32\left(-\frac{3\csc^2(a+bx)}{64b}-\frac{\csc^4(a+bx)}{128b}-\frac{3\log(\cos(a+bx))}{16b}+\frac{3\log(\sin(a+bx))}{16b}+\frac{3\sec^2(a+bx)}{64b}+\frac{\sec^4(a+bx)}{128b}\right)$$

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] 32*((-3*Csc[a + b*x]^2)/(64*b) - Csc[a + b*x]^4/(128*b) - (3*Log[Cos[a + b*
x]])/(16*b) + (3*Log[Sin[a + b*x]])/(16*b) + (3*Sec[a + b*x]^2)/(64*b) + Se
c[a + b*x]^4/(128*b))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{1}{4\sin(bx+a)^4\cos(bx+a)^4}-\frac{1}{2\cos(bx+a)^2\sin(bx+a)^4}+\frac{3}{2\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{\sin(bx+a)^2}+6\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\sin(bx+a)^4\cos(bx+a)^4}-\frac{1}{2\cos(bx+a)^2\sin(bx+a)^4}+\frac{3}{2\cos(bx+a)^2\sin(bx+a)^2}-\frac{3}{\sin(bx+a)^2}+6\ln(\tan(bx+a))}{b}$
risch	$\frac{12\,e^{14i(bx+a)}-44\,e^{10i(bx+a)}-44\,e^{6i(bx+a)}+12\,e^{2i(bx+a)}}{b(e^{2i(bx+a)}-1)^4(e^{2i(bx+a)}+1)^4}+\frac{6\ln(e^{2i(bx+a)}-1)}{b}-\frac{6\ln(e^{2i(bx+a)}+1)}{b}$
parallelrisch	$\frac{(-384\cos(2bx+2a)-96\cos(4bx+4a)-288)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-384\cos(2bx+2a)-96\cos(4bx+4a)-288)\ln\left(\tan\left(\frac{bx}{2}+\right.\right.}{}$

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4/sin(b*x+a)^4/cos(b*x+a)^4-1/2/cos(b*x+a)^2/sin(b*x+a)^4+3/2/cos(b*
x+a)^2/sin(b*x+a)^2-3/sin(b*x+a)^2+6*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.14

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{12\cos(bx+a)^6-18\cos(bx+a)^4+4\cos(bx+a)^2-12\left(\cos(bx+a)^8-2\cos(bx+a)^6+\cos(bx+a)^4\right)}{4\left(b\cos(bx+a)^8-2\,b\cos(\right.}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(12*cos(b*x + a)^6 - 18*cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 12*(cos(b*x
 + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(cos(b*x + a)^2) + 12*(cos(
b*x + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1
/4) + 1)/(b*cos(b*x + a)^8 - 2*b*cos(b*x + a)^6 + b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \int \frac{\sec^5(a+bx)}{\sin^5(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{12\sin(bx+a)^6-18\sin(bx+a)^4+4\sin(bx+a)^2+1}{\sin(bx+a)^8-2\sin(bx+a)^6+\sin(bx+a)^4}+12\log\left(\sin(bx+a)^2-1\right)-12\log\left(\sin(bx+a)^2\right)}{4b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((12*sin(b*x + a)^6 - 18*sin(b*x + a)^4 + 4*sin(b*x + a)^2 + 1)/(sin(b
*x + a)^8 - 2*sin(b*x + a)^6 + sin(b*x + a)^4) + 12*log(sin(b*x + a)^2 - 1)
 - 12*log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(65) = 130.

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.03

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{\frac{\left(\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{288(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{32\left(\frac{84(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{126(\cos(bx+a)}{(\cos(bx+a)+}\right.}{\left(\frac{\mathrm{co}}{\mathrm{co}}\right.}}{64b}$$

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 288*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 126*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 2
5*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(
b*x + a) + 1) + 1)^4 + 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)
) - 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{2\tan(a+b\,x)^2}{b} + \frac{\tan(a+b\,x)^4}{4\,b} + \frac{6\,\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\cot(a+b\,x)^4\,\left(2\tan(a+b\,x)^2+\frac{1}{4}\right)}{b}$$

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^5),x)
```

```
[Out] (2*tan(a + b*x)^2)/b + tan(a + b*x)^4/(4*b) + (6*log(tan(a + b*x)))/b - (co
t(a + b*x)^4*(2*tan(a + b*x)^2 + 1/4))/b
```

3.185 $\int \cot^2(x) \csc^4(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^2(x) \csc^4(x) \, dx = -\frac{1}{3} \cot^3(x) - \frac{\cot^5(x)}{5}$$

[Out] -1/3*cot(x)^3-1/5*cot(x)^5

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2687, 14}

$$\int \cot^2(x) \csc^4(x) \, dx = -\frac{1}{5} \cot^5(x) - \frac{\cot^3(x)}{3}$$

[In] Int[Cot[x]^2*Csc[x]^4,x]

[Out] -1/3*Cot[x]^3 - Cot[x]^5/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, -\cot(x)\right)\\
&= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, -\cot(x)\right)\\
&= -\frac{1}{3}\cot^3(x)-\frac{\cot^5(x)}{5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cot^2(x)\csc^4(x)\,dx=\frac{2\cot(x)}{15}+\frac{1}{15}\cot(x)\csc^2(x)-\frac{1}{5}\cot(x)\csc^4(x)$$

```
[In] Integrate[Cot[x]^2*Csc[x]^4,x]
[Out] (2*Cot[x])/15 + (Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{2(\cot^5(x))}{15}-\frac{(\cot^3(x))(\csc^2(x))}{3}$	18
default	$-\frac{\cos^3(x)}{5\sin(x)^5}-\frac{2(\cos^3(x))}{15\sin(x)^3}$	22
risch	$-\frac{4i(15\,e^{6ix}+5\,e^{4ix}+5\,e^{2ix}-1)}{15(e^{2ix}-1)^5}$	36
norman	$\frac{-\frac{1}{160}-\frac{(\tan^2(\frac{x}{2}))}{96}+\frac{(\tan^4(\frac{x}{2}))}{16}-\frac{(\tan^6(\frac{x}{2}))}{16}+\frac{(\tan^8(\frac{x}{2}))}{96}+\frac{(\tan^{10}(\frac{x}{2}))}{160}}{\tan(\frac{x}{2})^5}$	50

```
[In] int(cos(x)^2/sin(x)^6,x,method=_RETURNVERBOSE)
[Out] 2/15*cot(x)^5-1/3*cot(x)^3*csc(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{2\,\cos(x)^5 - 5\,\cos(x)^3}{15\,\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\sin(x)}$$

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="fricas")
[Out] 1/15*(2*cos(x)^5 - 5*cos(x)^3)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{2\cos(x)}{15\sin(x)} + \frac{\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

```
[In] integrate(cos(x)**2/sin(x)**6,x)
[Out] 2*cos(x)/(15*sin(x)) + cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{5\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="maxima")
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{5\,\tan(x)^2+3}{15\,\tan(x)^5}$$

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="giac")
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \cot^2(x)\csc^4(x)\,dx = -\cos(x)^3\left(\frac{2}{15\sin(x)^3}+\frac{1}{5\sin(x)^5}\right)$$

```
[In] int(cos(x)^2/sin(x)^6,x)
[Out] -cos(x)^3*(2/(15*sin(x)^3) + 1/(5*sin(x)^5))
```

3.186 $\int \cot^3(x) \csc^4(x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x) \csc^4(x) \, dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[Out] 1/4*csc(x)^4-1/6*csc(x)^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2686, 14}

$$\int \cot^3(x) \csc^4(x) \, dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\, dx, x, \csc(x)\right) \\ &= -\text{Subst}\left(\int \left(-x^3+x^5\right)\, dx, x, \csc(x)\right) \\ &= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx=\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\cos^4(x)}{6\sin(x)^6}-\frac{\cos^4(x)}{12\sin(x)^4}$	22
norman	$\frac{-\frac{1}{384}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{384}}{\tan\left(\frac{x}{2}\right)^6}$	34
risch	$\frac{4\,e^{8ix}+\frac{8\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}-1\right)^6}$	34
parallelrisch	$\frac{-\left(\tan^{12}\left(\frac{x}{2}\right)\right)+9\left(\tan^8\left(\frac{x}{2}\right)\right)+9\left(\tan^4\left(\frac{x}{2}\right)\right)-1}{384\tan\left(\frac{x}{2}\right)^6}$	35

```
[In] int(cos(x)^3/sin(x)^7,x,method=_RETURNVERBOSE)
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\cos(x)^2 - 1}{12\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

```
[In] integrate(cos(x)**3/sin(x)**7,x)
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\frac{\sin(x)^2}{4}-\frac{1}{6}}{\sin(x)^6}$$

```
[In] int(cos(x)^3/sin(x)^7,x)
[Out] (sin(x)^2/4 - 1/6)/sin(x)^6
```

3.187 $\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[Out] -2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^{3/2}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$	19
default	$-\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$	19

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2\sqrt{d\cos(bx+a)}d\cos(bx+a)^2}{5\,b}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
[Out] -2/5*sqrt(d*cos(b*x + a))*d*cos(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = \begin{cases} -\frac{2(d\cos(a+bx))^{\frac{3}{2}}\cos(a+bx)}{5b} & \text{for } b\neq 0 \\ x(d\cos(a))^{\frac{3}{2}}\sin(a) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a),x)
[Out] Piecewise((-2*(d*cos(a + b*x))**(3/2)*cos(a + b*x)/(5*b), Ne(b, 0)), (x*(d*
cos(a))**(3/2)*sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
[Out] -2/5*(d*cos(b*x + a))^(5/2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(3/2),x)
[Out] -(2*(d*cos(a + b*x))^(5/2))/(5*b*d)
```

3.188 $\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$
\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}
$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3bd}$	19
default	$-\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3bd}$	19

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$
\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{3b}
$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = \begin{cases} -\frac{2\sqrt{d\cos(a+bx)}\cos(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sqrt{d\cos(a)}\sin(a) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a),x)
[Out] Piecewise((-2*sqrt(d*cos(a + b*x))*cos(a + b*x)/(3*b), Ne(b, 0)), (x*sqrt(d
*cos(a))*sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,(d\cos(bx+a))^{\frac{3}{2}}}{3\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
[Out] -2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,\sqrt{d\cos(bx+a)}\cos(bx+a)}{3\,b}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}{3\,b\,d}$$

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(1/2),x)
[Out] -(2*(d*cos(a + b*x))^(3/2))/(3*b*d)
```

3.189 $\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] -2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[In] Int[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$	19
default	$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$	19
risch	$-\frac{2\cos(bx+a)}{\sqrt{d\cos(bx+a)}\,b}$	22

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(d*cos(b*x+a))^(1/2)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \begin{cases} -\frac{2\cos(a+bx)}{b\sqrt{d\cos(a+bx)}} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{\sqrt{d\cos(a)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(1/2),x)
[Out] Piecewise((-2*cos(a + b*x)/(b*sqrt(d*cos(a + b*x))), Ne(b, 0)), (x*sin(a)/s
qrt(d*cos(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}}{bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d}$$

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(1/2),x)
[Out] -(2*(d*cos(a + b*x))^(1/2))/(b*d)
```

3.190 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{2}{bd\sqrt{d\cos(bx+a)}}$	19
default	$\frac{2}{bd\sqrt{d\cos(bx+a)}}$	19

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d/(d*cos(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{bd^2\cos(bx+a)}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(d*cos(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \begin{cases} \frac{2\cos{(a+bx)}}{b(d\cos{(a+bx)})^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin{(a)}}{(d\cos{(a)})^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(3/2),x)
[Out] Piecewise((2*cos(a + b*x)/(b*(d*cos(a + b*x))**(3/2)), Ne(b, 0)), (x*sin(a)
/(d*cos(a))**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{\sqrt{d\cos{(bx+a)}}bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/(sqrt(d*cos(b*x + a))*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{4\,\cos\left(a+b\,x\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(3/2),x)
[Out] (4*cos(a + b*x)*(d*cos(a + b*x))^(1/2))/(b*d^2*(cos(2*a + 2*b*x) + 1))
```

3.191 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2645, 30}

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{3bd(d\cos(bx+a))^{\frac{3}{2}}}$	19
default	$\frac{2}{3bd(d\cos(bx+a))^{\frac{3}{2}}}$	19

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{3\,bd^3\cos(bx+a)^2}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*cos(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 3.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \begin{cases} \frac{2\cos{(a+bx)}}{3b(d\cos{(a+bx)})^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin{(a)}}{(d\cos{(a)})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(5/2),x)
[Out] Piecewise((2*cos(a + b*x)/(3*b*(d*cos(a + b*x))**(5/2)), Ne(b, 0)), (x*sin(
a)/(d*cos(a))**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3\,(d\cos{(bx+a)})^{\frac{3}{2}}\,bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] 2/3/((d*cos(b*x + a))^(3/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{8\left(\cos\left(2\,a+2\,b\,x\right)+1\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}{3\,b\,d^3\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+\cos\left(4\,a+4\,b\,x\right)+3\right)}$$

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] (8*(cos(2*a + 2*b*x) + 1)*(d*cos(a + b*x))^(1/2))/(3*b*d^3*(4*cos(2*a + 2*b
*x) + cos(4*a + 4*b*x) + 3))
```

3.192 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{5bd(d\cos(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{5bd(d\cos(bx+a))^{\frac{5}{2}}}$	19
default	$\frac{2}{5bd(d\cos(bx+a))^{\frac{5}{2}}}$	19

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{5\,bd^4\cos(bx+a)^3}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
[Out] 2/5*sqrt(d*cos(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 29.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \begin{cases} \frac{2\cos{(a+bx)}}{5b(d\cos{(a+bx)})^{\frac{7}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin{(a)}}{(d\cos{(a)})^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Piecewise((2*cos(a + b*x)/(5*b*(d*cos(a + b*x))**(7/2)), Ne(b, 0)), (x*sin(
a)/(d*cos(a))**(7/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5\,(d\cos{(bx+a)})^{\frac{5}{2}}\,bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5/((d*cos(b*x + a))^(5/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\cos{(bx+a)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{16\,\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}{5\,b\,d^4\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] (16*exp(a*3i + b*x*3i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(5*b*d^4*(exp(a*2i + b*x*2i) + 1)^3)
```

3.193 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2645, 30}

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{7bd(d\cos(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{7bd(d\cos(bx+a))^{\frac{7}{2}}}$	19
default	$\frac{2}{7bd(d\cos(bx+a))^{\frac{7}{2}}}$	19

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{7\,bd^5\cos(bx+a)^4}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] 2/7*sqrt(d*cos(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7\,(d\cos(bx+a))^{\frac{7}{2}}\,bd}$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] 2/7/((d*cos(b*x + a))^(7/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(bx+a)}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{32\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}{7\,b\,d^5\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(9/2),x)
[Out] (32*exp(a*4i + b*x*4i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(7*b*d^5*(exp(a*2i + b*x*2i) + 1)^4)
```

3.194 $\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \frac{28d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{195b\sqrt{\cos(a+bx)}} + \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}$$

```
[Out] 28/585*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+4/117*d*(d*cos(b*x+a))^(7/2)*s
in(b*x+a)/b-2/13*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d+28/195*d^4*(cos(1/2*a
+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))
*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2719}

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \frac{28d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{195b\sqrt{\cos(a+bx)}} + \frac{28d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{585b} - \frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd} + \frac{4d\sin(a+bx)(d\cos(a+bx))^{7/2}}{117b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

```
[Out] (28*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(195*b*Sqrt[Cos[a +
 b*x]]) + (28*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(585*b) + (4*d*(d*Co
s[a + b*x])^(7/2)*Sin[a + b*x])/(117*b) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a
+ b*x])/(13*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}+\frac{2}{13}\int(d\cos(a+bx))^{9/2}\,dx\\
&= \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b}-\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}\\
&\quad+\frac{1}{117}\left(14d^2\right)\int(d\cos(a+bx))^{5/2}\,dx\\
&= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b}+\frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b}\\
&\quad-\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}+\frac{1}{195}\left(14d^4\right)\int\sqrt{d\cos(a+bx)}\,dx
\end{aligned}
$$

$$= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd} + \frac{\left(14d^4\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{195\sqrt{\cos(a+bx)}}$$

$$= \frac{28d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{195b\sqrt{\cos(a+bx)}} + \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \frac{d^2(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]

[Out] (d^2*(d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.98

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^5\left(2880\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-11520\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+19280\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-17520\left(\text{co}\right.\right.}{585\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\right.}}$

[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 4/585*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(2880*cos(1/2*b*x+1/2*a)^15-11520*cos(1/2*b*x+1/2*a)^13+19280*cos(1/2*b*x+1/2*a)^11-17520*cos(1/2*b*x+1/2*a)^9+9284*cos(1/2*b*x+1/2*a)^7-2808*cos(1/2*b*x+1/2*a)^5+425*cos(1/2*b*x+1/2*a)^3+21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-21*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx =$$

$$-\frac{2\left(-21i\sqrt{2}d^{\frac{9}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+21i\sqrt{2}d^{\frac{9}{2}}\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/585*(-21*I*sqrt(2)*d^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(b*x + a) + I*sin(b*x + a))) + 21*I*sqrt(2)*d^(9/2)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + (45*d^
4*cos(b*x + a)^5 - 10*d^4*cos(b*x + a)^3 - 14*d^4*cos(b*x + a))*sqrt(d*cos(
b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{9/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2), x)
```

3.195 $\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \frac{20d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}$$

```
[Out] 4/77*d*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b-2/11*(d*cos(b*x+a))^(9/2)*sin(b*x+
a)/b/d+20/231*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF
(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+20/231
*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \frac{20d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{20d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b} - \frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd} + \frac{4d\sin(a+bx)(d\cos(a+bx))^{5/2}}{77b}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

```
[Out] (20*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
 b*x]]) + (20*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (4*d*(d*Cos[
a + b*x])^(5/2)*Sin[a + b*x])/(77*b) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*
x])/(11*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}+\frac{2}{11}\int (d\cos(a+bx))^{7/2}\,dx\\
&= \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b}-\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}\\
&\quad +\frac{1}{77}\left(10d^2\right)\int (d\cos(a+bx))^{3/2}\,dx\\
&= \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b}+\frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b}\\
&\quad -\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}+\frac{1}{231}\left(10d^4\right)\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx
\end{aligned}
$$

$$= \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd} + \frac{\left(10d^4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{231\sqrt{d\cos(a+bx)}}$$

$$= \frac{20d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \frac{d^2(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
[Out] (d^2*(d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4,
3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.87

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^4\left(672\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2352\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3312\left(\cos^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2400\left(\cos^{7}\left(\frac{bx}{2}+\right.\right.\right.}{231\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}}$

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 4/231*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(672*co
s(1/2*b*x+1/2*a)^13-2352*cos(1/2*b*x+1/2*a)^11+3312*cos(1/2*b*x+1/2*a)^9-24
00*cos(1/2*b*x+1/2*a)^7+922*cos(1/2*b*x+1/2*a)^5-159*cos(1/2*b*x+1/2*a)^3-5
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(co
s(1/2*b*x+1/2*a),2^(1/2))+5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4
-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2
-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(5i\sqrt{2}d^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-5i\sqrt{2}d^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -2/231*(5*I*sqrt(2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin
(b*x + a)) - 5*I*sqrt(2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) -
I*sin(b*x + a)) + (21*d^3*cos(b*x + a)^4 - 6*d^3*cos(b*x + a)^2 - 10*d^3)*s
qrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{7/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2), x)
```

3.196 $\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \frac{4d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} + \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b} - \frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}$$

```
[Out] 4/45*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-2/9*(d*cos(b*x+a))^(7/2)*sin(b*x+a
)/b/d+4/15*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(si
n(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2719}

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \frac{4d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}} - \frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd} + \frac{4d\sin(a+bx)(d\cos(a+bx))^{3/2}}{45b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b
*x]]) + (4*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(45*b) - (2*(d*Cos[a + b*
x])^(7/2)*Sin[a + b*x])/(9*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}+\frac{2}{9}\int(d\cos(a+bx))^{5/2}\,dx\\
&= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}\\
&\quad+\frac{1}{15}\left(2d^2\right)\int\sqrt{d\cos(a+bx)}\,dx\\
&= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}\\
&\quad+\frac{\left(2d^2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{15\sqrt{\cos(a+bx)}}\\
&= \frac{4d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}}\\
&\quad+\frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \frac{(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(110) = 220.

Time = 1.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^3\left(80\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-240\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+272\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-144\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}{45\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)}$

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 4/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(80*cos(
1/2*b*x+1/2*a)^11-240*cos(1/2*b*x+1/2*a)^9+272*cos(1/2*b*x+1/2*a)^7-144*cos
(1/2*b*x+1/2*a)^5+35*cos(1/2*b*x+1/2*a)^3+3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1
-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3*cos(
1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/si
n(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(-3i\sqrt{2}d^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+3i\sqrt{2}d^{\frac{5}{2}}\mathrm{we}\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/45*(-3*I*sqrt(2)*d^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*d^(5/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + (5*d^2*co
s(b*x + a)^3 - 2*d^2*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2), x)
```

3.197 $\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \frac{4d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b\sqrt{d\cos(a+bx)}} + \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b} - \frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}$$

```
[Out] -2/7*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b/d+4/21*d^2*(cos(1/2*a+1/2*b*x)^2)^(1
/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/
2)/b/(d*cos(b*x+a))^(1/2)+4/21*d*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2648, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \frac{4d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b\sqrt{d\cos(a+bx)}} - \frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd} + \frac{4d\sin(a+bx)\sqrt{d\cos(a+bx)}}{21b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[d*Cos[a + b
*x]]) + (4*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(21*b) - (2*(d*Cos[a + b*x]
)^(5/2)*Sin[a + b*x])/(7*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}+\frac{2}{7}\int(d\cos(a+bx))^{3/2}\,dx\\
&= \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b}-\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}\\
&\quad+\frac{1}{21}\left(2d^2\right)\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b}-\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}\\
&\quad+\frac{\left(2d^2\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{21\sqrt{d\cos(a+bx)}}\\
&= \frac{4d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b\sqrt{d\cos(a+bx)}}\\
&\quad+\frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b}-\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.12

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^2\left(24\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-60\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+50\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-15\left(\cos^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\sqrt{\cdots}\right.}{21\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\cdots\right.\right.\right.}}$

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/21*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(24*cos(
1/2*b*x+1/2*a)^9-60*cos(1/2*b*x+1/2*a)^7+50*cos(1/2*b*x+1/2*a)^5-15*cos(1/2
*b*x+1/2*a)^3-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)
*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b
*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b
*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = -\frac{2\left(i\sqrt{2}d^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-i\sqrt{2}d^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos\cdots\right.}{21\,b}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/21*(I*sqrt(2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*
x + a)) - I*sqrt(2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin
(b*x + a)) + (3*d*cos(b*x + a)^2 - 2*d)*sqrt(d*cos(b*x + a))*sin(b*x + a))/
b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2), x)
```

3.198 $\int \sqrt{d \cos(a+bx)} \sin^2(a+bx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \sqrt{d \cos(a+bx)} \sin^2(a+bx) \, dx = \frac{4\sqrt{d \cos(a+bx)} E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{2(d \cos(a+bx))^{3/2} \sin(a+bx)}{5bd}$$

```
[Out] -2/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d+4/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/c
os(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2
)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2648, 2721, 2719}

$$\int \sqrt{d \cos(a+bx)} \sin^2(a+bx) \, dx = \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right) \sqrt{d \cos(a+bx)}}{5b\sqrt{\cos(a+bx)}} - \frac{2 \sin(a+bx)(d \cos(a+bx))^{3/2}}{5bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]])
 - (2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}+\frac{2}{5}\int\sqrt{d\cos(a+bx)}\,dx\\
&= -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}+\frac{\left(2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}}\\
&= \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}-\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int\sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(85) = 170$.

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.81

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d\left(4\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+5\left(\cos^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{1-2\left(\cos^2\right.}}{5\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}\, b}$

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(4*cos(1/2*
b*x+1/2*a)^7-8*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))-cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)
^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(\sqrt{d\cos\left(bx+a\right)}\cos\left(bx+a\right)\sin\left(bx+a\right)-i\sqrt{2}\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(d*cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - I*sqrt(2)*sqrt(d)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x +
a))) + I*sqrt(2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos\left(a+bx\right)}\sin^2\left(a+bx\right)\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2), x)
```

3.199 $\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}$$

```
[Out] 4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-2/3*sin(b*x+a)*(d*co
s(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2648, 2721, 2720}

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]])
 - (2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd} + \frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx \\
&= -\frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd} + \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3\sqrt{d\cos(a+bx)}} \\
&= \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx \\
&= \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3b(d\cos(a+bx))^{3/2}}
\end{aligned}$$

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.36 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\right.}}{3\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}\,b}$

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*cos(1/2*b*
x+1/2*a)*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-(sin(
1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*
b*x+1/2*a),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/
2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx =$$
$$-\frac{2\left(i\sqrt{2}\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-i\sqrt{2}\sqrt{d}\text{weierstrassPInverse}(-4,0,\right.}{3\,bd}$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x
 + a)) - I*sqrt(2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(
b*x + a)) + sqrt(d*cos(b*x + a))*sin(b*x + a))/(b*d)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2), x)
```

3.200 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2*sin(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)-4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/
2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d
^2/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2646, 2721, 2719}

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
```

```
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{2\int \sqrt{d\cos(a+bx)}\,dx}{d^2} \\
&= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}} \\
&= -\frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 0.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{4\left(-\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{bx}{2}\right.\right.}\right.}{d\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}$

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/d*(-(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(1/2)*cos(1/2*b*x
+1/2*a)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(1/2)*El
lipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*
x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/
b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{2}\sqrt{d}\cos\left(bx+a\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)))-\right.}{}$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(2)*sqrt(d)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(d)*cos(b*x + a)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x
+ a))) - sqrt(d*cos(b*x + a))*sin(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2), x)
```

3.201 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)-4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/co
s(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d
^2/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2646, 2721, 2720}

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (2*Sin[a + b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
```

```
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2} \\
&= \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}} \\
&= -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 7/4, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(88) = 176$.

Time = 0.59 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.36

method	result
default	$-\frac{4\left(2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2}\right.}{3d^2\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}\right.}$

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellip
ticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-sin(1/2*b*x+1/2*a)^2*
cos(1/2*b*x+1/2*a)-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^
(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/d^2*(d*(2*cos(1/2*b*x+1/2*a)^2
-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2*sin(1/2*b
*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b
*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{2\left(-i\sqrt{2}\sqrt{d}\cos(bx+a)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{d}\cos(bx+a)^2\right.}{3\,bd^3\cos(bx+a)^2}$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(-I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x
+ a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*weierstrassPInver
se(-4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a))*sin(b*x + a
))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}^2}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}^2}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{{\sin{(a+b\,x)}}^2}{{(d\,\cos{(a+b\,x)})}^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2), x)
```

3.202 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/5*sin(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)-4/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))
^(1/2)+4/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/
2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2646, 2716, 2721, 2719}

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (4*Sin[a + b*x])/(
5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2} \\
&= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\int \sqrt{d\cos(a+bx)}\,dx}{5d^4} \\
&= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^4\sqrt{\cos(a+bx)}} \\
&= \frac{4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(2(a+bx))}{24b(d\cos(a+bx))^{7/2}}$$

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/2, 9/4, 5/2, Sin[a + b*x]^2]*Si
n[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(112) = 224.

Time = 1.03 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.65

method	result
default	$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(8\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-4\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(b}{2}}}{}$

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*b
*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1
/2*a)^2-1)*(8*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-4*(2*sin(1/2*b*x+1/2*
a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^
(1/2)*sin(1/2*b*x+1/2*a)^4-8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+4*(2*s
in(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2
*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+
1/2*a)-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipt
icE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1
/2*a)^2)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(-i\sqrt{2}\sqrt{d}\cos(bx+a)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))\right)}{}$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(-I*sqrt(2)*sqrt(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(d)*cos(b*x
+ a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*s
in(b*x + a))) + sqrt(d*cos(b*x + a))*(2*cos(b*x + a)^2 - 1)*sin(b*x + a))/(
b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(a+b\,x)^2}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2), x)
```

3.203 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21bd^4\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/7*sin(b*x+a)/b/d/(d*cos(b*x+a))^(7/2)-4/21*sin(b*x+a)/b/d^3/(d*cos(b*x+a)
)^(3/2)-4/21*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(
1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2646, 2716, 2721, 2720}

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*d^4*Sqrt[d*Cos[a +
b*x]]) + (2*Sin[a + b*x])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (4*Sin[a + b*x])
/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2} \\
&= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{21d^4} \\
&= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{21d^4\sqrt{d\cos(a+bx)}} \\
&= -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21bd^4\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(2(a+bx))}{24b(d\cos(a+bx))^{9/2}}$$

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 11/4, 5/2, Sin[a + b*x]^2]*S
in[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(112) = 224.

Time = 1.37 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.96

method	result
default	$\frac{4\left(-8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+12\sqrt{\frac{1}{2}-\frac{\cos(bx+a}{2}}\right.}{}$

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/21*(-8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-8*sin(1/2*b*x+1/2*a)
^6*cos(1/2*b*x+1/2*a)+12*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4+8*co
s(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-6*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin
(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*
x+1/2*a)^2+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(
1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))
)/d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^3/
sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{2}\sqrt{d}\cos(bx+a)^4 \text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{d}\cos(bx+a)^4\cdots\right.}{21\,bd^5\cos\cdots}$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/21*(-I*sqrt(2)*sqrt(d)*cos(b*x + a)^4*weierstrassPInverse(-4, 0, cos(b*x
 + a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(d)*cos(b*x + a)^4*weierstrassPInve
rse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + sqrt(d*cos(b*x + a))*(2*cos(b*x
 + a)^2 - 3)*sin(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(a+b\,x)^2}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2), x)
```

3.204 $\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}+\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}$$

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d+2/7*(d*cos(b*x+a))^(7/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2(d\cos(a+bx))^{7/2}}{7bd^3}-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d) + (2*(d*Cos[a + b*x])^(7/2))/(7*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x^2}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd} \\ &= -\frac{\text{Subst}\left(\int \left(\sqrt{x}-\frac{x^{5/2}}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd} \\ &= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}+\frac{2(d\cos(a+bx))^{7/2}}{7bd^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = -\frac{d\left(16\cos^2(a+bx)-16\sqrt[4]{\cos^2(a+bx)}+3\sin^2(2(a+bx))\right)}{42b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
[Out] -1/42*(d*(16*Cos[a + b*x]^2 - 16*(Cos[a + b*x]^2)^(1/4) + 3*Sin[2*(a + b*x)
]^2))/(b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{7}{2}}}{7}-\frac{2d^2(d\cos(bx+a))^{\frac{3}{2}}}{3}}{b\,d^3}$	37
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{7}{2}}}{7}-\frac{2d^2(d\cos(bx+a))^{\frac{3}{2}}}{3}}{b\,d^3}$	37

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/7*(d*cos(b*x+a))^(7/2)-1/3*d^2*(d*cos(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\,\cos\left(bx+a\right)^3-7\,\cos\left(bx+a\right)\right)\sqrt{d\cos\left(bx+a\right)}}{21\,b}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 2/21*(3*cos(b*x + a)^3 - 7*cos(b*x + a))*sqrt(d*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \begin{cases} -\frac{2\sqrt{d\cos\left(a+bx\right)}\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b}-\frac{8\sqrt{d\cos\left(a+bx\right)}\cos^3\left(a+bx\right)}{21b} & \text{for } b\neq 0 \\ x\sqrt{d\cos\left(a\right)}\sin^3\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**3,x)
[Out] Piecewise((-2*sqrt(d*cos(a + b*x))*sin(a + b*x)**2*cos(a + b*x)/(3*b) - 8*s
qrt(d*cos(a + b*x))*cos(a + b*x)**3/(21*b), Ne(b, 0)), (x*sqrt(d*cos(a))*si
n(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}-7\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^2\right)}{21\,bd^3}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 2/21*(3*(d*cos(b*x + a))^(7/2) - 7*(d*cos(b*x + a))^(3/2)*d^2)/(b*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\sqrt{d\cos(bx+a)}d^3\cos(bx+a)^3-7\sqrt{d\cos(bx+a)}d^3\cos(bx+a)\right)}{21\,bd^3}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
[Out] 2/21*(3*sqrt(d*cos(b*x + a))*d^3*cos(b*x + a)^3 - 7*sqrt(d*cos(b*x + a))*d^
3*cos(b*x + a))/(b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \int \sin(a+b\,x)^3\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

```
[In] int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2), x)
```

3.205 $\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}+\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}$$

```
[Out] 2/5*(d*cos(b*x+a))^(5/2)/b/d^3-2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2(d\cos(a+bx))^{5/2}}{5bd^3}-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d) + (2*(d*Cos[a + b*x])^(5/2))/(5*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-\frac{x^{3/2}}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2\sqrt{d\cos(a+bx)}}{bd}+\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\cos(a+bx)(-9+\cos(2(a+bx)))+8\cos^2(a+bx)^{3/4}\sec(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
[Out] (Cos[a + b*x]*(-9 + Cos[2*(a + b*x)]) + 8*(Cos[a + b*x]^2)^(3/4)*Sec[a + b*
x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5}-2d^2\sqrt{d\cos(bx+a)}}{b\,d^3}$	37
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5}-2d^2\sqrt{d\cos(bx+a)}}{b\,d^3}$	37

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/5*(d*cos(b*x+a))^(5/2)-d^2*(d*cos(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-5\right)}{5\,bd}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 5)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \begin{cases} -\frac{2\sin^2(a+bx)\cos(a+bx)}{b\sqrt{d\cos(a+bx)}} - \frac{8\cos^3(a+bx)}{5b\sqrt{d\cos(a+bx)}} & \text{for } b \neq 0 \\ \frac{x\sin^3(a)}{\sqrt{d\cos(a)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
[Out] Piecewise((-2*sin(a + b*x)**2*cos(a + b*x)/(b*sqrt(d*cos(a + b*x))) - 8*cos
(a + b*x)**3/(5*b*sqrt(d*cos(a + b*x))), Ne(b, 0)), (x*sin(a)**3/sqrt(d*cos
(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\left(5\sqrt{d\cos(bx+a)} - \frac{(d\cos(bx+a))^{\frac{5}{2}}}{d^2}\right)}{5\,bd}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/5*(5*sqrt(d*cos(b*x + a)) - (d*cos(b*x + a))^(5/2)/d^2)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\left(\sqrt{d\cos\left(bx+a\right)}d^2\cos\left(bx+a\right)^2-5\sqrt{d\cos\left(bx+a\right)}d^2\right)}{5\,bd^3}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2 - 5*sqrt(d*cos(b*x + a))*d^2)/
(b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2), x)
```

3.206 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}} + \frac{2(d\cos(a+bx))^{3/2}}{3bd^3}$$

```
[Out] 2/3*(d*cos(b*x+a))^(3/2)/b/d^3+2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2(d\cos(a+bx))^{3/2}}{3bd^3} + \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]]) + (2*(d*Cos[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{3/2}}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}-\frac{\sqrt{x}}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}}+\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{2\left(-4+4\sqrt[4]{\cos^2(a+bx)}+\sin^2(a+bx)\right)}{3bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(1/4) + Sin[a + b*x]^2))/(3*b*d*Sqrt[d*Cos[a +
 b*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3}+\frac{2d^2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	36
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3}+\frac{2d^2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	36

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/3*(d*cos(b*x+a))^(3/2)+d^2/(d*cos(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2+3\right)}{3\,bd^2\cos(bx+a)}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 3)/(b*d^2*cos(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \begin{cases} \frac{2\sin^2(a+bx)\cos(a+bx)}{b(d\cos(a+bx))^{\frac{3}{2}}} + \frac{8\cos^3(a+bx)}{3b(d\cos(a+bx))^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
[Out] Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(b*(d*cos(a + b*x))**(3/2)) + 8*c
os(a + b*x)**3/(3*b*(d*cos(a + b*x))**(3/2)), Ne(b, 0)), (x*sin(a)**3/(d*co
s(a))**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{3}{\sqrt{d\cos(bx+a)}} + \frac{(d\cos(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2/3*(3/sqrt(d*cos(b*x + a)) + (d*cos(b*x + a))^(3/2)/d^2)/(b*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2), x)
```

3.207 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}} + \frac{2\sqrt{d\cos(a+bx)}}{bd^3}$$

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)+2*(d*cos(b*x+a))^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}}{bd^3} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2)) + (2*Sqrt[d*Cos[a + b*x]])/(b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{5/2}}-\frac{1}{d^2\sqrt{x}}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{3bd(d\cos(a+bx))^{3/2}}+\frac{2\sqrt{d\cos(a+bx)}}{bd^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{2\left(-4+4\cos^2(a+bx)^{3/4}+3\sin^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(3/4) + 3*Sin[a + b*x]^2))/(3*b*d*(d*Cos[a + b
*x])^(3/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{d\cos(bx+a)}+\frac{2d^2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	35
default	$\frac{2\sqrt{d\cos(bx+a)}+\frac{2d^2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	35

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*((d*cos(b*x+a))^(1/2)+1/3*d^2/(d*cos(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(3\cos(bx+a)^2+1\right)}{3\,bd^3\cos(bx+a)^2}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)/(b*d^3*cos(b*x + a)^2)
```

Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \begin{cases} \frac{2\sin^2(a+bx)\cos(a+bx)}{3b(d\cos(a+bx))^{\frac{5}{2}}} + \frac{8\cos^3(a+bx)}{3b(d\cos(a+bx))^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
[Out] Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(3*b*(d*cos(a + b*x))**(5/2)) + 8
*cos(a + b*x)**3/(3*b*(d*cos(a + b*x))**(5/2)), Ne(b, 0)), (x*sin(a)**3/(d*
cos(a))**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\left(\frac{1}{(d\cos(bx+a))^{\frac{3}{2}}} + \frac{3\sqrt{d\cos(bx+a)}}{d^2}\right)}{3\,bd}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] 2/3*(1/(d*cos(b*x + a))^(3/2) + 3*sqrt(d*cos(b*x + a))/d^2)/(b*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}^3}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(a+b\,x)}\,(16\cos(2\,a+2\,b\,x)+3\cos(4\,a+4\,b\,x)+13)}{3\,b\,d^3\,(4\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3)}$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(5/2),x)
[Out] (2*(d*cos(a + b*x))^(1/2)*(16*cos(2*a + 2*b*x) + 3*cos(4*a + 4*b*x) + 13))/
(3*b*d^3*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

3.208 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx=\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)-2/b/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx=\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) - 2/(b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{7/2}}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{7/2}}-\frac{1}{d^2x^{3/2}}\right)\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= \frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\left(5-4\sqrt[4]{\cos^2(a+bx)}+4\left(-1+\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx)\right)\tan^2(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
[Out] (2*(5 - 4*(Cos[a + b*x]^2)^(1/4) + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{2d^2}{5(d\cos(bx+a))^{\frac{5}{2}}}-\frac{2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	37
default	$\frac{\frac{2d^2}{5(d\cos(bx+a))^{\frac{5}{2}}}-\frac{2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	37

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/5*d^2/(d*cos(b*x+a))^(5/2)-1/(d*cos(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(5\,\cos\left(bx+a\right)^2-1\right)}{5\,bd^4\cos\left(bx+a\right)^3}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
[Out] -2/5*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 1)/(b*d^4*cos(b*x + a)^3)
```

Sympy [A] (verification not implemented)

Time = 30.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \begin{cases} \frac{2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{5b(d\cos\left(a+bx\right))^{\frac{7}{2}}} - \frac{8\cos^3\left(a+bx\right)}{5b(d\cos\left(a+bx\right))^{\frac{7}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin^3\left(a\right)}{(d\cos\left(a\right))^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
[Out] Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(5*b*(d*cos(a + b*x))**(7/2)) - 8
*cos(a + b*x)**3/(5*b*(d*cos(a + b*x))**(7/2)), Ne(b, 0)), (x*sin(a)**3/(d*
cos(a))**(7/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(5\,d^2\cos\left(bx+a\right)^2-d^2\right)}{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\,bd^3}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] -2/5*(5*d^2*cos(b*x + a)^2 - d^2)/((d*cos(b*x + a))^(5/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,\left(6\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+5\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+5\right)}{5\,b\,d^4\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(7/2),x)
[Out] -(4*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2)*(6*exp(a*2i + b*x*2i) + 5*exp(a*4i + b*x*4i) + 5))/(5*b*d^4*(exp(a*2i
 + b*x*2i) + 1)^3)
```

3.209 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)-2/3/b/d^3/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) - 2/(3*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{9/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}-\frac{1}{d^2x^{5/2}}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= \frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\left(7-4\cos^2(a+bx)^{3/4}+4\left(-1+\cos^2(a+bx)^{3/4}\right)\csc^2(a+bx)\right)\tan^2(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
[Out] (2*(7 - 4*(Cos[a + b*x]^2)^(3/4) + 4*(-1 + (Cos[a + b*x]^2)^(3/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2d^2}{7(d\cos(bx+a))^{\frac{7}{2}}}-\frac{2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	37
default	$\frac{\frac{2d^2}{7(d\cos(bx+a))^{\frac{7}{2}}}-\frac{2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	37

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(1/7*d^2/(d*cos(b*x+a))^(7/2)-1/3/(d*cos(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\sqrt{d\cos\left(bx+a\right)}\left(7\cos\left(bx+a\right)^2-3\right)}{21\,bd^5\cos\left(bx+a\right)^4}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] -2/21*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 3)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\left(7\,d^2\cos\left(bx+a\right)^2-3\,d^2\right)}{21\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}bd^3}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] -2/21*(7*d^2*cos(b*x + a)^2 - 3*d^2)/((d*cos(b*x + a))^(7/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.07

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{8\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+7\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+7\right)}{21\,b\,d^5\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] -(8*exp(a*2i + b*x*2i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2)*(2*exp(a*2i + b*x*2i) + 7*exp(a*4i + b*x*4i) + 7))/(21*b*d^5*(exp(a*2
i + b*x*2i) + 1)^4)
```

3.210 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}} \, dx = \frac{2}{9bd(d\cos(a+bx))^{9/2}} - \frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/9/b/d/(d*cos(b*x+a))^(9/2)-2/5/b/d^3/(d*cos(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2645, 14}

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}} \, dx = \frac{2}{9bd(d\cos(a+bx))^{9/2}} - \frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] 2/(9*b*d*(d*Cos[a + b*x])^(9/2)) - 2/(5*b*d^3*(d*Cos[a + b*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{11/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{11/2}} - \frac{1}{d^2x^{7/2}}\right)\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{9bd(d\cos(a+bx))^{9/2}} - \frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 94 vs. $2(45) = 90$.

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\left(4\sqrt[4]{\cos^2(a+bx)} + \left(9 - 8\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx) + 4\left(-1 + \sqrt[4]{\cos^2(a+\right.\right.}{45bd^5\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
[Out] (2*(4*(Cos[a + b*x]^2)^(1/4) + (9 - 8*(Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^
2 + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^4)*Tan[a + b*x]^4)/(45*b*d
^5*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{2}{5(d\cos(bx+a))^{\frac{5}{2}}}+\frac{2d^2}{9(d\cos(bx+a))^{\frac{9}{2}}}}{b\,d^3}$	37
default	$\frac{-\frac{2}{5(d\cos(bx+a))^{\frac{5}{2}}}+\frac{2d^2}{9(d\cos(bx+a))^{\frac{9}{2}}}}{b\,d^3}$	37

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
[Out] 2/b/d^3*(-1/5/(d*cos(b*x+a))^(5/2)+1/9*d^2/(d*cos(b*x+a))^(9/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(9\,\cos\left(bx+a\right)^2-5\right)}{45\,bd^6\cos\left(bx+a\right)^5}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
[Out] -2/45*sqrt(d*cos(b*x + a))*(9*cos(b*x + a)^2 - 5)/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(11/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\left(9\,d^2\cos\left(bx+a\right)^2-5\,d^2\right)}{45\,\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}bd^3}$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
[Out] -2/45*(9*d^2*cos(b*x + a)^2 - 5*d^2)/((d*cos(b*x + a))^(9/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 6.20

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{16\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}{5\,b\,d^6\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^2} - \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,464\mathrm{i}}{45\,b\,d^6\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^3} - \frac{128\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}{9\,b\,d^6\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^4} + \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,64\mathrm{i}}{9\,b\,d^6\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^5}$$

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(11/2),x)
```

```
[Out] (16*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(5*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(d*(e
xp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2)*464i)/(45*b*d^6*(exp(a
*2i + b*x*2i)*1i + 1i)^3) - (128*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i
)/2 + exp(a*1i + b*x*1i)/2))^(1/2))/(9*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^4
) + (exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2)*64i)/(9*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^5)
```

3.211 $\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx=\frac{56d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{1105b\sqrt{\cos(a+bx)}}+\frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b}+\frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b}-\frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd}-\frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd}$$

```
[Out] 56/3315*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+8/663*d*(d*cos(b*x+a))^(7/2)*
sin(b*x+a)/b-12/221*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d-2/17*(d*cos(b*x+a)
)^(11/2)*sin(b*x+a)^3/b/d+56/1105*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*
a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos
(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2719}

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx=\frac{56d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{1105b\sqrt{\cos(a+bx)}}+\frac{56d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{3315b}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}-\frac{12\sin(a+bx)(d\cos(a+bx))^{11/2}}{221bd}+\frac{8d\sin(a+bx)(d\cos(a+bx))^{7/2}}{663b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

```
[Out] (56*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(1105*b*Sqrt[Cos[a
+ b*x]]) + (56*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(3315*b) + (8*d*(d*
Cos[a + b*x])^(7/2)*Sin[a + b*x])/(663*b) - (12*(d*Cos[a + b*x])^(11/2)*Sin
[a + b*x])/(221*b*d) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x]^3)/(17*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{6}{17}\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx \\
&= -\frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} \\
&\quad - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{12}{221}\int (d\cos(a+bx))^{9/2}\,dx \\
&= \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} \\
&\quad - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{1}{663}(28d^2)\int (d\cos(a+bx))^{5/2}\,dx
\end{aligned}$$

$$= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{(28d^4)\int\sqrt{d\cos(a+bx)}\,dx}{1105}$$

$$= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd} + \frac{\left(28d^4\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{1105\sqrt{\cos(a+bx)}}$$

$$= \frac{56d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{1105b\sqrt{\cos(a+bx)}} + \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{9/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\tan^5(a+bx)}{5b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Maple [A] (verified)

Time = 13.67 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.76

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^{5}\left(24960\left(\cos^{19}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-124800\left(\cos^{17}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+265440\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-312}{}$

[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] -8/3315*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(24960*cos(1/2*b*x+1/2*a)^19-124800*cos(1/2*b*x+1/2*a)^17+265440*cos(1/2*b*x+1/2*a)^15-312960*cos(1/2*b*x+1/2*a)^13+222520*cos(1/2*b*x+1/2*a)^11-96360*cos(1/2*b*x+1/2*a)^9+23866*cos(1/2*b*x+1/2*a)^7-2652*cos(1/2*b*x+1/2*a)^5-35*cos(1/2*b*x+1/2*a)^3-21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+21*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(-42i\sqrt{2}d^{\frac{9}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+42i\sqrt{2}d\right.}{}$$

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="fricas")

[Out] -2/3315*(-42*I*sqrt(2)*d^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 42*I*sqrt(2)*d^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) - (195*d^4*cos(b*x + a)^7 - 285*d^4*cos(b*x + a)^5 + 20*d^4*cos(b*x + a)^3 + 28*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x + a))/b

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{9/2}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2), x)
```

3.212 $\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \frac{8d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}$$

```
[Out] 8/385*d*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b-4/55*(d*cos(b*x+a))^(9/2)*sin(b*x
+a)/b/d-2/15*(d*cos(b*x+a))^(9/2)*sin(b*x+a)^3/b/d+8/231*d^4*(cos(1/2*a+1/2
*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos
(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+8/231*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1
/2)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \frac{8d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{8d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b} - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd} - \frac{4\sin(a+bx)(d\cos(a+bx))^{9/2}}{55bd} + \frac{8d\sin(a+bx)(d\cos(a+bx))^{5/2}}{385b}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
b*x]]) + (8*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (8*d*(d*Cos[a
+ b*x])^(5/2)*Sin[a + b*x])/(385*b) - (4*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x
])/(55*b*d) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^3)/(15*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}+\frac{2}{5}\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx \\
&= -\frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} \\
&\quad -\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}+\frac{4}{55}\int (d\cos(a+bx))^{7/2}\,dx \\
&= \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b}-\frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} \\
&\quad -\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}+\frac{1}{77}\left(4d^2\right)\int (d\cos(a+bx))^{3/2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} \\
&\quad - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd} \\
&\quad + \frac{1}{231}(4d^4)\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx \\
&= \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} \\
&\quad - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd} \\
&\quad + \frac{\left(4d^4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{231\sqrt{d\cos(a+bx)}} \\
&= \frac{8d^4\sqrt{\cos(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} \\
&\quad + \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} \\
&\quad - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$
\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{7/2}\cos^2(a+bx)^{3/4}\,\mathrm{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\tan^5(a+bx)}{5b}
$$

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
[Out] ((d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Maple [A] (verified)

Time = 11.61 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.68

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^4\left(4928\left(\cos^{17}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-22176\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+41216\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-40768\left(\cos\right.\right.}{1155\sqrt{-d\left(2\left(\sin^4\left(\right.\right.\right.}}$

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/1155*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(4928
*cos(1/2*b*x+1/2*a)^17-22176*cos(1/2*b*x+1/2*a)^15+41216*cos(1/2*b*x+1/2*a)
^13-40768*cos(1/2*b*x+1/2*a)^11+22868*cos(1/2*b*x+1/2*a)^9-6994*cos(1/2*b*x
+1/2*a)^7+926*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a)
,2^(1/2))-5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2
*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(10i\sqrt{2}d^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-10i\sqrt{2}d^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -2/1155*(10*I*sqrt(2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*s
in(b*x + a)) - 10*I*sqrt(2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a)
 - I*sin(b*x + a)) - (77*d^3*cos(b*x + a)^6 - 119*d^3*cos(b*x + a)^4 + 12*d
^3*cos(b*x + a)^2 + 20*d^3)*sqrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{7/2}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2), x)
```

3.213 $\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \frac{8d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{65b\sqrt{\cos(a+bx)}} + \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}$$

```
[Out] 8/195*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-4/39*(d*cos(b*x+a))^(7/2)*sin(b*x
+a)/b/d-2/13*(d*cos(b*x+a))^(7/2)*sin(b*x+a)^3/b/d+8/65*d^2*(cos(1/2*a+1/2*
b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*c
os(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2719}

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \frac{8d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{65b\sqrt{\cos(a+bx)}} - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd} - \frac{4\sin(a+bx)(d\cos(a+bx))^{7/2}}{39bd} + \frac{8d\sin(a+bx)(d\cos(a+bx))^{3/2}}{195b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(65*b*Sqrt[Cos[a + b
*x]]) + (8*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(195*b) - (4*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(39*b*d) - (2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^
3)/(13*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}+\frac{6}{13}\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx\\
&= -\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd}\\
&\quad -\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}+\frac{4}{39}\int (d\cos(a+bx))^{5/2}\,dx\\
&= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b}-\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd}\\
&\quad -\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}+\frac{1}{65}\left(4d^2\right)\int \sqrt{d\cos(a+bx)}\,dx
\end{aligned}
$$

$$= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd} + \frac{\left(4d^2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{65\sqrt{\cos(a+bx)}}$$

$$= \frac{8d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{65b\sqrt{\cos(a+bx)}} + \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int(d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\tan^3(a+}{5b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Maple [A] (verified)

Time = 11.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.95

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^3\left(480\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1920\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3040\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2400\left(\cos^9\left(\right.\right.\right.}{195\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin\right.\right.}}$

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/195*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(480*c
os(1/2*b*x+1/2*a)^15-1920*cos(1/2*b*x+1/2*a)^13+3040*cos(1/2*b*x+1/2*a)^11-
2400*cos(1/2*b*x+1/2*a)^9+958*cos(1/2*b*x+1/2*a)^7-156*cos(1/2*b*x+1/2*a)^5
-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*cos(1/2*b*x+1/2
*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/2*b*x+1/2*a))/(-
d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(
d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx =$$
$$-\frac{2\left(-6i\sqrt{2}d^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+6i\sqrt{2}d^{\frac{5}{2}}\mathrm{w}\right.}{}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="fricas")
[Out] -2/195*(-6*I*sqrt(2)*d^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(b*x + a) + I*sin(b*x + a))) + 6*I*sqrt(2)*d^(5/2)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) - (15*d^2*
cos(b*x + a)^5 - 25*d^2*cos(b*x + a)^3 + 4*d^2*cos(b*x + a))*sqrt(d*cos(b*x
 + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2), x)
```

3.214 $\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \frac{8d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{77b\sqrt{d\cos(a+bx)}} + \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}$$

```
[Out] -12/77*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b/d-2/11*(d*cos(b*x+a))^(5/2)*sin(b*
x+a)^3/b/d+8/77*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*Ellipti
cF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+8/77
*d*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2648, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \frac{8d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{77b\sqrt{d\cos(a+bx)}} - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd} - \frac{12\sin(a+bx)(d\cos(a+bx))^{5/2}}{77bd} + \frac{8d\sin(a+bx)\sqrt{d\cos(a+bx)}}{77b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(77*b*Sqrt[d*Cos[a + b
*x]]) + (8*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(77*b) - (12*(d*Cos[a + b*x
])^(5/2)*Sin[a + b*x])/(77*b*d) - (2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^3)
/(11*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}+\frac{6}{11}\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx\\
&= -\frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd}\\
&\quad -\frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}+\frac{12}{77}\int (d\cos(a+bx))^{3/2}\,dx\\
&= \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b}-\frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd}\\
&\quad -\frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}+\frac{1}{77}\left(4d^2\right)\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx
\end{aligned}$$

$$= \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd} + \frac{\left(4d^2\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{77\sqrt{d\cos(a+bx)}}$$

$$= \frac{8d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{77b\sqrt{d\cos(a+bx)}} + \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\tan^3(a}{5b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.99

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^2\left(112\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-280\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+228\left(\sin^8\left(\right.\right.\right.}{77\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\right.\right.}}$

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -8/77*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(112*co
s(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^12-280*sin(1/2*b*x+1/2*a)^10*cos(1/2*b*
x+1/2*a)+228*sin(1/2*b*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-62*sin(1/2*b*x+1/2*a)^
6*cos(1/2*b*x+1/2*a)+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1
/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a
),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/
2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx =$$
$$-\frac{2\left(2i\sqrt{2}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-2i\sqrt{2}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\text{c}\right.}{77\,}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="fricas")
[Out] -2/77*(2*I*sqrt(2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(
b*x + a)) - 2*I*sqrt(2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I
*sin(b*x + a)) - (7*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 + 4*d)*sqrt(d*co
s(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{3/2}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2), x)
```

3.215 $\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \frac{8\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} - \frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}$$

```
[Out] -4/15*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d-2/9*(d*cos(b*x+a))^(3/2)*sin(b*x+
a)^3/b/d+8/15*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin
(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2648, 2721, 2719}

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = -\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd} - \frac{4\sin(a+bx)(d\cos(a+bx))^{3/2}}{15bd} + \frac{8E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (8*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b*x]]
) - (4*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(15*b*d) - (2*(d*Cos[a + b*x])^
(3/2)*Sin[a + b*x]^3)/(9*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}+\frac{2}{3}\int\sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx\\
&= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd}-\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}+\frac{4}{15}\int\sqrt{d\cos(a+bx)}\,dx\\
&= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd}-\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}\\
&\quad+\frac{\left(4\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{15\sqrt{\cos(a+bx)}}\\
&= \frac{8\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}}-\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd}\\
&\quad-\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$$
$$=\frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.23

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d\left(40\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+118\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-36\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cdots}{45\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\cdots}$

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(40*cos(1
/2*b*x+1/2*a)^11-120*cos(1/2*b*x+1/2*a)^9+118*cos(1/2*b*x+1/2*a)^7-36*cos(1
/2*b*x+1/2*a)^5-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(1-2*
cos(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/2
*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1
/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$$
$$=\frac{2\left(\left(5\cos(bx+a)^3-11\cos(bx+a)\right)\sqrt{d\cos(bx+a)}\sin(bx+a)+6i\sqrt{2}\sqrt{d}\,\text{weierstrassZeta}(-4,0,\text{weiers}\cdots\right.}{\cdots}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 2/45*((5*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x + a
) + 6*I*sqrt(2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(b*x + a) + I*sin(b*x + a))) - 6*I*sqrt(2)*sqrt(d)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2), x)
```

3.216 $\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}$$

```
[Out] 8/7*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-4/7*sin(b*x+a)*(d*co
s(b*x+a))^(1/2)/b/d-2/7*sin(b*x+a)^3*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2648, 2721, 2720}

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd} - \frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{7bd} + \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]])
 - (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(7*b*d) - (2*Sqrt[d*Cos[a + b*x]]*
Sin[a + b*x]^3)/(7*b*d)
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}+\frac{6}{7}\int\frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd}-\frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}+\frac{4}{7}\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd}-\frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}\\
&\quad+\frac{\left(4\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{7\sqrt{d\cos(a+bx)}}\\
&= \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}}\\
&\quad-\frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd}-\frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5b(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.10

method	result
default	$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-6\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\right.\right.}{7\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}\right.\right.\right.}}$

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/7*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b
*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+si
n(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1
/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(
1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\left(\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-3\right)\sin(bx+a)-2i\sqrt{2}\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\mathrm{s}\right.}{7\,bd}$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*(sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 3)*sin(b*x + a) - 2*I*sqrt(2)*s
qrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + 2*I*sqrt
(2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/(b*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin{(bx+a)}^4}{\sqrt{d\cos{(bx+a)}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin{(bx+a)}^4}{\sqrt{d\cos{(bx+a)}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2), x)
```

3.217 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}} \, dx = -\frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^2\sqrt{\cos(a+bx)}} + \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 12/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d^3+2*sin(b*x+a)^3/b/d/(d*cos(b*x+a)
)^(1/2)-24/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^2/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2646, 2648, 2721, 2719}

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}} \, dx = \frac{12\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd^3} - \frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^2\sqrt{\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^2*Sqrt[Cos[a +
b*x]]) + (12*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d^3) + (2*Sin[a + b*
x]^3)/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{6\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx}{d^2} \\
&= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{12\int \sqrt{d\cos(a+bx)}\,dx}{5d^2} \\
&= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} \\
&\quad - \frac{\left(12\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^2\sqrt{\cos(a+bx)}} \\
&= -\frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^2\sqrt{\cos(a+bx)}} + \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.13

method	result
default	$\frac{8\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(2\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{b}{}\right.\right.}{5d\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{b}{}\right.\right.\right.}}$

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 8/5/d*(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b
*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+3*
sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*s
in(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d*(2*
sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*
cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx =$$

$$-\frac{2\left(6i\sqrt{2}\sqrt{d}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))\right.}{}$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(6*I*sqrt(2)*sqrt(d)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 6*I*sqrt(2)*sqrt(d)*cos(b*x
 + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*si
n(b*x + a))) - sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 5)*sin(b*x + a))/(b*d
^2*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2), x)
```

3.218 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*sin(b*x+a)^3/b/d/(d*cos(b*x+a))^(3/2)-8/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/
cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b
/d^2/(d*cos(b*x+a))^(1/2)+4/3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b/d^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2646, 2648, 2721, 2720}

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd^3} - \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d^3) + (2*Sin[a + b*x]^3
)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx}{d^2} \\
&= \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2} \\
&= \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}} \\
&= -\frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} \\
&\quad + \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5bd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(114) = 228.

Time = 0.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.80

method	result
default	$-\frac{8\left(-2\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)(\cdots}{3d^2\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\cdots\right.\right.}}$

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*(-2*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*cos(1/2*b*x+1/2*a)*sin(1
/2*b*x+1/2*a)^4+2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(
1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-sin(1/2*b*x
+1/2*a)^2*cos(1/2*b*x+1/2*a)-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/d^2*(d*(2*cos(1/2*b*
x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2
*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2
*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{2\left(-2i\sqrt{2}\sqrt{d}\cos(bx+a)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+2i\sqrt{2}\sqrt{d}\cos(bx+\right.}{3\,bd^3\,c}$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(-2*I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*
x + a) + I*sin(b*x + a)) + 2*I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*weierstrassPI
nverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a))*(cos(b*
x + a)^2 + 1)*sin(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2), x)
```

3.219 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{24\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} - \frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*sin(b*x+a)^3/b/d/(d*cos(b*x+a))^(5/2)-12/5*sin(b*x+a)/b/d^3/(d*cos(b*x+
a))^(1/2)+24/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(si
n(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2646, 2721, 2719}

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}} - \frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b
*x]]) - (12*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b*x]^
3)/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}\\
&=-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{12\int \sqrt{d\cos(a+bx)}\,dx}{5d^4}\\
&=-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{\left(12\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^4\sqrt{\cos(a+bx)}}\\
&=\frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}}-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$
\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx=\frac{\cos^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5b(d\cos(a+bx))^{7/2}}
$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 5/2, 7/2, Sin
[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(114) = 228$.

Time = 0.55 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.59

method	result
default	$\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(14\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\cdots}\right.}{}$

[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)

[Out] 8/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)*(14*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^4-14*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+3*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx =$$

$$-\frac{2\left(-6i\sqrt{2}\sqrt{d}\cos\left(bx+a\right)^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right)\cdots\right.}{}$$

[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")

[Out] -2/5*(-6*I*sqrt(2)*sqrt(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 6*I*sqrt(2)*sqrt(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 1)*sin(b*x + a))/(b*d^4*cos(b*x + a)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{{\sin\left(a+b\,x\right)}^4}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2), x)
```

3.220 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] -4/7*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(3/2)+2/7*sin(b*x+a)^3/b/d/(d*cos(b*x+
a))^(7/2)+8/7*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin
(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2646, 2721, 2720}

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*d^4*Sqrt[d*Cos[a + b*
x]]) - (4*Sin[a + b*x])/(7*b*d^3*(d*Cos[a + b*x])^(3/2)) + (2*Sin[a + b*x]^
3)/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2} \\
&= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{7d^4} \\
&= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{7d^4\sqrt{d\cos(a+bx)}} \\
&= \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{11}{4},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+b}{5b(d\cos(a+bx))^{9/2}}$$

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/2, 11/4, 7/2, Si
n[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(114) = 228$.

Time = 0.53 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.90

method	result
default	$\frac{8\left(8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-6\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}}{}$

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/7*(8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipt
icF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-6*sin(1/2*b*x+1/2*a)^6
*cos(1/2*b*x+1/2*a)-12*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2
-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4+6*cos(
1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+6*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1
/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+
1/2*a)^2-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-(sin(1/2*b*x+1/2*a)^2)^(1/
2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/
d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*
x+1/2*a)^2-1)^3/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/si
n(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\left(2i\sqrt{2}\sqrt{d}\cos(bx+a)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-2i\sqrt{2}\sqrt{d}\cos(bx+a)\right.}{7\,bd^5\,\mathrm{c}}$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(2*I*sqrt(2)*sqrt(d)*cos(b*x + a)^4*weierstrassPInverse(-4, 0, cos(b*x
 + a) + I*sin(b*x + a)) - 2*I*sqrt(2)*sqrt(d)*cos(b*x + a)^4*weierstrassPIn
verse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + sqrt(d*cos(b*x + a))*(3*cos(b
*x + a)^2 - 1)*sin(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin(a+b\,x)^4}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2), x)
```

3.221 $\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b} + \frac{4\cos^{\frac{9}{2}}(a+bx)}{9b} - \frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}$$

```
[Out] -2/5*cos(b*x+a)^(5/2)/b+4/9*cos(b*x+a)^(9/2)/b-2/13*cos(b*x+a)^(13/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 276}

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b} + \frac{4\cos^{\frac{9}{2}}(a+bx)}{9b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[In] Int[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

```
[Out] (-2*Cos[a + b*x]^(5/2))/(5*b) + (4*Cos[a + b*x]^(9/2))/(9*b) - (2*Cos[a + b
*x]^(13/2))/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^{3/2}(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^{3/2}-2x^{7/2}+x^{11/2}\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.13

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \frac{2\sqrt{\cos(a+bx)}\left(32-32\sqrt[4]{\cos^2(a+bx)}-8\sqrt[4]{\cos^2(a+bx)}\sin^2(a+bx)-5\sqrt[4]{\cos^2(a+bx)}\sin^4(a+bx)+4\right.}{585b\sqrt[4]{\cos^2(a+bx)}}$$

```
[In] Integrate[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
[Out] (2*Sqrt[Cos[a + b*x]]*(32 - 32*(Cos[a + b*x]^2)^(1/4) - 8*(Cos[a + b*x]^2)^
(1/4)*Sin[a + b*x]^2 - 5*(Cos[a + b*x]^2)^(1/4)*Sin[a + b*x]^4 + 45*(Cos[a
+ b*x]^2)^(1/4)*Sin[a + b*x]^6))/(585*b*(Cos[a + b*x]^2)^(1/4))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\frac{2\left(\cos^{\frac{13}{2}}(bx+a)\right)}{13}-\frac{4\left(\cos^{\frac{9}{2}}(bx+a)\right)}{9}+\frac{2\left(\cos^{\frac{5}{2}}(bx+a)\right)}{5}}{b}$	37
default	$-\frac{\frac{2\left(\cos^{\frac{13}{2}}(bx+a)\right)}{13}-\frac{4\left(\cos^{\frac{9}{2}}(bx+a)\right)}{9}+\frac{2\left(\cos^{\frac{5}{2}}(bx+a)\right)}{5}}{b}$	37

```
[In] int(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/b*(2/13*cos(b*x+a)^(13/2)-4/9*cos(b*x+a)^(9/2)+2/5*cos(b*x+a)^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\left(45\cos(bx+a)^6-130\cos(bx+a)^4+117\cos(bx+a)^2\right)\sqrt{\cos(bx+a)}}{585\,b}$$

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -2/585*(45*cos(b*x + a)^6 - 130*cos(b*x + a)^4 + 117*cos(b*x + a)^2)*sqrt(c
os(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**(3/2)*sin(b*x+a)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\left(45\cos(bx+a)^{\frac{13}{2}}-130\cos(bx+a)^{\frac{9}{2}}+117\cos(bx+a)^{\frac{5}{2}}\right)}{585\,b}$$

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -2/585*(45*cos(b*x + a)^(13/2) - 130*cos(b*x + a)^(9/2) + 117*cos(b*x + a)^
(5/2))/b
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\sin(bx+a)^5\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)*sin(b*x + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\cos(a+b\,x)^{5/2}\left(\frac{5\cos(a+b\,x)^4}{13}-\frac{10\cos(a+b\,x)^2}{9}+1\right)}{5\,b}$$

```
[In] int(cos(a + b*x)^(3/2)*sin(a + b*x)^5,x)
```

```
[Out] -(2*cos(a + b*x)^(5/2)*((5*cos(a + b*x)^4)/13 - (10*cos(a + b*x)^2)/9 + 1))
/(5*b)
```

3.222 $\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx=\frac{d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

```
[Out] d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(9/2)*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b+2/3*d^3*(d*cos(b*x+a))^(3/2)/b+2/7*d*(d*cos(b*x+a))^(7/
2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 327, 335, 304, 209, 212}

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx=\frac{d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

```
[Out] (d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(9/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*(d*Cos[a + b*x])^(3/2))/(3*b) + (2*d*(d*
Cos[a + b*x])^(7/2))/(7*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{9/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^3\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^5\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d^5\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \frac{d^4\sqrt{d\cos(a+bx)}\left(21\arctan\left(\sqrt{\cos(a+bx)}\right) - 21\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + 2\cos^{\frac{3}{2}}(a+bx)\,(7+\dots\right.}{21b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
[Out] (d^4*Sqrt[d*Cos[a + b*x]]*(21*ArcTan[Sqrt[Cos[a + b*x]]] - 21*ArcTanh[Sqrt[
Cos[a + b*x]]] + 2*Cos[a + b*x]^(3/2)*(7 + 3*Cos[a + b*x]^2)))/(21*b*Sqrt[C
os[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(80) = 160.

Time = 0.40 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.31

method	result
default	$-\frac{96d^4\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}\,\sqrt{-d}\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+21d^{\frac{9}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+21d^{\frac{9}{2}}\ln\left(\right.}{}$

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/(-d)^(1/2)*(96*d^4*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)*(-d)^(1/2)*sin
(1/2*b*x+1/2*a)^6+21*d^(9/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+
1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*(-d)^(1/2)+21*d^(9/2
)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2
*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)-144*d^4*(-2*d*sin(1/2*b*x+1/2*a)^2+d)
^(1/2)*(-d)^(1/2)*sin(1/2*b*x+1/2*a)^4+128*d^4*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)*(-d)^(1/2)*sin(1/2*b*x+1/2*a)^2-40*d^4*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)*(-d)^(1/2)+42*d^5*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2
*b*x+1/2*a)^2+d)^(1/2)-d)))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \left[\frac{42\sqrt{-d}d^4\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+21\sqrt{-d}d^4\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right.}{84b}\right.$$

$$-\frac{42d^{\frac{9}{2}}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-21d^{\frac{9}{2}}\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6d\cos(bx+a)+d}{\cos(bx+a)^2-2\cos(bx+a)+1}\right)-8\left(3d^4\right.}{84b}$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*d^4*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 21*sqrt(-d)*d^4*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*
sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos
(b*x + a) + 1)) + 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*cos(
b*x + a)))/b, -1/84*(42*d^(9/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*co
s(b*x + a) - d)) - 21*d^(9/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a
```

```
))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*c
os(b*x + a) + 1)) - 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*co
s(b*x + a)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \frac{42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + 21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) + 12\,(d\cos(bx+a))^{\frac{7}{2}}\,d^2 + 28\,(d\cos(bx+a))^{\frac{3}{2}}\,d^4}{42\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="maxima")
[Out] 1/42*(42*d^(11/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 21*d^(11/2)*log((s
qrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 12*(d*co
s(b*x + a))^(7/2)*d^2 + 28*(d*cos(b*x + a))^(3/2)*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \int \frac{\left(d\,\cos\left(a+b\,x\right)\right)^{9/2}}{\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x), x)
```

3.223 $\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

```
[Out] -d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(7/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2/5*d*(d*cos(b*x+a))^(5/2)/b+2*d^3*(d*cos(b*x+a))^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 327, 335, 218, 212, 209}

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(7/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*Sqrt[d*Cos[a + b*x]])/b + (2*d*(d*Cos
[a + b*x])^(5/2))/(5*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^4\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{d^4\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= -\frac{d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \frac{d^3\sqrt{d\cos(a+bx)}\left(-5\arctan\left(\sqrt{\cos(a+bx)}\right) - 5\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + \sqrt{\cos(a+bx)}(11+\dots}{5b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(-5*ArcTan[Sqrt[Cos[a + b*x]]] - 5*ArcTanh[Sqrt[C
os[a + b*x]]] + Sqrt[Cos[a + b*x]]*(11 + Cos[2*(a + b*x)])))/(5*b*Sqrt[Cos[
a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(81) = 162$.

Time = 0.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.95

method	result
default	$-\frac{5d^{\frac{7}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+5d^{\frac{7}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}-}{}$

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/(-d)^(1/2)*(5*d^(7/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/
2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)+5*d^(7/2)*l
n(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b
*x+1/2*a)^2+d)^(1/2)+d))*(-d)^(1/2)-16*d^3*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1
/2)*(-d)^(1/2)*sin(1/2*b*x+1/2*a)^4+16*d^3*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1
/2)*(-d)^(1/2)*sin(1/2*b*x+1/2*a)^2-24*d^3*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1
/2)*(-d)^(1/2)-10*d^4*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x
+1/2*a)^2+d)^(1/2)-d)))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.02

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \left[\frac{10\sqrt{-d}d^3\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+5\sqrt{-d}d^3\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\,c}{\cos(bx+a)^2+2\cos(bx+a)+1}\right.}{20\,b}\right.$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*d^3*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 5*sqrt(-d)*d^3*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(
b*x + a) + 1)) + 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b, 1/
20*(10*d^(7/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d))
+ 5*d^(7/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*
x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
+ 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 4\,(d\cos(bx+a))^{\frac{5}{2}}\,d^2 - 20\,\sqrt{d\cos(bx+a)}d^4}{10\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="maxima")
[Out] -1/10*(10*d^(9/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sqr
t(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*(d*cos(b
*x + a))^(5/2)*d^2 - 20*sqrt(d*cos(b*x + a))*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx=\int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x), x)
```

3.224 $\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \frac{d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

```
[Out] d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(5/2)*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b+2/3*d*(d*cos(b*x+a))^(3/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 327, 335, 304, 209, 212}

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \frac{d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] (d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(5/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d*(d*Cos[a + b*x])^(3/2))/(3*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{(2d)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d^3\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d^3\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \frac{(d\cos(a+bx))^{5/2}\left(3\arctan\left(\sqrt{\cos(a+bx)}\right) - 3\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + 2\cos^{\frac{3}{2}}(a+bx)\right)}{3b\cos^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(3*ArcTan[Sqrt[Cos[a + b*x]]] - 3*ArcTanh[Sqrt[Cos[
a + b*x]]] + 2*Cos[a + b*x]^(3/2)))/(3*b*Cos[a + b*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(62) = 124$.

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.24

method	result
default	$-\frac{3d^{\frac{5}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+3d^{\frac{5}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}}{6}$

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(-d)^(1/2)*(3*d^(5/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2
*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)+3*d^(5/2)*ln
```

```
(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*
x+1/2*a)^2+d)^(1/2)+d))*(-d)^(1/2)+8*d^2*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2
)*(-d)^(1/2)*sin(1/2*b*x+1/2*a)^2-4*d^2*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
*(-d)^(1/2)+6*d^3*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2
*a)^2+d)^(1/2)-d)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(62) = 124$.

Time = 0.42 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.60

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \left[\frac{6\sqrt{-d}d^2\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+8\sqrt{d\cos(bx+a)}d^2\cos(bx+a)+3\sqrt{-d}d^2\log\left(-\frac{d\cos(bx+a}{\ }\right.}{12b}\right.$$

$$-\frac{6d^{\frac{5}{2}}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-8\sqrt{d\cos(bx+a)}d^2\cos(bx+a)-3d^{\frac{5}{2}}\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(}{\cos(bx+a)^2-2\cos(}\right.}{12b}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*d^2*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a
) + d)) + 8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) + 3*sqrt(-d)*d^2*log(-(d*
cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/b, -1/12*(6*d^(5/2
)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 8*sqrt(d*co
s(b*x + a))*d^2*cos(b*x + a) - 3*d^(5/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*
cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
 a)^2 - 2*cos(b*x + a) + 1)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \frac{6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)+4\,(d\cos(bx+a))^{\frac{3}{2}}\,d^2}{6\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*d^(7/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*d^(7/2)*log((sqrt(d
*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 4*(d*cos(b*x
+ a))^(3/2)*d^2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x), x)
```

3.225 $\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}$$

```
[Out] -d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(3/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2*d*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2645, 327, 335, 218, 212, 209}

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(3/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d*Sqrt[d*Cos[a + b*x]])/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d^2\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{d^2\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= -\frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \frac{\left(-\arctan\left(\sqrt{\cos(a+bx)}\right) - \text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + 2\sqrt{\cos(a+bx)}\right)(d\cos(a+bx))^{3/2}}{b\cos^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
[Out] ((-ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]] + 2*Sqrt[Cos[a
+ b*x]])*(d*Cos[a + b*x])^(3/2))/(b*Cos[a + b*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(63) = 126.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.75

method	result
default	$\frac{-d^{\frac{3}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}-d^{\frac{3}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+4d\sqrt{}}{2\sqrt{-d}\,b}$

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(-d^(3/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(
-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)-d^(3/2)*ln(-2/(cos(1/2*b*
```

```
x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(
1/2)+d))*(-d)^(1/2)+4*d*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)*(-d)^(1/2)+2*d^
2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d
)))/(-d)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.36

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \left[\frac{2\sqrt{-d}d\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+\sqrt{-d}d\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)+8\sqrt{d\cos(bx+a)}d}{4b}, \ldots\right]$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*d*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
 d)) + sqrt(-d)*d*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*
(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a)
 + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b, 1/4*(2*d^(3/2)*arctan(2*sqrt(d*cos(b*
x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + d^(3/2)*log(-(d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 4\sqrt{d\cos(bx+a)}d^2}{2\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="maxima")
[Out] -1/2*(2*d^(5/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d*
cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*sqrt(d*cos(b
*x + a))*d^2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x), x)
```

3.226 $\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx=\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))*d^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2645, 335, 304, 209, 212}

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx=\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (Sqrt[d]*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd} \\
&= -\frac{d\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} \\
&= \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{\left(\arctan\left(\sqrt{\cos(a+bx)}\right)-\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
[Out] ((ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[d*Cos[a +
b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(46) = 92.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.14

method	result
default	$-\frac{\sqrt{d}\,\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+\sqrt{d}\,\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+}{2\sqrt{-d}\,b}$

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2/(-d)^(1/2)*(d^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a
)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)+d^(1/2)*ln(-2/
(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/
2*a)^2+d)^(1/2)+d))*(-d)^(1/2)+2*d*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.

Time = 0.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.10

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)+1}\right.}{4\,b}\right.$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) + sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/b, 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(
b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + sqrt(d)*log((d*cos(b*x + a)^2 - 4*s
qrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos
(b*x + a)^2 - 2*cos(b*x + a) + 1)))/b]
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \int \sqrt{d\cos{(a+bx)}}\csc{(a+bx)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a),x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^(3/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*c
os(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}} + \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}}\right)}{b}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/sqrt(-d) + arctan(sqrt(d*cos(b*x +
 a))/sqrt(d))/sqrt(d))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x), x)
```

3.227 $\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2645, 335, 218, 212, 209}

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

```
[In] Int[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*Sqrt[d])) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*Sqrt[d])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} \\
&= -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\left(\arctan\left(\sqrt{\cos(a+bx)}\right)+\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\right)\sqrt{\cos(a+bx)}}{b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Cos[a + b*x]]] + ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[Cos[a + b*x]])/(b*Sqrt[d*Cos[a + b*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(47) = 94$.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.07

method	result
default	$-\frac{\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}-2\ln\left(\frac{2\sqrt{-d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\sqrt{d}+\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\cdots}{\cos\cdots}\right.}{2\sqrt{-d}\sqrt{d}\,b}$

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-d)^(1/2)/d^(1/2)*(ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*(-d)^(1/2)-2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d^(1/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 4.17

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= \left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4\,bd}\right.$$
$$\left.-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-\sqrt{d}\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6\,d\cos(bx+a)+d}{\cos(bx+a)^2-2\cos(bx+a)+1}\right)}{4\,bd}\right]$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) - sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/(b*d), -1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*
(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) - sqrt(d)*log((d*cos(b*x + a)^2
- 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)
/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)))/(b*d)]
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(d*cos(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-\sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(d)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - sqrt(d)*log((sqrt(d*
cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d} - \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)}{b}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d) - arctan(sqrt(d*cos(b
*x + a))/sqrt(d))/d^(3/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\sqrt{d\,\cos(a+b\,x)}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)),x)
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)), x)
```

3.228 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))/b/d^(3/2)+2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 331, 335, 304, 209, 212}

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(3/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(3/2)) + 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^3} \\
&= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}
\end{aligned}$$

$$= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd}$$

$$= \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} + \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2+\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)} - \operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+}}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2 + ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - ArcTanh[Sqrt[Cos[a + b
*x]]]*Sqrt[Cos[a + b*x]])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.65

method	result
default	$-\frac{4d^{\frac{5}{2}}\ln\left(\frac{2\sqrt{-d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\sqrt{-d}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\left(\sin^2\right.}{}$

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-d)^(1/2)/d^(7/2)/(2*sin(1/2*b*x+1/2*a)^2-1)*(4*d^(5/2)*ln(2/cos(1/2*
b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*sin(1/2*b*x+
1/2*a)^2+2*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-
d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*sin(1/2*b*x+1/2*a)^2*d^2+2*
(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^2*d^2-2*ln(2/cos(1/2
*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d^(5/2)+4*(
-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)*(-d)^(1/2)*d^(3/2)-ln(-2/(cos(1/2*b*x+1/
2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
```

```
+d))*(-d)^(1/2)*d^2-ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(
1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*(-d)^(1/2)*d^2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(64) = 128$.

Time = 0.32 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.96

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a) - \sqrt{-d}\cos(bx+a)\log\left(\right.}{4\,bd^2\cos(bx}\right.$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a)))*cos(b*x + a) - sqrt(-d)*cos(b*x + a)*log((d*cos(b*x + a
)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a)
 + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(b*d
^2*cos(b*x + a)), 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x +
 a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a) + sqrt(d)*cos(b*x + a)*log((d
*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x
+ a)))/(b*d^2*cos(b*x + a))]
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(a+bx)}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*cos(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\frac{2\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{\sqrt{d}} + \frac{4}{\sqrt{d\cos(bx+a)}}}{2\,bd}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + log((sqrt(d*cos(b*x +
 a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4/sqrt(d*cos(b*
x + a)))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)), x)
```

3.229 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(5/2)+2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 331, 335, 218, 212, 209}

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(5/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(5/2)) + 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^2} - \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^2}$$

$$= -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{-2+3\arctan\left(\sqrt{\cos(a+bx)}\right)\cos^{\frac{3}{2}}(a+bx)+3\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)\cos^{\frac{3}{2}}(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
[Out] -1/3*(-2 + 3*ArcTan[Sqrt[Cos[a + b*x]]]*Cos[a + b*x]^(3/2) + 3*ArcTanh[Sqrt
[Cos[a + b*x]]]*Cos[a + b*x]^(3/2))/(b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(65) = 130.

Time = 0.13 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.01

method	result
default	$\frac{24d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{-d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{-d}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\left(\sin^4\left(\frac{bx}{2}+\right.\right.}{}$

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d^(7/2)/(-d)^(1/2)/(4*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)*(2
4*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)
^(1/2)-d))*sin(1/2*b*x+1/2*a)^4-12*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(
2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*sin(
1/2*b*x+1/2*a)^4*d-12*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*
b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*sin(1/2*b*x+1/2*
a)^4*d-24*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2
*a)^2+d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^2+12*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2
*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-
d))*sin(1/2*b*x+1/2*a)^2*d+12*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*
cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*sin(1/2*
b*x+1/2*a)^2*d+6*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*
b*x+1/2*a)^2+d)^(1/2)-d))+4*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)-3*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^
(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d-3*(-d)^(1/2)*ln(-2/(cos(1/2
*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)+d))*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(65) = 130$.

Time = 0.33 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.93

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \left[\frac{6\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a)^2 - 3\sqrt{-d}\cos(bx+a)^2 \cdots}{12\,bd^3\cos \cdots}\right.$$

$$-\frac{6\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)\cos(bx+a)^2 - 3\sqrt{d}\cos(bx+a)^2\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(\cdots}{\cos(bx+a)^2-2\cos(\cdots}\right.\cdots}{12\,bd^3\cos(bx+a)^2}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
1)/(d*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(-d)*cos(b*x + a)^2*log((d*cos(
b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b
*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)
))/(b*d^3*cos(b*x + a)^2), -1/12*(6*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))
*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(d)*cos(
b*x + a)^2*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x
+ a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) -
8*sqrt(d*cos(b*x + a)))/(b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\frac{6\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{3\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{4}{(d\cos(bx+a))^{\frac{3}{2}}}}{6\,bd}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -1/6*(6*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) - 3*log((sqrt(d*cos(b*
x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(3/2) - 4/(d*cos(b*x
 + a))^(3/2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)),x)
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)), x)
```

3.230 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))/b/d^(7/2)+2/5/b/d/(d*cos(b*x+a))^(5/2)+2/b/d^3/(d*cos(b*x+a))^(1/
2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2645, 331, 335, 304, 209, 212}

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(7/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(7/2)) + 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) + 2/(b*d^3*Sqr
t[d*Cos[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^5}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^5}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3} + \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}$$

$$= \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{5\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)} - 5\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+b}}{5bd^3\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (5*ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - 5*ArcTanh[Sqrt[Cos[a + b
*x]]]*Sqrt[Cos[a + b*x]] + 2*(5 + Sec[a + b*x]^2))/(5*b*d^3*Sqrt[d*Cos[a +
b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(82) = 164.

Time = 0.14 (sec) , antiderivative size = 862, normalized size of antiderivative = 8.62

method	result	size
default	Expression too large to display	862

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/d^(9/2)/(-d)^(1/2)/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*s
in(1/2*b*x+1/2*a)^2-1)*(10*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))-24*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x
+1/2*a)^2+d)^(1/2)+5*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*
x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d+5*(-d)^(1/2)*ln(
-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x
+1/2*a)^2+d)^(1/2)+d))*d-40*(2*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*
(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a
)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d)
)*d+(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*
(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^6-20*(-6*d^(3
/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
-d))+4*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-3*(-d)^(1/2)*
ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*
b*x+1/2*a)^2+d)^(1/2)+d))*d-3*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*c
os(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/
2*b*x+1/2*a)^4+10*(-6*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin
(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+8*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a
)^2+d)^(1/2)-3*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2
*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d-3*(-d)^(1/2)*ln(2/(co
s(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a
)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(82) = 164$.

Time = 0.35 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.42

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \left[\frac{10\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a)^3 - 5\sqrt{-d}\cos(bx+a)^3 \mathrm{l}}{2}\right.$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^3 - 5*sqrt(-d)*cos(b*x + a)^3*log((d*cos
(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(
b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a
))*(5*cos(b*x + a)^2 + 1))/(b*d^4*cos(b*x + a)^3), 1/20*(10*sqrt(d)*arctan(
1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x
 + a)^3 + 5*sqrt(d)*cos(b*x + a)^3*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x
 + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 -
 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 + 1))/(b*d
^4*cos(b*x + a)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{4\left(5d^2\cos(bx+a)^2+d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}d^2}}{10\,bd}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] 1/10*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(5/2) + 5*log((sqrt(d*cos(b
*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(5/2) + 4*(5*d^2*co
s(b*x + a)^2 + d^2)/((d*cos(b*x + a))^(5/2)*d^2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{7/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)),x)
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)), x)
```

3.231 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} + \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(9/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(9/2)+2/7/b/d/(d*cos(b*x+a))^(7/2)+2/3/b/d^3/(d*cos(b*x+a))^
(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2645, 331, 335, 218, 212, 209}

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} + \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(9/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(9/2)) + 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) + 2/(3*b*d^
3*(d*Cos[a + b*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$=\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{\text{Subst}\left(\int\frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}dx,x,d\cos(a+bx)\right)}{bd^3}$$

$$=\frac{2}{7bd(d\cos(a+bx))^{7/2}}+\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}dx,x,d\cos(a+bx)\right)}{bd^5}$$

$$=\frac{2}{7bd(d\cos(a+bx))^{7/2}}+\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{2\text{Subst}\left(\int\frac{1}{1-\frac{x^4}{d^2}}dx,x,\sqrt{d\cos(a+bx)}\right)}{bd^5}$$

$$=\frac{2}{7bd(d\cos(a+bx))^{7/2}}+\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{\text{Subst}\left(\int\frac{1}{d-x^2}dx,x,\sqrt{d\cos(a+bx)}\right)}{bd^4}-\frac{\text{Subst}\left(\int\frac{1}{d+x^2}dx,x,\sqrt{d\cos(a+bx)}\right)}{bd^4}$$

$$=-\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}+\frac{2}{7bd(d\cos(a+bx))^{7/2}}+\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int\frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}dx=\frac{-21\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)}-21\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a-}{21bd^4\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
[Out] (-21*ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - 21*ArcTanh[Sqrt[Cos[a
+ b*x]]]*Sqrt[Cos[a + b*x]] + 14*Sec[a + b*x] + 6*Sec[a + b*x]^3)/(21*b*d^4
*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. 2(83) = 166.

Time = 0.16 (sec) , antiderivative size = 1057, normalized size of antiderivative = 10.26

method	result	size
default	Expression too large to display	1057

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42/d^(11/2)/(-d)^(1/2)/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+2
4*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(42*d^(3/2)*ln(2/cos(1/2*b
*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+40*(-d)^(1/2)
*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-21*(-d)^(1/2)*ln(2/(cos(1/2*b*
x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(
1/2)-d))*d-21*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*
a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d-336*(-2*d^(3/2)*ln(2/c
os(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+(-d)^
(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*si
n(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d+(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2
*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*si
n(1/2*b*x+1/2*a)^8+672*(-2*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)
*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d+
(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^6-56*(6*d^(3/2)*l
n(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+
2*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-3*(-d)^(1/2)*ln(-2
/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1
/2*a)^2+d)^(1/2)+d))*d-3*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/
2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x
+1/2*a)^2+56*(18*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*
b*x+1/2*a)^2+d)^(1/2)-d))+2*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)-9*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d
^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d-9*(-d)^(1/2)*ln(2/(cos(1/2
*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d
)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^4)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.32

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \left[\frac{42\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos\left(bx+a\right)^4 - 21\sqrt{-d}\cos\left(bx+a\right.}{}\right.$$

$$-\frac{42\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)\cos\left(bx+a\right)^4 - 21\sqrt{d}\cos\left(bx+a\right)^4\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(c}{\cos(bx+a)^2-2\,c}\right.}{84\,bd^5\cos\left(bx+a\right)^4}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^4 - 21*sqrt(-d)*cos(b*x + a)^4*log((d*co
s(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos
(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x +
a))*(7*cos(b*x + a)^2 + 3))/(b*d^5*cos(b*x + a)^4), -1/84*(42*sqrt(d)*arcta
n(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b
*x + a)^4 - 21*sqrt(d)*cos(b*x + a)^4*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(
b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^
2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 + 3))/(
b*d^5*cos(b*x + a)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\frac{42\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}} - \frac{21\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{7}{2}}} - \frac{4\left(7\,d^2\cos(bx+a)^2+3\,d^2\right)}{(d\cos(bx+a))^{\frac{7}{2}}\,d^2}}{42\,bd}$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/42*(42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(7/2) - 21*log((sqrt(d*cos
(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(7/2) - 4*(7*d^2*
cos(b*x + a)^2 + 3*d^2)/((d*cos(b*x + a))^(7/2)*d^2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\csc\left(bx+a\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{9/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)),x)
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)), x)
```

3.232 $\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^6\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b}$$

```
[Out] -d*(d*cos(b*x+a))^(9/2)*csc(b*x+a)/b-9/7*d^3*(d*cos(b*x+a))^(5/2)*sin(b*x+a
)/b-15/7*d^6*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(
1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-15/7*d^5*si
n(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2647, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = -\frac{15d^6\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{15d^5\sin(a+bx)\sqrt{d\cos(a+bx)}}{7b} - \frac{9d^3\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b} - \frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(9/2)*Csc[a + b*x])/b) - (15*d^6*Sqrt[Cos[a + b*x]]*E
llipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]]) - (15*d^5*Sqrt[d*Cos[a
 + b*x]]*Sin[a + b*x])/(7*b) - (9*d^3*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/
(7*b)
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{1}{2}\left(9d^2\right)\int (d\cos(a+bx))^{7/2}\,dx \\
&= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b} \\
&\quad - \frac{1}{14}\left(45d^4\right)\int (d\cos(a+bx))^{3/2}\,dx \\
&= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} \\
&\quad - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b} - \frac{1}{14}\left(15d^6\right)\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx
\end{aligned}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b} - \frac{\left(15d^6\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{14\sqrt{d\cos(a+bx)}}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^6\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int(d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \frac{d^5\sqrt{d\cos(a+bx)}\csc(a+bx)\left(\sqrt{\cos(a+bx)}(-45+16\cos(2(a+bx))+\cos(4(a+bx)))-60\,\text{Elli}\right.}{28b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

```
[Out] (d^5*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]*(Sqrt[Cos[a + b*x]]*(-45 + 16*Cos[2*
(a + b*x)] + Cos[4*(a + b*x)]) - 60*EllipticF[(a + b*x)/2, 2]*Sin[a + b*x])
)/(28*b*Sqrt[Cos[a + b*x]])
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^7\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-128\left(\sin^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+384\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-576\left(\sin^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+30\,c\right.}{14\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\right.\right.\right.}$

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^7/(-2*sin
(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/
2*b*x+1/2*a)*(-128*sin(1/2*b*x+1/2*a)^12+384*sin(1/2*b*x+1/2*a)^10-576*sin(
1/2*b*x+1/2*a)^8+30*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*Ell
ipticF(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+512*sin(1/2
*b*x+1/2*a)^6-204*sin(1/2*b*x+1/2*a)^4+12*sin(1/2*b*x+1/2*a)^2+7)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \frac{15i\sqrt{2}d^{\frac{11}{2}}\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) - 15i\sqrt{2}d^{\frac{11}{2}}\sin(b$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/14*(15*I*sqrt(2)*d^(11/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x
 + a) + I*sin(b*x + a)) - 15*I*sqrt(2)*d^(11/2)*sin(b*x + a)*weierstrassPIn
verse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*(2*d^5*cos(b*x + a)^4 + 6*d
^5*cos(b*x + a)^2 - 15*d^5)*sqrt(d*cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{11/2}}{\sin(a+b\,x)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2, x)
```

3.233 $\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{21d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b}$$

```
[Out] -d*(d*cos(b*x+a))^(7/2)*csc(b*x+a)/b-7/5*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a
)/b-21/5*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2647, 2715, 2721, 2719}

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{21d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}} - \frac{7d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b} - \frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x])/b) - (21*d^4*Sqrt[d*Cos[a + b*x]]
*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (7*d^3*(d*Cos[a + b*
x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{1}{2}(7d^2)\int (d\cos(a+bx))^{5/2}\,dx \\
&= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b} \\
&\quad - \frac{1}{10}(21d^4)\int \sqrt{d\cos(a+bx)}\,dx \\
&= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b} \\
&\quad - \frac{\left(21d^4\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{10\sqrt{\cos(a+bx)}} \\
&= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{21d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} \\
&\quad - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{d^4\sqrt{d\cos(a+bx)}\left(21E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}(5\cot(a+bx)+\sin(2(a+bx)))\right)}{5b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -1/5*(d^4*Sqrt[d*Cos[a + b*x]]*(21*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a +
 b*x]]*(5*Cot[a + b*x] + Sin[2*(a + b*x)])))/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(110) = 220.

Time = 2.85 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.39

method	result
default	$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^6\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-64\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+160\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+42\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}\right.\right.\right.\right.}{10\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\cos\left(\frac{b}{}\right.}$

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^6/(-2*sin(
1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-64*sin(1/2*b*x+1/2*a)^10+160*sin(1/2*b*x+1/2*a)^8+42*cos(1/2*
b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticE(cos(1/2*b*x+1/2*a),2^
(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)-104*sin(1/2*b*x+1/2*a)^6-4*sin(1/2*b*x+
1/2*a)^4+22*sin(1/2*b*x+1/2*a)^2-5)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.26

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \frac{-21i\sqrt{2}d^{\frac{9}{2}}\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx}{}$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(-21*I*sqrt(2)*d^(9/2)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 21*I*sqrt(2)*d^(9/2)*sin(
b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I
*sin(b*x + a))) + 2*(2*d^4*cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(
b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos\left(a+b\,x\right))^{9/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2, x)
```

3.234 $\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b}$$

```
[Out] -d*(d*cos(b*x+a))^(5/2)*csc(b*x+a)/b-5/3*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/c
os(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/
(d*cos(b*x+a))^(1/2)-5/3*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2647, 2715, 2721, 2720}

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = -\frac{5d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{5d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b} - \frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x])/b) - (5*d^4*Sqrt[Cos[a + b*x]]*El
lipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (5*d^3*Sqrt[d*Cos[a +
 b*x]]*Sin[a + b*x])/(3*b)
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b}-\frac{1}{2}\left(5d^2\right)\int(d\cos(a+bx))^{3/2}\,dx\\
&= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b}\\
&\quad-\frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b}-\frac{1}{6}\left(5d^4\right)\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b}-\frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b}\\
&\quad-\frac{\left(5d^4\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{6\sqrt{d\cos(a+bx)}}\\
&= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b}-\frac{5d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}\\
&\quad-\frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(-4+\cos(2(a+bx)))\csc(a+bx)-5\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)}{3b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(Sqrt[Cos[a + b*x]]*(-4 + Cos[2*(a + b*x)])*Csc[a
 + b*x] - 5*EllipticF[(a + b*x)/2, 2]))/(3*b*Sqrt[Cos[a + b*x]])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^5\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-32\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}F\left(\cos\left(\frac{bx}{2}\right.\right.\right.}{6\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d(}}$

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5/(-2*sin(
1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-32*sin(1/2*b*x+1/2*a)^8+10*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+
1/2*a)^2-1)^(3/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)
^2)^(1/2)+64*sin(1/2*b*x+1/2*a)^6-28*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2
*a)^2+3)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \frac{5i\sqrt{2}d^{\frac{7}{2}}\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-5i\sqrt{2}d^{\frac{7}{2}}\sin(bx+a}{6\,b\,\text{si}}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(5*I*sqrt(2)*d^(7/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x +
a) + I*sin(b*x + a)) - 5*I*sqrt(2)*d^(7/2)*sin(b*x + a)*weierstrassPInverse
(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*(2*d^3*cos(b*x + a)^2 - 5*d^3)*s
qrt(d*cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2, x)
```

3.235 $\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{3d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b-3*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos
(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/
b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2721, 2719}

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}} - \frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/b) - (3*d^2*Sqrt[d*Cos[a + b*x]]*
EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{1}{2}\left(3d^2\right)\int\sqrt{d\cos(a+bx)}\,dx \\
&= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{\left(3d^2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}} \\
&= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{3d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int(d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{5/2}\left(\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)+3E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b\cos^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
[Out] -(((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + 3*EllipticE[(a
 + b*x)/2, 2]))/(b*Cos[a + b*x]^(5/2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(86) = 172.

Time = 2.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.08

method	result
default	$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^4 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}} E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a}{2}}}{2\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right.}}$

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4/(-2*sin(1
/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2*
b*x+1/2*a)*(6*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticE
(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \frac{-3i\sqrt{2}d^{\frac{5}{2}}\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+}{}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*d^(5/2)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*d^(5/2)*sin(b*x
 + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*si
n(b*x + a))) - 2*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2, x)
```

3.236 $\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b} - \frac{d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

```
[Out] -d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-d*csc(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2721, 2720}

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}} - \frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/b) - (d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b} - \frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\ &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b} - \frac{\left(d^2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}\\ &= -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b} - \frac{d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{3/2}\left(\sqrt{\cos(a+bx)}\csc(a+bx)+\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
[Out] -(((d*Cos[a + b*x])^(3/2)*(Sqrt[Cos[a + b*x]]*Csc[a + b*x] + EllipticF[(a +
 b*x)/2, 2]))/(b*Cos[a + b*x]^(3/2)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.88

method	result
default	$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^3 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}} F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx}{2}}\right.}{2\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}$

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3/(-2*sin(
1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*Elliptic
F(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+4*sin(1/2*b*x+1/
2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \frac{i\sqrt{2}d^{\frac{3}{2}}\sin(bx+a)\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) - i\sqrt{2}d^{\frac{3}{2}}\sin(bx+a)\,\mathrm{w}}{2\,b\sin(bx+a)}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(2)*d^(3/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a)
 + I*sin(b*x + a)) - I*sqrt(2)*d^(3/2)*sin(b*x + a)*weierstrassPInverse(-4,
 0, cos(b*x + a) - I*sin(b*x + a)) - 2*sqrt(d*cos(b*x + a))*d)/(b*sin(b*x +
 a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2, x)
```

3.237 $\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b/d-(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a
+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(
b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2721, 2719}

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = -\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd} - \frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/(b*d)) - (Sqrt[d*Cos[a + b*x]]*Elli
pticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{1}{2}\int \sqrt{d\cos(a+bx)}\,dx \\
&= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}} \\
&= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = -\frac{\sqrt{d\cos(a+bx)}\left(\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)+E\left(\frac{1}{2}(a+bx)\middle|\,2\right)\right)}{b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
[Out] -((Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + EllipticE[(a + b
*x)/2, 2]))/(b*Sqrt[Cos[a + b*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.

Time = 0.43 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.12

method	result
default	$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^2 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}} E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a}{2}}}{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}}$

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2/cos(1/2*b
*x+1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)*sin(1/2*
b*x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticE
(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.57

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{d}\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+i}{}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(2)*sqrt(d)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(d)*sin(b*x + a
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*
x + a))) - 2*sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**2,x)
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \frac{\sqrt{d\,\cos(a+b\,x)}}{\sin(a+b\,x)^2}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2, x)
```

3.238 $\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

```
[Out] (cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x
),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-csc(b*x+a)*(d*cos(b*x+a)
)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2650, 2721, 2720}

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/(b*d)) + (Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
```

```
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{1}{2}\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}\\
&= -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int\frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx=\frac{-\cot(a+bx)+\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
[Out] (-Cot[a + b*x] + Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[d*Co
s[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(84) = 168.

Time = 0.39 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

method	result
default	$\frac{\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}} F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}}{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}\, b}$

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1
/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)*d*sin(1/2*b*
x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticF(c
os(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)-4*sin(1/2*b*x+1/2*a
)^4+4*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.45

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\, dx = \frac{-i\sqrt{2}\sqrt{d}\sin(bx+a)\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{d}\sin(bx+a)\,\mathrm{weier}}{2\,bd\sin(bx+a)}$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(2)*sqrt(d)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a
) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(d)*sin(b*x + a)*weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a)) - 2*sqrt(d*cos(b*x + a)))/(b*d*sin(b*x
+ a))
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
[Out] Integral(csc(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,\sqrt{d\,\cos(a+b\,x)}}\,dx$$

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)),x)
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)), x)
```

3.239 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{3\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)+3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)-
3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^2/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2650, 2716, 2721, 2719}

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{3E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*Sqrt[d*Cos[a + b*x]])) - (3*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (3*Sin[a + b*x])/(b*d*Sqr
t[d*Cos[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3}{2}\int\frac{1}{(d\cos(a+bx))^{3/2}}\,dx\\
&= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{3\int\sqrt{d\cos(a+bx)}\,dx}{2d^2}\\
&= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\left(3\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}}\\
&= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{3\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-\cos(a+bx)\cot(a+bx)-3\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-(Cos[a + b*x]*Cot[a + b*x]) - 3*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2,
 2] + 2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\left(6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{2(\sin\cdots}\right.}{2d^3\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^5\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^2\sqrt{d\left(2\left(\cos^2\left(\frac{b}{2}\cdots\right.\right.\right.}}$

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^3/cos(1/2*
b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(2*sin(1/2*b*x+1/2*a)^2-1)^2*(-2*sin(1/2*b*
x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)*(6*cos(1/2*b*x+1/2*a)*EllipticE(
cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1
/2*a)^2)^(1/2)+12*sin(1/2*b*x+1/2*a)^4-12*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{d}\cos(bx+a)\sin(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4\cdots}{\cdots}$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*sqrt(d)*cos(b*x + a)*sin(b*x + a)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*sq
rt(d)*cos(b*x + a)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a))) - 2*sqrt(d*cos(b*x + a))*(3*cos(b*x
+ a)^2 - 2))/(b*d^2*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
[Out] Integral(csc(a + b*x)**2/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)),x)
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)), x)
```

3.240 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)+5/3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(3/2
)+5/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1
/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2650, 2716, 2721, 2720}

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(3/2))) + (5*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (5*Sin[a + b*x])/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5}{2}\int\frac{1}{(d\cos(a+bx))^{5/2}}\,dx\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}+\frac{5\int\frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{6d^2}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}+\frac{\left(5\sqrt{\cos(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{6d^2\sqrt{d\cos(a+bx)}}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{-3\cot(a+bx)+5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\tan(a+bx)}{3bd^2\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-3*Cot[a + b*x] + 5*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2*Tan[a
 + b*x])/(3*b*d^2*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.94

method	result
default	$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(10\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}-20\left(\sin^4\left(\right.\right.\right.}{6d\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}\,b}$

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d/(-2*sin(1/2
*b*x+1/2*a)^4*d+d*sin(1/2*b*x+1/2*a)^2)^(3/2)/cos(1/2*b*x+1/2*a)*(10*cos(1/
2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticF(cos(1/2*b*x+1/2*a),
2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)-20*sin(1/2*b*x+1/2*a)^4+20*sin(1/2*b*
x+1/2*a)^2-3)*sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{-5i\sqrt{2}\sqrt{d}\cos(bx+a)^2\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin}{}$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(-5*I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*sin(b*x + a)*weierstrassPInverse(-
4, 0, cos(b*x + a) + I*sin(b*x + a)) + 5*I*sqrt(2)*sqrt(d)*cos(b*x + a)^2*s
in(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) - 2*s
qrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 2))/(b*d^3*cos(b*x + a)^2*sin(b*x +
 a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)),x)
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)), x)
```

3.241 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} - \frac{21\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)+7/5*sin(b*x+a)/b/d/(d*cos(b*x+a))^(5/2
)+21/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(1/2)-21/5*(cos(1/2*a+1/2*b*x)^2)^(1
/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))
^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2650, 2716, 2721, 2719}

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{21E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(5/2))) - (21*Sqrt[d*Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*x]]) + (7*Sin[a + b*x])/(5*
```

```
b*d*(d*Cos[a + b*x])^(5/2)) + (21*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x
]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7}{2}\int\frac{1}{(d\cos(a+bx))^{7/2}}\,dx\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{21\int\frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{10d^2}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\\
&\quad+\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{21\int\sqrt{d\cos(a+bx)}\,dx}{10d^4}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\\
&\quad+\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{\left(21\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{10d^4\sqrt{\cos(a+bx)}}
\end{aligned}
$$

$$= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} - \frac{21\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{-5\cos(a+bx)\cot(a+bx) - 21\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + 16\sin(a+bx) +}{5bd^3\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (-5*Cos[a + b*x]*Cot[a + b*x] - 21*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2] + 16*Sin[a + b*x] + 2*Sec[a + b*x]*Tan[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a
 + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(136) = 272.

Time = 1.06 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(168\cos\left(\frac{bx}{2}+\frac{a}{2}\right)E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\left(\sin^4\left(\frac{bx}{2}\right.\right.\right.}{}$

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^5/(2*sin(
1/2*b*x+1/2*a)^2-1)/cos(1/2*b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(8*sin(1/2*b*x+
1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)*(168*cos(1/2*b*x
+1/2*a)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)*
(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*sin(1/2*b*x+1/2*a)^4+336*sin(1/2*b*x+1/2*a
)^8-168*cos(1/2*b*x+1/2*a)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b
*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*sin(1/2*b*x+1/2*a)^2-67
2*sin(1/2*b*x+1/2*a)^6+42*cos(1/2*b*x+1/2*a)*EllipticE(cos(1/2*b*x+1/2*a),2
^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)+448*s
in(1/2*b*x+1/2*a)^4-112*sin(1/2*b*x+1/2*a)^2+5)*(-2*sin(1/2*b*x+1/2*a)^4*d+
d*sin(1/2*b*x+1/2*a)^2)^(3/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{-21i\sqrt{2}\sqrt{d}\cos{(bx+a)}^3\sin{(bx+a)}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/10*(-21*I*sqrt(2)*sqrt(d)*cos(b*x + a)^3*sin(b*x + a)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 21*I*sqrt(
2)*sqrt(d)*cos(b*x + a)^3*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) - 2*(21*cos(b*x + a)^4 - 14*c
os(b*x + a)^2 - 2)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(b*x + a)^3*sin(b*x + a)
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc{(bx+a)}^2}{(d\cos{(bx+a)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{7/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)),x)
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)), x)
```

3.242 $\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{9d^{11/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b}$$

```
[Out] 9/4*d^(11/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+9/4*d^(11/2)*arctanh((d
*cos(b*x+a))^(1/2)/d^(1/2))/b-9/10*d^3*(d*cos(b*x+a))^(5/2)/b-1/2*d*(d*cos(
b*x+a))^(9/2)*csc(b*x+a)^2/b-9/2*d^5*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 294, 327, 335, 218, 212, 209}

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{9d^{11/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d\csc^2(a+bx)(d\cos(a+bx))^{9/2}}{2b}$$

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (9*d^(11/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (9*d^(11/2)*ArcTa
nh[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (9*d^5*Sqrt[d*Cos[a + b*x]])/(2*b
) - (9*d^3*(d*Cos[a + b*x])^(5/2))/(10*b) - (d*(d*Cos[a + b*x])^(9/2)*Csc[a
 + b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{11/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d)\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d^3)\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d^5)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d^6)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{(9d^6)\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= \frac{9d^{11/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{d(d\cos(a+bx))^{9/2}\left(45\arctan\left(\sqrt{\cos(a+bx)}\right)+24\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)-2\sqrt{\cos(a+bx)}(2\,\mathrm{c}\ldots\right.}{20\ldots}$$

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(9/2)*(45*ArcTan[Sqrt[Cos[a + b*x]]] + 24*ArcTanh[Sqrt[
Cos[a + b*x]]] - 2*Sqrt[Cos[a + b*x]]*(2*Cos[2*(a + b*x)] + 5*Csc[a + b*x]^
2) - (21*(8*Sqrt[Cos[a + b*x]] + Log[1 - Sqrt[Cos[a + b*x]]] - Log[1 + Sqrt
[Cos[a + b*x]]]))/2))/(20*b*Cos[a + b*x]^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(107) = 214.

Time = 6.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.01

method	result
default	$\frac{-\frac{d^5\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{9d^6\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-6d^5\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}-\frac{8d^5\left(\cos^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}\ldots\right.\right.}}{5}\ldots}{\ldots}$

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*d^5/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-9/4*d^6/(
-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2
*b*x+1/2*a))-6*d^5*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)-8/5*d^5*cos(1/2*b*x
+1/2*a)^4*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5*d^5*cos(1/2*b*x+1/2*a)^2*(
2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5*d^5*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2
)+9/8*d^(11/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a
)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+9/8*d^(11/2)*ln((-4*d*cos(1/2*b*x
+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2
*a)+1))+1/16*d^5/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
-1/16*d^5/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.10

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \left[-\frac{90\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-45\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\log\cdots}{\cdots}\right.$$

$$-\frac{90\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-45\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{d}\log\left(-\frac{d\cos(bx+a)^2+4\cdots}{\cdots}\right.\cdots}{80\left(b\cos\left(b\cdots\right.\right.}$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 45*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*l
og(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^5*c
os(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*x + a)))/(b*co
s(b*x + a)^2 - b), -1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(d)*arctan(2*sq
rt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 45*(d^5*cos(b*x + a)^2 -
 d^5)*sqrt(d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(
b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)
) + 8*(4*d^5*cos(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*
x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{\frac{20\sqrt{d\cos(bx+a)}d^8}{d^2\cos(bx+a)^2-d^2}+90\,d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-45\,d^{\frac{13}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-16\,(d\cos(bx+a))^{\frac{5}{2}}\,d}{40\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(20*sqrt(d*cos(b*x + a))*d^8/(d^2*cos(b*x + a)^2 - d^2) + 90*d^(13/2)*
arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 45*d^(13/2)*log((sqrt(d*cos(b*x + a)
) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(5/2)*
d^4 - 160*sqrt(d*cos(b*x + a))*d^6)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^{11/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3, x)
```

3.243 $\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = -\frac{7d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{7d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b}$$

```
[Out] -7/4*d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+7/4*d^(9/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-7/6*d^3*(d*cos(b*x+a))^(3/2)/b-1/2*d*(d*cos(b*
x+a))^(7/2)*csc(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 294, 327, 335, 304, 209, 212}

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = -\frac{7d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{7d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d\csc^2(a+bx)(d\cos(a+bx))^{7/2}}{2b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-7*d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (7*d^(9/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (7*d^3*(d*Cos[a + b*x])^(3/2))/(6*
b) - (d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{(7d)\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b} \\
&= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} \\
&\quad + \frac{(7d^3)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b} \\
&= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} \\
&\quad + \frac{(7d^3)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b} \\
&= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} \\
&\quad + \frac{(7d^5)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} \\
&\quad - \frac{(7d^5)\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} \\
&= -\frac{7d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{7d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} \\
&\quad - \frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.77 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \frac{d^5\left(\left(-5+2\cos(2(a+bx))\right)\cot^2(a+bx)+21\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a\right.\right.}{6b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^5*((-5 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2 + 21*(-Cot[a + b*x]^2)^(1/4)
*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2]))/(6*b*Sqrt[d*Cos[a + b*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(89) = 178.

Time = 6.06 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.28

method	result
default	$\frac{2d^4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}+\frac{7d^5\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}+\frac{d^4\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{4d^4\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\right.\right.}}{3}}{}$

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+7/4*d^5/(-d)^(1/2)*ln((-2*d+2*(
-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))+1/8*d^4/c
os(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-4/3*d^4*cos(1/2*b*x+
1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-4/3*d^4*(2*cos(1/2*b*x+1/2*a)^2
*d-d)^(1/2)+7/8*d^(9/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*
b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+7/8*d^(9/2)*ln((-4*d*cos
(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2
*b*x+1/2*a)+1))+1/16*d^4/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+
d)^(1/2)-1/16*d^4/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2
))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(89) = 178.

Time = 0.42 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.58

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \left[-\frac{42\left(d^4\cos(bx+a)^2-d^4\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-21\left(d^4\cos(bx+a)^2-d^4\right)\sqrt{-d}\log}{4}\right.$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*l
og(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^4*c
os(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2
 - b), 1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)*arctan(2*sqrt(d*cos(b*x
+ a))*sqrt(d)/(d*cos(b*x + a) - d)) + 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)
*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1)
 + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(4*d^4*
cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^
2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \frac{\frac{12\,(d\cos(bx+a))^{\frac{3}{2}}d^6}{d^2\cos(bx+a)^2-d^2}-42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-16\,(d\cos(bx+a))}{24\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(12*(d*cos(b*x + a))^(3/2)*d^6/(d^2*cos(b*x + a)^2 - d^2) - 42*d^(11/2
)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 21*d^(11/2)*log((sqrt(d*cos(b*x +
a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(3/2
)*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{9/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3, x)
```

3.244 $\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \frac{5d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{5d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b}$$

```
[Out] 5/4*d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+5/4*d^(7/2)*arctanh((d*c
os(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(5/2)*csc(b*x+a)^2/b-5/2*d
^3*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 294, 327, 335, 218, 212, 209}

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \frac{5d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{5d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d\csc^2(a+bx)(d\cos(a+bx))^{5/2}}{2b}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

```
[Out] (5*d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (5*d^(7/2)*ArcTanh
[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (5*d^3*Sqrt[d*Cos[a + b*x]])/(2*b)
- (d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d)\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d^4)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{(5d^4)\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= \frac{5d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{5d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \frac{(d\cos(a+bx))^{7/2}\left(5\arctan\left(\sqrt{\cos(a+bx)}\right)+3\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)-8\sqrt{\cos(a+bx)}-2\sqrt{c}\right.}{4b\cos^{\frac{7}{2}}(a+bx)}$$

[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]

[Out] ((d*Cos[a + b*x])^(7/2)*(5*ArcTan[Sqrt[Cos[a + b*x]]] + 3*ArcTanh[Sqrt[Cos[a + b*x]]] - 8*Sqrt[Cos[a + b*x]] - 2*Sqrt[Cos[a + b*x]]*Csc[a + b*x]^2 - Log[1 - Sqrt[Cos[a + b*x]]] + Log[1 + Sqrt[Cos[a + b*x]]]))/(4*b*Cos[a + b*x]^(7/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(89) = 178.

Time = 6.02 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.74

method	result
default	$\frac{-\frac{d^3\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{5d^4\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-2d^3\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}+\frac{5d^{\frac{7}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-}}{\cos\left(\frac{bx}{2}\right.}\right.}{8}}{}$

[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] (-1/8*d^3/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-5/4*d^4/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))-2*d^3*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+5/8*d^(7/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+5/8*d^(7/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16*d^3/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-1/16*d^3/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2))/b

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(89) = 178.

Time = 0.42 (sec) , antiderivative size = 393, normalized size of antiderivative = 3.48

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \left[-\frac{10\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-5\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-d}\log\left(\cdots\right.}{16\left(b\cos\cdots\right.}\right.$$

$$-\frac{10\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-5\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{d}\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{\cdots}}{\cdots}\right.}{16\left(b\cos\left(bx+a\right)^2-b\right)\cdots}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*lo
g(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^3*co
s(b*x + a)^2 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b), -1/16*(
10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)
/(d*cos(b*x + a) - d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*log(-(d*cos(b
*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
 + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4*d^3*cos(b*x + a)^2
 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^6}{d^2\cos(bx+a)^2-d^2} + 10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\,\sqrt{d\cos(bx+a)}d^4}{8\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^6/(d^2*cos(b*x + a)^2 - d^2) + 10*d^(9/2)*arc
tan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sqrt(d*cos(b*x + a)) - s
qrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*sqrt(d*cos(b*x + a))*d^4)/(b
*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3, x)
```

3.245 $\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{3d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b}$$

```
[Out] -3/4*d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+3/4*d^(5/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2645, 294, 335, 304, 209, 212}

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{3d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-3*d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (3*d^(5/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*(d*Cos[a + b*x])^(3/2)*Csc[a +
b*x]^2)/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} - \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= -\frac{3d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{d^3\left(\cot^2(a+bx) - 3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
[Out] -1/2*(d^3*(Cot[a + b*x]^2 - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4
, 1/4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.

Time = 5.92 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.14

method	result
default	$\frac{\frac{3d^3\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}} + \frac{d^2\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2} + \frac{3d^{\frac{5}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8} + 3d^{\frac{5}{2}}}{b}$

[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] (3/4*d^3/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))+1/8*d^2/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+3/8*d^(5/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+3/8*d^(5/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16*d^2/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-1/16*d^2/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2))/b

Fricas [B] (verification not implemented)

Leave count of result is larger than twice the leaf count of optimal. 184 vs. 2(71) = 142.

Time = 0.38 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.18

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \left[\frac{8\sqrt{d\cos(bx+a)}d^2\cos(bx+a) - 6\left(d^2\cos(bx+a)^2 - d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+}{2\,d\cos(bx+a)}\right.}{16\left(b\cos(bx\right.}\right.$$

[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="fricas")

[Out] [1/16*(8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b), 1/16*(8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^2)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b)]

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**3,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^4}{d^2\cos(bx+a)^2-d^2} - 6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="maxima")
[Out] 1/8*(4*(d*cos(b*x + a))^(3/2)*d^4/(d^2*cos(b*x + a)^2 - d^2) - 6*d^(7/2)*ar
ctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 3*d^(7/2)*log((sqrt(d*cos(b*x + a)) -
sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3,x)
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

3.246 $\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b}$$

```
[Out] 1/4*d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+1/4*d^(3/2)*arctanh((d*c
os(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*csc(b*x+a)^2*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2645, 294, 335, 218, 212, 209}

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (d^(3/2)*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2)
/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b} + \frac{d^2\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{d^2\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= \frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\left(-\cot^2(a+bx)\right)^{3/4}\left(3\sqrt[4]{-\cot^2(a+bx)} + \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\csc^2(a+\right.\right.}{6b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
[Out] ((d*Cos[a + b*x])^(3/2)*(-Cot[a + b*x]^2)^(3/4)*(3*(-Cot[a + b*x]^2)^(1/4)
+ Hypergeometric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(6*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(71) = 142.

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.08

method	result
default	$\frac{-\frac{d\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2} - \frac{d^2\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}} + \frac{d^{\frac{3}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8} + \frac{d^{\frac{3}{2}}\ln\left(\right.}{}}{b}$

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-1/4*d^2/(-d
)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b
*x+1/2*a))+1/8*d^(3/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b
*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))+1/8*d^(3/2)*ln((-4*d*cos(
1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*
b*x+1/2*a)+1))+1/16*d/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^
(1/2)-1/16*d/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(71) = 142$.

Time = 0.33 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.81

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \left[-\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{-d}\,\mathrm{l}}{16\left(b\cos\left(bx+a\right)^2\right.}\right.$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(d*cos(b*x + a)^2 - d)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - (d*cos(b*x + a)^2 - d)*sqrt(
-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) -
 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) - 8*sqrt
(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 - b), 1/16*(2*(d*cos(b*x + a)^2 - d)*
sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x
 + a))) + (d*cos(b*x + a)^2 - d)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*c
os(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 -
 b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^4}{d^2\cos(bx+a)^2-d^2} + 2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^4/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(5/2)*arct
an(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d*cos(b*x + a)) - sqrt
(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

3.247 $\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd}$$

```
[Out] -1/2*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b/d+1/4*arctan((d*cos(b*x+a))^(1/2)/
d^(1/2))*d^(1/2)/b-1/4*arctanh((d*cos(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2645, 296, 335, 304, 209, 212}

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (Sqrt[d]*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - ((d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2)
/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{d\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} + \frac{d\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = -\frac{d\left(\cot^2(a+bx) + \sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
[Out] -1/2*(d*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/
4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.96

method	result
default	$\frac{\frac{\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2} - \frac{d\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}} + \frac{\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}}{16\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-16} - \frac{\sqrt{d}\,\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2} \cdots\right.\right.}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8}}{b}$

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-1/4*d/(-d)^(1/
2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/
2*a))+1/16/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-1/8*d
^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(
1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2
*b*x+1/2*a)^2+d)^(1/2)-1/8*d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-
2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.66

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$$
$$= \left[\frac{2\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+}{}\right.}{16\left(b\cos\left(bx+a\right)^2-b\right)}\right.$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(-d)*l
og((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*co
s(b*x + a))*cos(b*x + a))/(b*cos(b*x + a)^2 - b), 1/16*(2*(cos(b*x + a)^2 -
 1)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos
(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d
*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x
+ a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*cos
(b*x + a)^2 - b)]
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \int \sqrt{d\cos\left(a+bx\right)}\csc^3\left(a+bx\right)dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^2}{d^2\cos(bx+a)^2-d^2} + 2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(d*cos(b*x + a))^(3/2)*d^2/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(3/2)*ar
ctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*cos(b*x + a)) - sq
rt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{d^3\left(\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{\left(d^2\cos(bx+a)^2-d^2\right)d} + \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)}{4\,b}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*d^3*(2*sqrt(d*cos(b*x + a))*cos(b*x + a)/((d^2*cos(b*x + a)^2 - d^2)*d)
 + arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^2) + arctan(sqrt(d*cos
(b*x + a))/sqrt(d))/d^(5/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{{\sin\left(a+b\,x\right)}^3}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

3.248 $\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd}$$

```
[Out] -3/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-3/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(1/2)-1/2*csc(b*x+a)^2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2645, 296, 335, 218, 212, 209}

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2bd}$$

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (3*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]
^2)/(2*b*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d(-\cot^2(a+bx))^{3/4}\left(\sqrt[4]{-\cot^2(a+bx)} - \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\csc^2(a+bx)\right)\right)}{2b(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
[Out] (d*(-Cot[a + b*x]^2)^(3/4)*((-Cot[a + b*x]^2)^(1/4) - Hypergeometric2F1[3/4
, 3/4, 7/4, Csc[a + b*x]^2]))/(2*b*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.04

method	result
default	$\frac{-\frac{\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{3\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-\frac{3\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8\sqrt{d}}-\frac{3\ln\left(\frac{-4d\cos \cdots}{\cdots}\right)}{\cdots}}{b}$

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8/d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+3/4/(-d)^(1
/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1
/2*a))-3/8/d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1
/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-3/8/d^(1/2)*ln((-4*d*cos(1/2*
b*x+1/2*a)+2*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-2*d)/(cos(1/2*b*x+
1/2*a)+1))+1/16/d/(cos(1/2*b*x+1/2*a)-1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2
)-1/16/d/(cos(1/2*b*x+1/2*a)+1)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.59

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \left[\frac{6\left(\cos(bx+a)^2-1\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-3\left(\cos(bx+a)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+\cdots}{\cdots}\right.\cdots}{16\left(bd\cos(bx+a)^2-bd\right)}\right.$$
$$-\frac{6\left(\cos(bx+a)^2-1\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-3\left(\cos(bx+a)^2-1\right)\sqrt{d}\log\left(\frac{d\cos(bx+a)^2-4\sqrt{\cdots}}{\cdots}\right.\cdots}{16\left(bd\cos(bx+a)^2-bd\right)}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(-d)
*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
 - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*
cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b*d), -1/16*(6*(cos(b*x + a)^2 - 1)*sq
rt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
 a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos
(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)
^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b
*d)]
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
[Out] Integral(csc(a + b*x)**3/sqrt(d*cos(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^2}{d^2\cos(bx+a)^2-d^2} - 6\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + 3\sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^2/(d^2*cos(b*x + a)^2 - d^2) - 6*sqrt(d)*arct
an(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*sqrt(d)*log((sqrt(d*cos(b*x + a)) - sq
rt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d^3\left(\frac{2\sqrt{d\cos(bx+a)}}{\left(d^2\cos(bx+a)^2-d^2\right)d^2} + \frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^3} - \frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)}{4\,b}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/4*d^3*(2*sqrt(d*cos(b*x + a))/((d^2*cos(b*x + a)^2 - d^2)*d^2) + 3*arctan
(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^3) - 3*arctan(sqrt(d*cos(b*x +
a))/sqrt(d))/d^(7/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)), x)
```

3.249 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 5/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-5/4*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b/d^(3/2)+5/2/b/d/(d*cos(b*x+a))^(1/2)-1/2*csc(b*x+a)^2/b
/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 296, 331, 335, 304, 209, 212}

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (5*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) - (5*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) + 5/(2*b*d*Sqrt[d*Cos[a + b*x]]) - Cs
c[a + b*x]^2/(2*b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^3}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd} + \frac{5\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd}$$

$$= \frac{5\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-(-\cot^2(a+bx))^{3/4}\,(-4+\cot^2(a+bx)) + 5\cot^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\right.}{2bd\sqrt{d\cos(a+bx)}\,(-\cot^2(a+bx))^{3/4}}$$

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-((-Cot[a + b*x]^2)^(3/4)*(-4 + Cot[a + b*x]^2)) + 5*Cot[a + b*x]^2*Hyperg
eometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2])/(2*b*d*Sqrt[d*Cos[a + b*x]]*(-C
ot[a + b*x]^2)^(3/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 689, normalized size of antiderivative = 5.99

method	result
default	$-\frac{\sqrt{-d}\,\sqrt{d}\,\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(-20d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2d\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)-10\sqrt{-d}\,\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\right.\right.}{}\right.\right.}{}$

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d^(5/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(2*sin(1/2*b*x+1/2*a)^4-3*sin(
1/2*b*x+1/2*a)^2+1)*((-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
-sin(1/2*b*x+1/2*a)^6*(-20*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))-10*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)
+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))
*d-10*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2
)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)-5*(6*d^(3/2)*ln(2/cos(1/2*b*x+
1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))-4*(-d)^(1/2)*d^(
1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+3*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/
2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
+d))*d+3*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(
1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^4+5*(2*d
^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1
/2)-d))-4*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+(-d)^(1/2)
*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2
*b*x+1/2*a)^2+d)^(1/2)+d))*d+(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*co
s(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2
*b*x+1/2*a)^2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(91) = 182$.

Time = 0.34 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.53

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \left[\frac{10\left(\cos\left(bx+a\right)^3-\cos\left(bx+a\right)\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-5\,(\right.$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*
x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 5*(cos(b*x + a)^3 -
```

```
cos(b*x + a))*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x
 + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 4))/(b*d^2*cos(b*x
 + a)^3 - b*d^2*cos(b*x + a)), 1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqr
t(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
a))) + 5*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(d)*log((d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a
)^2 - 4))/(b*d^2*cos(b*x + a)^3 - b*d^2*cos(b*x + a))]
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc^3\left(a+bx\right)}{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*cos(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{\sqrt{d}} + \frac{4\left(5\,d^2\cos(bx+a)^2-4\,d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}-\sqrt{d\cos(bx+a)}d^2}}{8\,bd}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + 5*log((sqrt(d*cos(b*
x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4*(5*d^2*cos
(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(5/2) - sqrt(d*cos(b*x + a))*d^2))/(
b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)), x)
```

3.250 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -7/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-7/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(5/2)+7/6/b/d/(d*cos(b*x+a))^(3/2)-1/2*csc(b*x+a)^2/
b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 296, 331, 335, 218, 212, 209}

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-7*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) - (7*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) + 7/(6*b*d*(d*Cos[a + b*x])^(3/2)) -
 Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd^2} - \frac{7\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd^2}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt[4]{-\cot^2(a+bx)}(4-3\cot^2(a+bx)) + 7\cot^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4}\right.}{6bd(d\cos(a+bx))^{3/2}\sqrt[4]{-\cot^2(a+bx)}}$$

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
[Out] ((-Cot[a + b*x]^2)^(1/4)*(4 - 3*Cot[a + b*x]^2) + 7*Cot[a + b*x]^2*Hypergeo
metric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])/(6*b*d*(d*Cos[a + b*x])^(3/2)*(-C
ot[a + b*x]^2)^(1/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 885, normalized size of antiderivative = 7.70

method	result	size
default	Expression too large to display	885

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d^(7/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(4*sin(1/2*b*x+1/2*a)^6-8*sin
(1/2*b*x+1/2*a)^4+5*sin(1/2*b*x+1/2*a)^2-1)*(-3*(-d)^(1/2)*d^(1/2)*(-2*d*si
n(1/2*b*x+1/2*a)^2+d)^(1/2)+84*(-2*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1
/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1
/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2
)+d))*d+(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1
/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^8-168*(-2
*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^
(1/2)-d))+(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d
^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d+(-d)^(1/2)*ln(2/(cos(1/2*b
*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^
(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^6+7*(6*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d
)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+4*(-d)^(1/2)*d^(1/2)*(-2*d*
sin(1/2*b*x+1/2*a)^2+d)^(1/2)-3*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*
d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d-3*(-
d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*
sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x+1/2*a)^2-7*(30*d^(3/2)*ln(
2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+4*
(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-15*(-d)^(1/2)*ln(-2/
(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/
2*a)^2+d)^(1/2)+d))*d-15*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/
2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(1/2*b*x
+1/2*a)^4)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.63

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \left[\frac{42\left(\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-21}{}\right.$$

$$-\frac{42\left(\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-21\left(\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2\right)}{48\left(bd^3\cos\left(bx+a\right)^4-\right.}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 21*(cos(b*x + a)^
4 - cos(b*x + a)^2)*sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 4))/(b*d^3*co
s(b*x + a)^4 - b*d^3*cos(b*x + a)^2), -1/48*(42*(cos(b*x + a)^4 - cos(b*x +
 a)^2)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*
cos(b*x + a))) - 21*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(d)*log((d*cos(b*
x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
+ a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(
7*cos(b*x + a)^2 - 4))/(b*d^3*cos(b*x + a)^4 - b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\frac{4\left(7\,d^2\cos(bx+a)^2-4\,d^2\right)}{(d\cos(bx+a))^{\frac{7}{2}}-(d\cos(bx+a))^{\frac{3}{2}}d^2} - \frac{42\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{21\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{24\,bd}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*(7*d^2*cos(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(7/2) - (d*cos(b*x
 + a))^(3/2)*d^2) - 42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) + 21*lo
g((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(3/2
))/(b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)), x)
```

3.251 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{9\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 9/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-9/4*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b/d^(7/2)+9/10/b/d/(d*cos(b*x+a))^(5/2)-1/2*csc(b*x+a)^2/
b/d/(d*cos(b*x+a))^(5/2)+9/2/b/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2645, 296, 331, 335, 304, 209, 212}

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{9\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (9*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) - (9*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) + 9/(10*b*d*(d*Cos[a + b*x])^(5/2)) +
```

```
9/(2*b*d^3*Sqrt[d*Cos[a + b*x]]) - Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^
(5/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd^5}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^5}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd^3} + \frac{9\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd^3}$$

$$= \frac{9\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{45\cot^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right)+\left(-\cot^2(a+bx)\right)^{3/4}}{10bd^3\sqrt{d\cos(a+bx)}\left(-\cot^2(a+bx)\right)^{3/4}}$$

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (45*Cot[a + b*x]^2*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2] + (-Cot
[a + b*x]^2)^(3/4)*(40 - 5*Cot[a + b*x]^2 + 4*Sec[a + b*x]^2))/(10*b*d^3*Sq
rt[d*Cos[a + b*x]]*(-Cot[a + b*x]^2)^(3/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. 2(109) = 218.

Time = 0.18 (sec) , antiderivative size = 1140, normalized size of antiderivative = 8.32

method	result	size
default	Expression too large to display	1140

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40/d^(9/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(8*sin(1/2*b*x+1/2*a)^8-20*si
n(1/2*b*x+1/2*a)^6+18*sin(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)*(5*(-d
)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+360*(2*d^(3/2)*ln(2/cos
(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+(-d)^(1
/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(
1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d+(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d
*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d)*sin(
1/2*b*x+1/2*a)^10+180*(-10*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+4*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+
1/2*a)^2+d)^(1/2)-5*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x
+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d-5*(-d)^(1/2)*ln(-
2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+
1/2*a)^2+d)^(1/2)+d))*d)*sin(1/2*b*x+1/2*a)^8-90*(-18*d^(3/2)*ln(2/cos(1/2*
b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+16*(-d)^(1/2
)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-9*(-d)^(1/2)*ln(2/(cos(1/2*b*
x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(
1/2)-d))*d-9*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a
)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d)*sin(1/2*b*x+1/2*a)^6+9
*(-70*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^
2+d)^(1/2)-d))+104*(-d)^(1/2)*d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-3
```

```
5*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-
2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d-35*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/
2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)
+d))*d)*sin(1/2*b*x+1/2*a)^4-9*(-10*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(
1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))+24*(-d)^(1/2)*d^(1/2)*(-2*d*si
n(1/2*b*x+1/2*a)^2+d)^(1/2)-5*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*c
os(1/2*b*x+1/2*a)+d^(1/2)*(-2*d*sin(1/2*b*x+1/2*a)^2+d)^(1/2)-d))*d-5*(-d)^
(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*d*si
n(1/2*b*x+1/2*a)^2+d)^(1/2)+d))*d)*sin(1/2*b*x+1/2*a)^2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.20

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \left[\frac{90\left(\cos\left(bx+a\right)^5-\cos\left(bx+a\right)^3\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-45}{}\right.$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/80*(90*(cos(b*x + a)^5 - cos(b*x + a)^3)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 45*(cos(b*x + a)^
5 - cos(b*x + a)^3)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*(45*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos
(b*x + a)))/(b*d^4*cos(b*x + a)^5 - b*d^4*cos(b*x + a)^3), 1/80*(90*(cos(b*
x + a)^5 - cos(b*x + a)^3)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x
 + a) - 1)/(sqrt(d)*cos(b*x + a))) + 45*(cos(b*x + a)^5 - cos(b*x + a)^3)*s
qrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a)
 + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4
5*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(
b*x + a)^5 - b*d^4*cos(b*x + a)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\frac{4\left(45\,d^4\cos(bx+a)^4-36\,d^4\cos(bx+a)^2-4\,d^4\right)}{(d\cos(bx+a))^{\frac{9}{2}}d^2-(d\cos(bx+a))^{\frac{5}{2}}d^4}+\frac{90\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{45\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}}}{40\,bd}$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] 1/40*(4*(45*d^4*cos(b*x + a)^4 - 36*d^4*cos(b*x + a)^2 - 4*d^4)/((d*cos(b*x
 + a))^(9/2)*d^2 - (d*cos(b*x + a))^(5/2)*d^4) + 90*arctan(sqrt(d*cos(b*x +
 a))/sqrt(d))/d^(5/2) + 45*log((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos
(b*x + a)) + sqrt(d)))/d^(5/2))/(b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\,\cos\left(a+b\,x\right)\right)^{7/2}}\,dx$$

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)),x)
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)), x)
```

3.252 $\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

```
[Out] -5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 30}

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

```
[In] Int[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
```

```
[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\text{Subst}\left(\int \sqrt[5]{x}\,dx, x, d\cos(a+bx)\right)}{bd} \\ &= -\frac{5(d\cos(a+bx))^{6/5}}{6bd} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

```
[In] Integrate[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{5(d\cos(bx+a))^{\frac{6}{5}}}{6bd}$	19
default	$-\frac{5(d\cos(bx+a))^{\frac{6}{5}}}{6bd}$	19

```
[In] int((d*cos(b*x+a))^(1/5)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{1}{5}}\cos{(bx+a)}}{6\,b}$$

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="fricas")
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = \begin{cases} -\frac{5\sqrt[5]{d\cos{(a+bx)}}\cos{(a+bx)}}{6b} & \text{for } b\neq 0 \\ x\sqrt[5]{d\cos{(a)}}\sin{(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*cos(b*x+a))**(1/5)*sin(b*x+a),x)
[Out] Piecewise((-5*(d*cos(a + b*x))**(1/5)*cos(a + b*x)/(6*b), Ne(b, 0)), (x*(d*
cos(a))**(1/5)*sin(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{6}{5}}}{6\,bd}$$

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="maxima")
[Out] -5/6*(d*cos(b*x + a))^(6/5)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{1}{5}}\cos{(bx+a)}}{6\,b}$$

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="giac")
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx=-\frac{5\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{6/5}}{6\,b\,d}$$

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(1/5),x)
[Out] -(5*(d*cos(a + b*x))^(6/5))/(6*b*d)
```

3.253 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2644, 14}

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx, x, \sin(x)\right)\\ &= \frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx=\frac{1}{21}(11+3\cos(2x))\sin^{\frac{3}{2}}(x)$$

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}$	14
default	$\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}$	14

```
[In] int(cos(x)^3*sin(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{21}\left(3\,\cos\left(x\right)^2+4\right)\sin\left(x\right)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(19) = 38.

Time = 3.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 8.10

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^5\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{8\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^3\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}$$

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
[Out] 28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**5/(21*tan(x/2)**6 + 6
3*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2
 + 1))*tan(x/2)**3/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21)
+ 28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)/(21*tan(x/2)**6 + 63
*tan(x/2)**4 + 63*tan(x/2)**2 + 21)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{\cos(x)^4\sin(x)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{3/4}}$$

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
[Out] -(cos(x)^4*sin(x)^(3/2)*hypergeom([1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(3/
4))
```

3.254 $\int \cos^3(x) \sin^{\frac{3}{2}}(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x) \sin^{\frac{3}{2}}(x) \, dx = \frac{2}{5} \sin^{\frac{5}{2}}(x) - \frac{2}{9} \sin^{\frac{9}{2}}(x)$$

```
[Out] 2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2644, 14}

$$\int \cos^3(x) \sin^{\frac{3}{2}}(x) \, dx = \frac{2}{5} \sin^{\frac{5}{2}}(x) - \frac{2}{9} \sin^{\frac{9}{2}}(x)$$

```
[In] Int[Cos[x]^3*Sin[x]^(3/2),x]
```

```
[Out] (2*Sin[x]^(5/2))/5 - (2*Sin[x]^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^{3/2}\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^{3/2}-x^{7/2}\right)\,dx, x, \sin(x)\right)\\ &= \frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \frac{1}{45}(13+5\cos(2x))\sin^{\frac{5}{2}}(x)$$

```
[In] Integrate[Cos[x]^3*Sin[x]^(3/2),x]
[Out] ((13 + 5*Cos[2*x])*Sin[x]^(5/2))/45
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}-\frac{2\left(\sin^{\frac{9}{2}}(x)\right)}{9}$	14
default	$\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}-\frac{2\left(\sin^{\frac{9}{2}}(x)\right)}{9}$	14

```
[In] int(cos(x)^3*sin(x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{45}\left(5\,\cos\left(x\right)^4 - \cos\left(x\right)^2 - 4\right)\sqrt{\sin\left(x\right)}$$

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="fricas")
[Out] -2/45*(5*cos(x)^4 - cos(x)^2 - 4)*sqrt(sin(x))
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \frac{8\sin^{\frac{9}{2}}\left(x\right)}{45} + \frac{2\sin^{\frac{5}{2}}\left(x\right)\cos^2\left(x\right)}{5}$$

```
[In] integrate(cos(x)**3*sin(x)**(3/2),x)
[Out] 8*sin(x)**(9/2)/45 + 2*sin(x)**(5/2)*cos(x)**2/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{9}\,\sin\left(x\right)^{\frac{9}{2}} + \frac{2}{5}\,\sin\left(x\right)^{\frac{5}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="maxima")
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{9}\,\sin\left(x\right)^{\frac{9}{2}} + \frac{2}{5}\,\sin\left(x\right)^{\frac{5}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="giac")
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{\cos(x)^4\sin(x)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{5/4}}$$

```
[In] int(cos(x)^3*sin(x)^(3/2),x)
[Out] -(cos(x)^4*sin(x)^(5/2)*hypergeom([-1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(5
/4))
```

3.255 $\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \frac{2}{7}\sin^{\frac{7}{2}}(x) - \frac{2}{11}\sin^{\frac{11}{2}}(x)$$

```
[Out] 2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2644, 14}

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \frac{2}{7}\sin^{\frac{7}{2}}(x) - \frac{2}{11}\sin^{\frac{11}{2}}(x)$$

```
[In] Int[Cos[x]^3*Sin[x]^(5/2),x]
```

```
[Out] (2*Sin[x]^(7/2))/7 - (2*Sin[x]^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^{5/2}\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^{5/2}-x^{9/2}\right)\,dx, x, \sin(x)\right)\\ &= \frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \frac{1}{77}(15+7\cos(2x))\sin^{\frac{7}{2}}(x)$$

```
[In] Integrate[Cos[x]^3*Sin[x]^(5/2),x]
[Out] ((15 + 7*Cos[2*x])*Sin[x]^(7/2))/77
```

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}-\frac{2\left(\sin^{\frac{11}{2}}(x)\right)}{11}$	14
default	$\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}-\frac{2\left(\sin^{\frac{11}{2}}(x)\right)}{11}$	14

```
[In] int(cos(x)^3*sin(x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{77}\left(7\,\cos\left(x\right)^4 - 3\,\cos\left(x\right)^2 - 4\right)\sin\left(x\right)^{\frac{3}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="fricas")
[Out] -2/77*(7*cos(x)^4 - 3*cos(x)^2 - 4)*sin(x)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 32.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \frac{8\sin^{\frac{11}{2}}(x)}{77} + \frac{2\sin^{\frac{7}{2}}(x)\cos^2(x)}{7}$$

```
[In] integrate(cos(x)**3*sin(x)**(5/2),x)
[Out] 8*sin(x)**(11/2)/77 + 2*sin(x)**(7/2)*cos(x)**2/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{11}\,\sin\left(x\right)^{\frac{11}{2}} + \frac{2}{7}\,\sin\left(x\right)^{\frac{7}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="maxima")
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{11}\,\sin\left(x\right)^{\frac{11}{2}} + \frac{2}{7}\,\sin\left(x\right)^{\frac{7}{2}}$$

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="giac")
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{\cos(x)^4\sin(x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{7/4}}$$

```
[In] int(cos(x)^3*sin(x)^(5/2),x)
[Out] -(cos(x)^4*sin(x)^(7/2)*hypergeom([-3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(7
/4))
```

3.256 $\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx = 2\sqrt{\sin(x)} - \frac{2}{5}\sin^{\frac{5}{2}}(x)$$

```
[Out] -2/5*sin(x)^(5/2)+2*sin(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2644, 14}

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx = 2\sqrt{\sin(x)} - \frac{2}{5}\sin^{\frac{5}{2}}(x)$$

```
[In] Int[Cos[x]^3/Sqrt[Sin[x]],x]
```

```
[Out] 2*Sqrt[Sin[x]] - (2*Sin[x]^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1-x^2}{\sqrt{x}}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{\sqrt{x}} - x^{3/2}\right)\,dx, x, \sin(x)\right)\\
&= 2\sqrt{\sin(x)} - \frac{2}{5}\sin^{\frac{5}{2}}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \frac{1}{5}(9+\cos(2x))\sqrt{\sin(x)}$$

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]],x]
[Out] ((9 + Cos[2*x])*Sqrt[Sin[x]])/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}+2\left(\sqrt{\sin}(x)\right)$	14
default	$-\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}+2\left(\sqrt{\sin}(x)\right)$	14

```
[In] int(cos(x)^3/sin(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5*sin(x)^(5/2)+2*sin(x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \frac{2}{5}\left(\cos(x)^2+4\right)\sqrt{\sin(x)}$$

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(cos(x)^2 + 4)*sqrt(sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(17) = 34.

Time = 4.34 (sec) , antiderivative size = 323, normalized size of antiderivative = 17.00

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \frac{10\sqrt{2}\tan^5\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right)+5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}} + \frac{12\sqrt{2}\tan^3\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right)+5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}} + \frac{10\sqrt{2}\tan\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right)+15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right)+5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$

```
[In] integrate(cos(x)**3/sin(x)**(1/2),x)
[Out] 10*sqrt(2)*tan(x/2)**5/(5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 15
*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2)*
*2 + 1))*tan(x/2)**2 + 5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))) + 12*sqrt(2)*tan
(x/2)**3/(5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 15*sqrt(tan(x/2)
/(tan(x/2)**2 + 1))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x
/2)**2 + 5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))) + 10*sqrt(2)*tan(x/2)/(5*sqrt(
tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 15*sqrt(tan(x/2)/(tan(x/2)**2 + 1
))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**2 + 5*sqrt(t
an(x/2)/(tan(x/2)**2 + 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{2}{5}\sin(x)^{\frac{5}{2}} + 2\sqrt{\sin(x)}$$

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="maxima")
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{2}{5}\sin(x)^{\frac{5}{2}} + 2\sqrt{\sin(x)}$$

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="giac")
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{\cos(x)^4\sqrt{\sin(x)}\,{}_2\mathrm{F}_1\left(\frac{3}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{1/4}}$$

```
[In] int(cos(x)^3/sin(x)^(1/2),x)
[Out] -(cos(x)^4*sin(x)^(1/2)*hypergeom([3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(1/
4))
```

3.257 $\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc} + \frac{7d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 7/30*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c+1/5*d*(d*cos(b*x+a))
^(7/2)*(c*sin(b*x+a))^(3/2)/b/c-7/20*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*si
n(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2649, 2652, 2719}

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{7d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}} + \frac{7d^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{30bc} + \frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (7*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(30*b*c) + (d*(d*Cos[
a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^4*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=\frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}+\frac{1}{10}\left(7d^2\right)\int(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx\\
&=\frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc}+\frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}\\
&\quad+\frac{1}{20}\left(7d^4\right)\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx\\
&=\frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc}+\frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}\\
&\quad+\frac{\left(7d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{20\sqrt{\sin(2a+2bx)}}\\
&=\frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc}+\frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc}\\
&\quad+\frac{7d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{2d^4\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{}}{3b}$$

[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]

[Out] (2*d^4*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(137) = 274.

Time = 0.91 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.20

method	result
default	$-\frac{\sqrt{2}\,\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,\left(12\sqrt{2}\,(\cos^6(bx+a))+2\sqrt{2}\,(\cos^4(bx+a))-21\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+}\right.}{}$

[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/120/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*(12*2^(1/2)*cos(b*x+a)^6+2*2^(1/2)*cos(b*x+a)^4-21*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+42*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)-21*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+42*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+7*2^(1/2)*cos(b*x+a)^2-21*2^(1/2)*cos(b*x+a))*d^4*sec(b*x+a)*csc(b*x+a)

Fricas [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^4*cos(b*x + a)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\,\cos(a+b\,x))^{9/2}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2), x)
```

3.258 $\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc} + \frac{d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/3*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c-1/2*d^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(
b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2649, 2652, 2719}

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}} + \frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc} + \frac{1}{2}d^2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx \\ &= \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc} \\ &\quad + \frac{\left(d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}} \\ &= \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc} + \frac{d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \frac{2d^2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{}}{3b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(106) = 212$.

Time = 0.34 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.31

method	result
default	$-\frac{\sqrt{2}\,\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,\left(6\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{-\cot \cdots}\right.\right.}{}$

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*(6*(-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^
(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*
(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a
)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))
*cos(b*x+a)+2*2^(1/2)*cos(b*x+a)^4+6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(
b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*
x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(c
ot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot
(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)^2-3*2^(1/2)*cos
(b*x+a))*d^2*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^2*cos(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\,\cos(a+b\,x))^{5/2}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2), x)
```

3.259 $\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx=\frac{\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2652, 2719}

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx=\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b
*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}}\\ &= \frac{\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$$
$$=\frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\tan(a+bx)}{3b}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 7.42

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\cos\right.}{}$

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/b*2^(1/2)*(2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1
)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^
(1/2),1/2*2^(1/2))*cos(b*x+a)-(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-
csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2
)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((
```

```
-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-cot(b*x+a)+csc(b*x+a)+1)^(1/
2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF(
(-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)^2-2^(1/2)*
cos(b*x+a))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin\left(a+bx\right)}\sqrt{d\cos\left(a+bx\right)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\,\cos\left(a+b\,x\right)}\,\sqrt{c\,\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2), x)
```

3.260 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(1/2)+2*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^
(1/2)*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2651, 2652, 2719}

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a +
 b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{\left(2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}\\ &= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(}}{3bd^2}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 5/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(108) = 216.

Time = 0.50 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.84

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{c(\csc(bx+a)-\cot(bx+a))}{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)+1}}\left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\sqrt{2+2\cot(bx+a)-2\csc(bx+a)}\sqrt{\cot(bx+a)-\csc(bx+a)}\,E(\right.}{}$

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*2^(1/2)*(c/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(csc(b*x+a)-cot(b*x+a)))^
(1/2)/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)
*(2+2*cot(b*x+a)-2*csc(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elliptic
E((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-cot(b*x+a)+csc(b*x+a)+1)^
(1/2)*(2+2*cot(b*x+a)-2*csc(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ell
ipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*(1-cos(b*x+a))^2*csc
(b*x+a)^2)*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/(1-cos(b*x+a))*sin(b*x+a)/(-d*
((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/((1-cos(b*x+a))^2*csc(b*x+a)^2+1))^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-i\sqrt{i\,cd}\cos(bx+a)\,E(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+i\sqrt{-i\,cd}\cos}{}$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(I*c*d)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a
)), -1) + I*sqrt(-I*c*d)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) - I*si
n(b*x + a)), -1) + I*sqrt(I*c*d)*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a
) + I*sin(b*x + a)), -1) - I*sqrt(-I*c*d)*cos(b*x + a)*elliptic_f(arcsin(co
s(b*x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
 a))*sin(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin{(a+bx)}}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(3/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin{(bx+a)}}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin{(bx+a)}}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\,\sin{(a+b\,x)}}}{{(d\,\cos{(a+b\,x)})}^{3/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2), x)
```

3.261 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2/5*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(5/2)+4/5*(c*sin(b*x+a))^(3/2
)/b/c/d^3/(d*cos(b*x+a))^(1/2)+4/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi
+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+
a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2651, 2652, 2719}

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} + \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (4*(c*Sin[a +
 b*x])^(3/2))/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]]) - (4*Sqrt[d*Cos[a + b*x]]*El
```

```
lipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*
b*x]])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{2\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}\\
&=\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}-\frac{4\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{5d^4}\\
&=\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}\\
&\quad-\frac{\left(4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{5d^4\sqrt{\sin(2a+2bx)}}\\
&=\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}\\
&\quad-\frac{4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin}}{3bd^4}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 9/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(139) = 278.

Time = 0.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.07

method	result
default	$\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\left(4\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{-\cot(bx+a)+\csc(bx+}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*2^(1/2)*(c*sin(b*x+a))^(1/2)/d^3/(d*cos(b*x+a))^(1/2)*(4*(-cot(b*x+a)
+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cot(b*x+a)-
2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x
+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2
))*cot(b*x+a)+4*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^
(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1
/2),1/2*2^(1/2))*csc(b*x+a)-2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-
csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2),1/2*2^(1/2))*csc(b*x+a)-2*2^(1/2)*cot(b*x+a)+2^(1/2)*csc(
b*x+a)+2^(1/2)*sec(b*x+a)^2*csc(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(i\sqrt{icd}\cos\left(bx+a\right)^{3}E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)-i\sqrt{-icd}\cos\left(bx+a\right)^{3}E(\arcsin\left(\cos\right.\right.}{}$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(I*sqrt(I*c*d)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1) - I*sqrt(-I*c*d)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a
) - I*sin(b*x + a)), -1) - I*sqrt(I*c*d)*cos(b*x + a)^3*elliptic_f(arcsin(c
os(b*x + a) + I*sin(b*x + a)), -1) + I*sqrt(-I*c*d)*cos(b*x + a)^3*elliptic
_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*(2*cos
(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))*sin(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2),x)
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2), x)
```

3.262 $\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 320

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx &= -\frac{\sqrt{c}d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}\\ &+\frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b}\\ &-\frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b}\\ &+\frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc}\end{aligned}$$

```
[Out] -1/8*d^(3/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b
*x+a))^(1/2))*c^(1/2)/b*2^(1/2)+1/8*d^(3/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin
(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*c^(1/2)/b*2^(1/2)+1/16*d^(3/2)
*ln(c^(1/2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/
2)*tan(b*x+a))*c^(1/2)/b*2^(1/2)-1/16*d^(3/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c
*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))*c^(1/2)/b*2^(1/
2)+1/2*d*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2649, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = -\frac{\sqrt{c}d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{8\sqrt{2}b}-\frac{\sqrt{c}d^{3/2}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{8\sqrt{2}b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -1/4*(Sqrt[c]*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sq
rt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b) + (Sqrt[c]*d^(3/2)*ArcTan[1 + (Sq
rt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqr
t[2]*b) + (Sqrt[c]*d^(3/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*
x]])/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) - (Sqrt[c]
*d^(3/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a
+ b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) + (d*Sqrt[d*Cos[a + b*x]]*(c
*Sin[a + b*x])^(3/2))/(2*b*c)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{1}{4}d^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{(cd^3)\,\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2b}$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} - \frac{(cd^2)\,\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b} + \frac{(cd^2)\,\text{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b}$$

$$= \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{(cd)\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8b} + \frac{(cd)\text{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8b} + \frac{\left(\sqrt{c}d^{3/2}\right)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}+2x}{-\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8\sqrt{2}b} + \frac{\left(\sqrt{c}d^{3/2}\right)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}-2x}{-\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8\sqrt{2}b}$$

$$= \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b} - \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b} + \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc} + \frac{\left(\sqrt{c}d^{3/2}\right)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b} - \frac{\left(\sqrt{c}d^{3/2}\right)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}$$

$$= -\frac{\sqrt{c}d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b} + \frac{\sqrt{c}d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}$$
$$+ \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b}$$
$$- \frac{\sqrt{c}d^{3/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b}$$
$$+ \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.22

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \frac{2d^2\cos^2(a+bx)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\,\text{t}}{3b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, Sin[a + b*x
]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{2}\left(4\sin(bx+a)\sqrt{2}\,\cos(bx+a)\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}+4\sin(bx+a)\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}+\ln\left(-2\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\,\cot\right.\right.}{}$

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*2^(1/2)*(4*sin(b*x+a)*2^(1/2)*cos(b*x+a)*(-sin(b*x+a)*cos(b*x+a)/(1+
cos(b*x+a))^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*
x+a))^2)^(1/2)+ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2
)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(
b*x+a)+2-2*cot(b*x+a))-ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^
```

```
2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/
2)*csc(b*x+a)+2-2*cot(b*x+a))+2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos
(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))+2*arctan((sin
(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-
1)/(cos(b*x+a)-1)))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*d/(1+cos(b*x+
a))/(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 1033, normalized size of antiderivative = 3.23

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*sin(b*x + a) - (-c^2*d
^6/b^4)^(1/4)*b*log(1/2*c^2*d^5*cos(b*x + a)*sin(b*x + a) + 1/2*((-c^2*d^6/
b^4)^(1/4)*b*c*d^3*sin(b*x + a) - (-c^2*d^6/b^4)^(3/4)*b^3*cos(b*x + a))*sq
rt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 1/4*(2*b^2*c*d^2*cos(b*x + a)^2 -
 b^2*c*d^2)*sqrt(-c^2*d^6/b^4)) + (-c^2*d^6/b^4)^(1/4)*b*log(1/2*c^2*d^5*co
s(b*x + a)*sin(b*x + a) - 1/2*((-c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b*x + a) -
(-c^2*d^6/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x
+ a)) - 1/4*(2*b^2*c*d^2*cos(b*x + a)^2 - b^2*c*d^2)*sqrt(-c^2*d^6/b^4)) -
I*(-c^2*d^6/b^4)^(1/4)*b*log(1/2*c^2*d^5*cos(b*x + a)*sin(b*x + a) + 1/2*(I
*(-c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b*x + a) + I*(-c^2*d^6/b^4)^(3/4)*b^3*cos
(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + 1/4*(2*b^2*c*d^2*cos
(b*x + a)^2 - b^2*c*d^2)*sqrt(-c^2*d^6/b^4)) + I*(-c^2*d^6/b^4)^(1/4)*b*log
(1/2*c^2*d^5*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*(-c^2*d^6/b^4)^(1/4)*b*c*d
^3*sin(b*x + a) - I*(-c^2*d^6/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x +
 a))*sqrt(c*sin(b*x + a)) + 1/4*(2*b^2*c*d^2*cos(b*x + a)^2 - b^2*c*d^2)*sq
rt(-c^2*d^6/b^4)) - (-c^2*d^6/b^4)^(1/4)*b*log(c^2*d^5 + 2*((-c^2*d^6/b^4)^
(1/4)*b*c*d^3*cos(b*x + a) - (-c^2*d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*
cos(b*x + a))*sqrt(c*sin(b*x + a))) + (-c^2*d^6/b^4)^(1/4)*b*log(c^2*d^5 -
2*((-c^2*d^6/b^4)^(1/4)*b*c*d^3*cos(b*x + a) - (-c^2*d^6/b^4)^(3/4)*b^3*sin
(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + I*(-c^2*d^6/b^4)^(1
/4)*b*log(c^2*d^5 - 2*(I*(-c^2*d^6/b^4)^(1/4)*b*c*d^3*cos(b*x + a) + I*(-c^
2*d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)
)) - I*(-c^2*d^6/b^4)^(1/4)*b*log(c^2*d^5 - 2*(-I*(-c^2*d^6/b^4)^(1/4)*b*c*
d^3*cos(b*x + a) - I*(-c^2*d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a))))/b
```

Sympy [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\sin{(a+bx)}}(d\cos{(a+bx)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx=\int (d\cos{(bx+a)})^{\frac{3}{2}}\sqrt{c\sin{(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx=\int (d\cos{(bx+a)})^{\frac{3}{2}}\sqrt{c\sin{(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx=\int (d\,\cos{(a+b\,x)})^{3/2}\,\sqrt{c\,\sin{(a+b\,x)}}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2), x)
```

3.263 $\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 280

$$\begin{aligned}\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = &-\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}\\
&+\frac{\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}\\
&+\frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}}\\
&-\frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}}\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(
1/2))*c^(1/2)/b*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*d^(1/2)*(c*sin(b*x+a))
^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*c^(1/2)/b*2^(1/2)/d^(1/2)+1/4*ln(c^(1/
2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*
x+a))*c^(1/2)/b*2^(1/2)/d^(1/2)-1/4*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c*sin(b*x+a
))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))*c^(1/2)/b*2^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{d}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[
d*Cos[a + b*x]])])/(Sqrt[2]*b*Sqrt[d])) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*Sqrt[
d]) + (Sqrt[c]*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*
Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d]) - (Sqrt[c]*Log
[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sq
rt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{(2cd)\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$= -\frac{c\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b} + \frac{c\text{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$= \frac{c\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd} + \frac{\sqrt{c}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}+2x}{-\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}-2x}{-\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2\sqrt{2}b\sqrt{d}}$$

$$= \frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}$$

$$= -\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\tan(a+bx)}{3b\sqrt{d\cos(a+bx)}}$$

[In] Integrate[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]

[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{2}\left(\ln\left(-2\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\cot(bx+a)-2\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\csc(bx+a)+2-2\cot(bx+a)\right)+2\arctan\left(\frac{\sin(bx+a)\sqrt{}}{}\right.\right.}{}$

[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/b*2^(1/2)*(ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))+2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))-ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))+2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1)))*(c*sin(b*x+a))^(1/2)*cos(b*x+a)/(1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \text{Too large to display}$$

[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")

[Out] 1/8*(-c^2/(b^4*d^2))^(1/4)*log(1/2*c^2*cos(b*x + a)*sin(b*x + a) + 1/2*(b^3*d*(-c^2/(b^4*d^2))^(3/4)*cos(b*x + a) - b*c*(-c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 1/4*(2*b^2*c*d*cos(b*x + a)^2 - b^2*c*d)*sqrt(-c^2/(b^4*d^2))) - 1/8*(-c^2/(b^4*d^2))^(1/4)*log(1/2*c^2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d*(-c^2/(b^4*d^2))^(3/4)*cos(b*x + a) - b*c*(-c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 1/4*(2*b^2*c*d*cos(b*x + a)^2 - b^2*c*d)*sqrt(-c^2/(b^4*d^2))) - 1/8*I*(-c^2/(b^4*d^2))^(1/4)*log(1/2*c^2*cos(b*x + a)*sin(b*x + a) +

```
 1/2*(I*b^3*d*(-c^2/(b^4*d^2))^(3/4)*cos(b*x + a) + I*b*c*(-c^2/(b^4*d^2))^
(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + 1/4*(2*b^2*
c*d*cos(b*x + a)^2 - b^2*c*d)*sqrt(-c^2/(b^4*d^2))) + 1/8*I*(-c^2/(b^4*d^2)
)^(1/4)*log(1/2*c^2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d*(-c^2/(b^4*d^
2))^(3/4)*cos(b*x + a) - I*b*c*(-c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*
cos(b*x + a))*sqrt(c*sin(b*x + a)) + 1/4*(2*b^2*c*d*cos(b*x + a)^2 - b^2*c*
d)*sqrt(-c^2/(b^4*d^2))) + 1/8*(-c^2/(b^4*d^2))^(1/4)*log(2*(b^3*d*(-c^2/(b
^4*d^2))^(3/4)*sin(b*x + a) - b*c*(-c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt
(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + c^2) - 1/8*(-c^2/(b^4*d^2))^(1/4)*l
og(-2*(b^3*d*(-c^2/(b^4*d^2))^(3/4)*sin(b*x + a) - b*c*(-c^2/(b^4*d^2))^(1/
4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + c^2) + 1/8*I*(
-c^2/(b^4*d^2))^(1/4)*log(-2*(I*b^3*d*(-c^2/(b^4*d^2))^(3/4)*sin(b*x + a) +
 I*b*c*(-c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin
(b*x + a)) + c^2) - 1/8*I*(-c^2/(b^4*d^2))^(1/4)*log(-2*(-I*b^3*d*(-c^2/(b^
4*d^2))^(3/4)*sin(b*x + a) - I*b*c*(-c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + c^2)
```

Sympy [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin{(a+bx)}}}{\sqrt{d\cos{(a+bx)}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin{(bx+a)}}}{\sqrt{d\cos{(bx+a)}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2), x)
```

3.264 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2643}

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,\tan(bx+a)}{3b\,d^{2}\sqrt{d\cos(bx+a)}}$	35

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/b*(c*sin(b*x+a))^(1/2)/d^2/(d*cos(b*x+a))^(1/2)*tan(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{3\,bd^{3}\cos(bx+a)^{2}}$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^3*cos(b*x +
 a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{c\,\sin\left(a+b\,x\right)}}{3\,b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(5/2),x)
[Out] (2*sin(2*a + 2*b*x)*(c*sin(a + b*x))^(1/2))/(3*b*d^2*(cos(2*a + 2*b*x) + 1)
*(d*cos(a + b*x))^(1/2))
```

3.265 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(7/2)+8/21*(c*sin(b*x+a))^(3/
2)/b/c/d^3/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2651, 2643}

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}} + \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a +
 b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2} \\ &= \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(5+2\cos(2(a+bx)))\sec^4(a+bx)(c\sin(a+bx))^{3/2}}{21bcd^5}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(5 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^4*(c*Sin[a +
b*x])^(3/2))/(21*b*c*d^5)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,(4\tan(bx+a)+3\tan(bx+a)(\sec^2(bx+a)))}{21b\,d^4\sqrt{d\cos(bx+a)}}$	54

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b*(c*sin(b*x+a))^(1/2)/d^4/(d*cos(b*x+a))^(1/2)*(4*tan(b*x+a)+3*tan(b*
x+a)*sec(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(4\,\cos\left(bx+a\right)^{2}+3\right)\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{21\,bd^{5}\cos\left(bx+a\right)^{4}}$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] 2/21*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 3)*sqrt(c*sin(b*x + a))*sin(b
*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8\,\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(11\,\sin\left(2\,a+2\,b\,x\right)+7\,\sin\left(4\,a+4\,b\,x\right)+\sin\left(6\,a+6\,b\,x\right)\right)}{21\,b\,d^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] (8*(c*sin(a + b*x))^(1/2)*(11*sin(2*a + 2*b*x) + 7*sin(4*a + 4*b*x) + sin(6
*a + 6*b*x)))/(21*b*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*cos
(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

3.266 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/11*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(11/2)+16/77*(c*sin(b*x+a))^
(3/2)/b/c/d^3/(d*cos(b*x+a))^(7/2)+64/231*(c*sin(b*x+a))^(3/2)/b/c/d^5/(d*c
os(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2651, 2643}

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (16*(c*Sin[
a + b*x])^(3/2))/(77*b*c*d^3*(d*Cos[a + b*x])^(7/2)) + (64*(c*Sin[a + b*x])
^(3/2))/(231*b*c*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{11d^2} \\
&= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{32\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{77d^4} \\
&= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(45+28\cos(2(a+bx))+4\cos(4(a+bx)))\sec^6(a+bx)(c\sin(a+}{231bcd^7}$$

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(45 + 28*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^6*(c*Sin[a + b*x])^(3/2))/(231*b*c*d^7)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(32\left(\cos^{4}\left(bx+a\right)\right)+24\left(\cos^{2}\left(bx+a\right)\right)+21\right)\sqrt{c\sin\left(bx+a\right)}\,\tan\left(bx+a\right)\left(\sec^{4}\left(bx+a\right)\right)}{231b\,d^{6}\sqrt{d\cos\left(bx+a\right)}}$	65

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x,method=_RETURNVERBOSE)
[Out] 2/231/b*(32*cos(b*x+a)^4+24*cos(b*x+a)^2+21)*(c*sin(b*x+a))^(1/2)/d^6/(d*co
s(b*x+a))^(1/2)*tan(b*x+a)*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2\left(32\cos\left(bx+a\right)^{4}+24\cos\left(bx+a\right)^{2}+21\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(\right.}{231\,bd^{7}\cos\left(bx+a\right)^{6}}$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
[Out] 2/231*(32*cos(b*x + a)^4 + 24*cos(b*x + a)^2 + 21)*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(13/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(2\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)\,\left(2\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2+\sin\left(5\,a+5\,b\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{1984\,\sin(a+b\,x)\,\left(-2\sin\left(\frac{5}{\cdots}\right.\right.}{2\cdots}\right.}{32\,\left(\sin\left(a+b\,x\right)^2\cdots\right.}$$

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(13/2),x)
[Out] -((c*sin(a + b*x))^(1/2)*(2*sin(a/2 + (b*x)/2)^2 - 1)*(sin(5*a + 5*b*x)*1i
+ 2*sin((5*a)/2 + (5*b*x)/2)^2 - 1)*((1984*sin(a + b*x)*(sin(5*a + 5*b*x)*1
i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(231*b*d^6) + (256*sin(3*a + 3*b*x)*
(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(77*b*d^6) + (128
*sin(5*a + 5*b*x)*(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))
/(231*b*d^6)))/(32*(sin(a + b*x)^2 - 1)^3*(-d*(2*sin(a/2 + (b*x)/2)^2 - 1))
^(1/2))
```

3.267 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b} - \frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{c^2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] -1/3*c*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2)/b/d+1/6*c*d*(d*cos(b*x+a))
^(1/2)*(c*sin(b*x+a))^(1/2)/b-1/12*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(
d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2648, 2649, 2653, 2720}

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \frac{c^2d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} - \frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd} + \frac{cd\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6b}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c*d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b) - (c*(d*Cos[a + b*x])
^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*d) + (c^2*d^2*EllipticF[a - Pi/4 + b*x, 2
]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}+\frac{1}{6}c^2\int\frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx\\
&= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\\
&\quad+\frac{1}{12}\left(c^2d^2\right)\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx\\
&= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\\
&\quad+\frac{\left(c^2d^2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{12\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\
&= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\\
&\quad+\frac{c^2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \frac{2cd\sqrt{d\cos(a+bx)}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
[Out] (2*c*d*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4,
5/4, 9/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 1744, normalized size of antiderivative = 13.31

method	result	size
default	Expression too large to display	1744

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/48/b*2^(1/2)*(6*I*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a
)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)
+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-6*I*(-cot(b*x+a)+csc(b*x+a)+1)^
(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellipti
cPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*2^(1/2)*cos(b
*x+a)^3*sin(b*x+a)-6*I*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x
+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+
a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*(-cot(b*x+a)+csc(b*x+a)+1
)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellip
ticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*(-cot(b*x+a
)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))
*cos(b*x+a)+8*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1
/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2
),1/2*2^(1/2))*cos(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-cs
c(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc
(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a
)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*(-cot(b*
```

```
x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-6*(-cot
(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc
(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^
(1/2))-4*2^(1/2)*cos(b*x+a)*sin(b*x+a)+6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))*cos(b*x+a)+6*(-sin(b*x+a)*cos(b
*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b
*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))*cos(b*x+a)+3*(-
sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*ln(2*2^(1/2)*(-sin(b*x+a)*cos
(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a
)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))*cos(b*x+a)-3*(-sin(b*x
+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a
)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+c
os(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))*cos(b*x+a)+6*(-sin(b*x+a)*co
s(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*co
s(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))+6*(-sin(b*x+
a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+
a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))+3*(-sin
(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*
x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(
1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))-3*(-sin(b*x+a)*cos(b*x+a)
/(1+cos(b*x+a))^2)^(1/2)*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a
))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^
(1/2)*csc(b*x+a)+2-2*cot(b*x+a)))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)
*c*d*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*d*cos(b*x + a)*sin(b*x
 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\,\cos(a+b\,x))^{3/2}\,(c\,\sin(a+b\,x))^{3/2}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2), x)
```

3.268 $\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd} + \frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] -c*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d-1/2*c^2*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+
2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2648, 2653, 2720}

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} - \frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((c*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*d)) + (c^2*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Si
n[a + b*x]])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{1}{2}c^2\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx\\
&= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\
&= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int\frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx=\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{3/2}\tan(}{5b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.27

method	result
default	$-\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\,c\left(-\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(b}\right.\right.}{}$

[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)*(-(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cot(b*x+a)-(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*csc(b*x+a)+2^(1/2)*cos(b*x+a))

Fricas [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d*cos(b*x + a)), x)

Sympy [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(a+bx))^{\frac{3}{2}}}{\sqrt{d\cos(a+bx)}}\,dx$$

[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(1/2),x)

[Out] Integral((c*sin(a + b*x))**(3/2)/sqrt(d*cos(a + b*x)), x)

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{3/2}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2), x)
```

3.269 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] 2/3*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(3/2)+1/3*c^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b
*x+2*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2646, 2653, 2720}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (c^2*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sq
rt[c*Sin[a + b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
```

```
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int\frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\,c\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(bx-}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^2*((-cot(b*x+a
)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cot(b*x+a)
+(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+
a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2)
)*csc(b*x+a)-2^(1/2)*sec(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{i\,cd}c\cos{(bx+a)}^2\,F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+\sqrt{-i\,cd}c\cos{(b}}{3\,bd^3}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(I*c*d)*c*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*c*d)*c*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a)
 - I*sin(b*x + a)), -1) + 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c)/(b
*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin\left(bx+a\right))^{\frac{3}{2}}}{(d\cos\left(bx+a\right))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin\left(bx+a\right))^{\frac{3}{2}}}{(d\cos\left(bx+a\right))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{3/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2), x)
```

3.270 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{21bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] 2/7*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(7/2)-2/21*c*(c*sin(b*x+a))^(
1/2)/b/d^3/(d*cos(b*x+a))^(3/2)+2/21*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^4
/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2646, 2651, 2653, 2720}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{21bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(21*b*d^3*(d*Cos[a + b*x])^(3/2)) - (2*c^2*EllipticF[a - Pi/4 +
```

```
 b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(21*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[
a + b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{c^2\int\frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx}{7d^2}\\
&= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{(2c^2)\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{21d^4}\\
&= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2c^2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{21d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\
&= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}\\
&\quad - \frac{2c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{21bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\cos^2(a+bx)^{7/4}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+}{5bc^2(d\cos(a+bx))^{9/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(7/4)*Cot[a + b*x]*Hypergeometric2F1[5/4, 11/4, 9/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.71

method	result
default	$-\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\,c\left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(bx}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^4*(2*(-cot(b*
x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cot(b*x
+a)+2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot
(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^
(1/2))*csc(b*x+a)+2^(1/2)*sec(b*x+a)-3*2^(1/2)*sec(b*x+a)^3)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\left(\sqrt{i\,cd}c\cos(bx+a)^4\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,cd}c\cos\right.}{}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(I*c*d)*c*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1) + sqrt(-I*c*d)*c*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a
) - I*sin(b*x + a)), -1) - (c*cos(b*x + a)^2 - 3*c)*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a)))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{3/2}}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2), x)
```

3.271 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 320

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \frac{c^{3/2}\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} + \frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} - \frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}$$

```
[Out] -1/8*c^(3/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(
b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)-1/8*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*(d*co
s(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)-1/16*c^(3/2
)*ln(d^(1/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin
(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)+1/16*c^(3/2)*ln(d^(1/2)+cot(b*x+a)*d^(1/2
)+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1
/2)-1/2*c*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2648, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \frac{c^{3/2}\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{8\sqrt{2}b} + \frac{c^{3/2}\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{8\sqrt{2}b} - \frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c^(3/2)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]
*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2]*b) - (c^(3/2)*Sqrt[d]*ArcTan[1 + (Sqrt[
2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2
]*b) - (c^(3/2)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[
c]*Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) + (c^(3/2)*Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a +
b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) - (c*(d*Cos[a + b*x])^(3/2)*Sqr
t[c*Sin[a + b*x]])/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} + \frac{1}{4}c^2\int\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} - \frac{(c^3d)\,\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2b}$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} + \frac{(c^2d)\,\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{4b} - \frac{(c^2d)\,\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{4b}$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} - \frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} - \frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2x}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} - \frac{(cd)\text{Subst}\left(\int \frac{1}{\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8b} - \frac{(cd)\text{Subst}\left(\int \frac{1}{\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8b}$$

$$= -\frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} + \frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} - \frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} - \frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} + \frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b}$$

$$= \frac{c^{3/2}\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} + \frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}b} - \frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{3/2}\tan$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/4,
9/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\sqrt{2}\left(\cos^2(bx+a)\right)+4\sqrt{2}\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\cos(bx+a)+2\arctan\left(\frac{\sin(bx+a)\sqrt{2}\sqrt{-\frac{\sin(bx+a)\cos(bx}{(1+\cos(bx+a))}}}{\cos(bx+a)-1}\right.\right.}{}$

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/b*2^(1/2)*(4*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2)*
cos(b*x+a)^2+4*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cos(
b*x+a)+2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^
2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))-ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a
)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+c
os(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))+ln(2*2^(1/2)*(-sin(b*x+a)*co
```

```
s(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+
a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))+2*arctan((sin(b*x+a)*
2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(
b*x+a)-1)))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*c/(1+cos(b*x+a))/(-si
n(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.17

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*cos(b*x + a) - (-c^6*
d^2/b^4)^(1/4)*b*log(-2*c^5*d^2*cos(b*x + a)^2 + 2*sqrt(-c^6*d^2/b^4)*b^2*c
^2*d*cos(b*x + a)*sin(b*x + a) + c^5*d^2 + 2*((-c^6*d^2/b^4)^(1/4)*b*c^3*d*
sin(b*x + a) + (-c^6*d^2/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*
sqrt(c*sin(b*x + a))) + (-c^6*d^2/b^4)^(1/4)*b*log(-2*c^5*d^2*cos(b*x + a)^
2 + 2*sqrt(-c^6*d^2/b^4)*b^2*c^2*d*cos(b*x + a)*sin(b*x + a) + c^5*d^2 - 2*
((-c^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x + a) + (-c^6*d^2/b^4)^(3/4)*b^3*cos(b
*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + I*(-c^6*d^2/b^4)^(1/4
)*b*log(-2*c^5*d^2*cos(b*x + a)^2 - 2*sqrt(-c^6*d^2/b^4)*b^2*c^2*d*cos(b*x
+ a)*sin(b*x + a) + c^5*d^2 - 2*(I*(-c^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x + a
) - I*(-c^6*d^2/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*si
n(b*x + a))) - I*(-c^6*d^2/b^4)^(1/4)*b*log(-2*c^5*d^2*cos(b*x + a)^2 - 2*s
qrt(-c^6*d^2/b^4)*b^2*c^2*d*cos(b*x + a)*sin(b*x + a) + c^5*d^2 - 2*(-I*(-c
^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x + a) + I*(-c^6*d^2/b^4)^(3/4)*b^3*cos(b*x
 + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + (-c^6*d^2/b^4)^(1/4)*b*
log(-c^5*d^2 + 2*((-c^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x + a) - (-c^6*d^2/b^4
)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - (-c^
6*d^2/b^4)^(1/4)*b*log(-c^5*d^2 - 2*((-c^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x +
 a) - (-c^6*d^2/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*si
n(b*x + a))) - I*(-c^6*d^2/b^4)^(1/4)*b*log(-c^5*d^2 - 2*(I*(-c^6*d^2/b^4)^
(1/4)*b*c^3*d*sin(b*x + a) + I*(-c^6*d^2/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(
d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + I*(-c^6*d^2/b^4)^(1/4)*b*log(-c^5*d
^2 - 2*(-I*(-c^6*d^2/b^4)^(1/4)*b*c^3*d*sin(b*x + a) - I*(-c^6*d^2/b^4)^(3/
4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int (c\sin(a+bx))^{\frac{3}{2}}\sqrt{d\cos(a+bx)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**(3/2)*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\,\cos(a+b\,x)}\,(c\,\sin(a+b\,x))^{3/2}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2), x)
```

3.272 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 313

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 1/2*c^(3/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b
*x+a))^(1/2))/b/d^(3/2)*2^(1/2)+1/2*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*(d*cos
(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))/b/d^(3/2)*2^(1/2)+1/4*c^(3/2)*
ln(d^(1/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b
*x+a))^(1/2))/b/d^(3/2)*2^(1/2)-1/4*c^(3/2)*ln(d^(1/2)+cot(b*x+a)*d^(1/2)+2
^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))/b/d^(3/2)*2^(1/2)
+2*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2646, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}bd^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[
c*Sin[a + b*x]])])/(Sqrt[2]*b*d^(3/2))) + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*d^(3/
2)) + (c^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d
*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) - (c^(3/2)*Log
[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sq
rt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) + (2*c*Sqrt[c*Sin[a + b*x]])/(b*
d*Sqrt[d*Cos[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{(2c^3)\,\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd} + \frac{c^2\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{c^{3/2}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2x}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2bd} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2bd}$$

$$= \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{c^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}}$$

$$= -\frac{c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}bd^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}}{5bcd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. 2(237) = 474.

Time = 0.25 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{2}\left(\frac{c(\csc(bx+a)-\cot(bx+a))}{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)+1}\right)^{\frac{3}{2}}\left(\sin^2(bx+a)\right)\left(\ln\left(-\frac{-(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{(1-\cos(bx+a))\left((1-\cos(bx+a))^2\left(\csc^2(bx+\right.\right.}}{1-\cos(bx+a)}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*2^(1/2)*(c/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(csc(b*x+a)-cot(b*x+a))
)^(3/2)/(1-cos(b*x+a))^2*sin(b*x+a)^2*(ln(-1/(1-cos(b*x+a))*(-(1-cos(b*x+a)
)^2*csc(b*x+a)+2*((1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+
a))^(1/2)*sin(b*x+a)-2+2*cos(b*x+a)+sin(b*x+a)))*((1-cos(b*x+a))*((1-cos(b*
x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)+2*arctan(1/(1-cos(b*x+a))*(((1-co
s(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+co
s(b*x+a)-1))*((1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^
(1/2)-ln(1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*((1-cos(b*x+a))*((
1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)
-sin(b*x+a)))*((1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))
^(1/2)+2*arctan(1/(1-cos(b*x+a))*(((1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x
+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+1-cos(b*x+a)))*((1-cos(b*x+a))*((1-co
s(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)+8*csc(b*x+a)-8*cot(b*x+a))*((
1-cos(b*x+a))^2*csc(b*x+a)^2-1)/(-d*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/((1-c
os(b*x+a))^2*csc(b*x+a)^2+1))^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 1092, normalized size of antiderivative = 3.49

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(-I*b*d^2*(-c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(2*b^2*c^2*d^3*sqrt(-
c^6/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*c^5*cos(b*x + a)^2 - c^5 - 2*(
I*b^3*d^4*(-c^6/(b^4*d^6))^(3/4)*cos(b*x + a) - I*b*c^3*d*(-c^6/(b^4*d^6))^
(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + I*b*d^2*(-
c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(2*b^2*c^2*d^3*sqrt(-c^6/(b^4*d^6))*co
s(b*x + a)*sin(b*x + a) + 2*c^5*cos(b*x + a)^2 - c^5 - 2*(-I*b^3*d^4*(-c^6/
(b^4*d^6))^(3/4)*cos(b*x + a) + I*b*c^3*d*(-c^6/(b^4*d^6))^(1/4)*sin(b*x +
a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - b*d^2*(-c^6/(b^4*d^6))^(1/
4)*cos(b*x + a)*log(-2*b^2*c^2*d^3*sqrt(-c^6/(b^4*d^6))*cos(b*x + a)*sin(b*
x + a) + 2*c^5*cos(b*x + a)^2 - c^5 + 2*(b^3*d^4*(-c^6/(b^4*d^6))^(3/4)*cos
(b*x + a) + b*c^3*d*(-c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a
))*sqrt(c*sin(b*x + a))) + b*d^2*(-c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(-2
*b^2*c^2*d^3*sqrt(-c^6/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*c^5*cos(b*x
 + a)^2 - c^5 - 2*(b^3*d^4*(-c^6/(b^4*d^6))^(3/4)*cos(b*x + a) + b*c^3*d*(-
c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)
)) + b*d^2*(-c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(-c^5 + 2*(b^3*d^4*(-c^6/
(b^4*d^6))^(3/4)*cos(b*x + a) - b*c^3*d*(-c^6/(b^4*d^6))^(1/4)*sin(b*x + a)
)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - b*d^2*(-c^6/(b^4*d^6))^(1/4)
*cos(b*x + a)*log(-c^5 - 2*(b^3*d^4*(-c^6/(b^4*d^6))^(3/4)*cos(b*x + a) - b
*c^3*d*(-c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin
(b*x + a))) + I*b*d^2*(-c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(-c^5 - 2*(I*b
^3*d^4*(-c^6/(b^4*d^6))^(3/4)*cos(b*x + a) + I*b*c^3*d*(-c^6/(b^4*d^6))^(1/
4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - I*b*d^2*(-c^6
/(b^4*d^6))^(1/4)*cos(b*x + a)*log(-c^5 - 2*(-I*b^3*d^4*(-c^6/(b^4*d^6))^(3
/4)*cos(b*x + a) - I*b*c^3*d*(-c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*co
s(b*x + a))*sqrt(c*sin(b*x + a))) - 16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x
+ a))*c)/(b*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(a+bx))^{\frac{3}{2}}}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**(3/2)/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{3/2}}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2), x)
```

3.273 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*(c*sin(b*x+a))^(5/2)/b/c/d/(d*cos(b*x+a))^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2643}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx=\frac{2\cot(a+bx)(c\sin(a+bx))^{7/2}}{5bc^2(d\cos(a+bx))^{7/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c\left(\tan^2(bx+a)\right)}{5b\,d^3\sqrt{d\cos(bx+a)}}$	38

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b*(c*sin(b*x+a))^(1/2)*c/d^3/(d*cos(b*x+a))^(1/2)*tan(b*x+a)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx=-\frac{2\left(c\cos\left(bx+a\right)^2-c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{5\,bd^4\cos\left(bx+a\right)^3}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
[Out] -2/5*(c*cos(b*x + a)^2 - c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^
4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\,c\,(\cos(4\,a+4\,b\,x)-1)\,\sqrt{c\,\sin(a+b\,x)}}{5\,b\,d^3\,\sqrt{d\,\cos(a+b\,x)}\,(4\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3)}$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(7/2),x)
[Out] -(2*c*(cos(4*a + 4*b*x) - 1)*(c*sin(a + b*x))^(1/2))/(5*b*d^3*(d*cos(a + b*
x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

3.274 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/9*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(9/2)-2/45*c*(c*sin(b*x+a))^(
1/2)/b/d^3/(d*cos(b*x+a))^(5/2)-8/45*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d*cos(b*
x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2646, 2651, 2643}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(45*b*d^3*(d*Cos[a + b*x])^(5/2)) - (8*c*Sqrt[c*Sin[a + b*x]])/
(45*b*d^5*Sqrt[d*Cos[a + b*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2\int\frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{(4c^2)\int\frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{45d^4} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int\frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(7+2\cos(2(a+bx)))\sec^5(a+bx)(c\sin(a+bx))^{5/2}}{45bcd^6}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(7 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^5*(c*Sin[a +
b*x])^(5/2))/(45*b*c*d^6)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2c\sqrt{c\sin(bx+a)}\,(4(\tan^2(bx+a))+5(\tan^2(bx+a))\,(\sec^2(bx+a)))}{45b\,d^5\sqrt{d\cos(bx+a)}}$	59

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/b*c*(c*sin(b*x+a))^(1/2)/d^5/(d*cos(b*x+a))^(1/2)*(4*tan(b*x+a)^2+5*ta
n(b*x+a)^2*sec(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\left(4\,c\cos\left(bx+a\right)^4+c\cos\left(bx+a\right)^2-5\,c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{45\,bd^6\cos\left(bx+a\right)^5}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*c*cos(b*x + a)^4 + c*cos(b*x + a)^2 - 5*c)*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.95

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{\sqrt{c\sin(a+bx)}\left(2\sin(2a+2bx)^2+\sin(4a+4bx)\,1\mathrm{i}-1\right)\left(\frac{32c\left(-2\sin(2a+2bx)^2+\sin(4a+4bx)\,1\mathrm{i}+1\right)}{15bd^5}+\frac{16c\left(2\mathrm{s}\cdots\right.}{\cdots}\right.}{16\left(\sin(a+bx)^2-1\right)^2\sqrt{-d\left(\cdots\right.}}$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(11/2),x)
[Out] -((c*sin(a + b*x))^(1/2)*(sin(4*a + 4*b*x)*1i + 2*sin(2*a + 2*b*x)^2 - 1)*(
(32*c*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 + 1))/(15*b*d^5) + (16*c*
(2*sin(2*a + 2*b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 + 1)
)/(45*b*d^5) + (16*c*(2*sin(a + b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*sin(2*
a + 2*b*x)^2 + 1))/(9*b*d^5)))/(16*(sin(a + b*x)^2 - 1)^2*(-d*(2*sin(a/2 +
(b*x)/2)^2 - 1))^(1/2))
```

3.275 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/13*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(13/2)-2/117*c*(c*sin(b*x+a)
)^(1/2)/b/d^3/(d*cos(b*x+a))^(9/2)-16/585*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d*c
os(b*x+a))^(5/2)-64/585*c*(c*sin(b*x+a))^(1/2)/b/d^7/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2646, 2651, 2643}

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = -\frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(13*b*d*(d*Cos[a + b*x])^(13/2)) - (2*c*Sqrt[c*S
in[a + b*x]])/(117*b*d^3*(d*Cos[a + b*x])^(9/2)) - (16*c*Sqrt[c*Sin[a + b*x
]])/(585*b*d^5*(d*Cos[a + b*x])^(5/2)) - (64*c*Sqrt[c*Sin[a + b*x]])/(585*b
*d^7*Sqrt[d*Cos[a + b*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx}{13d^2} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{(8c^2)\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{117d^4} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} \\
&\quad - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{(32c^2)\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{585d^6} \\
&= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} \\
&\quad - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(77+36\cos(2(a+bx))+4\cos(4(a+bx)))\sec^7(a+bx)(c\sin(a+bx))^{5/2}}{585bcd^8}$$

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(77 + 36*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^7*(c*Sin[a + b*x])^(5/2))/(585*b*c*d^8)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2c\left(32\left(\cos^4(bx+a)\right)+40\left(\cos^2(bx+a)\right)+45\right)\sqrt{c\sin(bx+a)}\,\left(\tan^2(bx+a)\right)\left(\sec^4(bx+a)\right)}{585b\,d^7\sqrt{d\cos(bx+a)}}$	68

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585/b*c*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*(c*sin(b*x+a))^(1/2)/d^7/(d*
cos(b*x+a))^(1/2)*tan(b*x+a)^2*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = -\frac{2\left(32\,c\cos\left(bx+a\right)^6+8\,c\cos\left(bx+a\right)^4+5\,c\cos\left(bx+a\right)^2-45\,c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{585\,bd^8\cos\left(bx+a\right)^7}$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*c*cos(b*x + a)^6 + 8*c*cos(b*x + a)^4 + 5*c*cos(b*x + a)^2 - 45*
c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^8*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(15/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{15}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{15}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = -\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\sqrt{c\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(-\frac{3776\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}}{585\,b\,d^7}+\frac{2752\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,\cos(2\,a+2\,b\,x)}{585\,b\,d^7}+\frac{896\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,\cos(4\,a+4\,b\,}{585\,b\,d^7}\right.}{64\cos\left(a+b\,x\right)^6\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}$$

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(15/2),x)
```

```
[Out] -(exp(- a*6i - b*x*6i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((2752*c*exp(a*6i + b*x*6i)*cos(2*a + 2*b*x))/(585*b*d^7) -
 (3776*c*exp(a*6i + b*x*6i))/(585*b*d^7) + (896*c*exp(a*6i + b*x*6i)*cos(4*
a + 4*b*x))/(585*b*d^7) + (128*c*exp(a*6i + b*x*6i)*cos(6*a + 6*b*x))/(585*
b*d^7)))/(64*cos(a + b*x)^6*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)
/2))^(1/2))
```

3.276 $\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b} + \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b} - \frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} + \frac{3c^2d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/20*c*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b+3/70*c*d*(d*cos(b*x+
a))^(7/2)*(c*sin(b*x+a))^(3/2)/b-1/7*c*(d*cos(b*x+a))^(11/2)*(c*sin(b*x+a))
^(3/2)/b/d-3/40*c^2*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b
/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2648, 2649, 2652, 2719}

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \frac{3c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{40b\sqrt{\sin(2a+2bx)}} + \frac{cd^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{20b} - \frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd} + \frac{3cd(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{70b}$$

```
[In] Int[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(20*b) + (3*c*d*(d*Co
s[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(70*b) - (c*(d*Cos[a + b*x])^(11/
2)*(c*Sin[a + b*x])^(3/2))/(7*b*d) + (3*c^2*d^4*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(40*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} \\ &\quad+\frac{1}{14}\left(3c^2\right)\int(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx \\ &= \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}-\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} \\ &\quad+\frac{1}{20}\left(3c^2d^2\right)\int(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx\end{aligned}$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b} + \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b} - \frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} + \frac{1}{40}\left(3c^2d^4\right)\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b} + \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b} - \frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} + \frac{\left(3c^2d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{40\sqrt{\sin(2a+2bx)}}$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b} + \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b} - \frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} + \frac{3c^2d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \frac{2(d\cos(a+bx))^{9/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)}{7bc}$$

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
[Out] (2*(d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 7/
4, 11/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*(c*Sin[a + b*x])^(7/2))/(7*b*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(165) = 330.

Time = 1.70 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\sqrt{2}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\left(-40\sqrt{2}\left(\cos^8(bx+a)\right)+52\sqrt{2}\left(\cos^6(bx+a)\right)+42\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\sqrt{\cot(bx+a)-\csc(b}\right.}{}$

```
[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/560/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*(-40*2^(1/2)*cos
(b*x+a)^8+52*2^(1/2)*cos(b*x+a)^6+42*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(
b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*
x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)-21*(-cot(b*x+a)+csc(b*x+a)
+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ell
ipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*2^(1/2)*c
os(b*x+a)^4+42*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(
1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/
2),1/2*2^(1/2))-21*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+
1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)
^(1/2),1/2*2^(1/2))+7*2^(1/2)*cos(b*x+a)^2-21*2^(1/2)*cos(b*x+a))*d^4*c^2*s
ec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*d^4*cos(b*x + a)^6 - c^2*d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\,\cos(a+b\,x))^{9/2}\,(c\,\sin(a+b\,x))^{5/2}\,dx$$

```
[In] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2), x)
```

3.277 $\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} + \frac{3c^2d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/10*c*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b-1/5*c*(d*cos(b*x+a))^(
7/2)*(c*sin(b*x+a))^(3/2)/b/d-3/20*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*
sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2648, 2649, 2652, 2719}

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \frac{3c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}} - \frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd} + \frac{cd(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{10b}$$

```
[In] Int[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(10*b) - (c*(d*Cos[a +
b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*d) + (3*c^2*d^2*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule bi steps

$$\begin{aligned}
\text{integral} &= -\frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} \\
&\quad + \frac{1}{10}\left(3c^2\right)\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx \\
&= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} \\
&\quad + \frac{1}{20}\left(3c^2d^2\right)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx \\
&= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} \\
&\quad + \frac{\left(3c^2d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{20\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} + \frac{3c^2d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \frac{2d^2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx\right.}{7b}$$

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
7/4, 11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(136) = 272.

Time = 1.42 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

method	result
default	$\frac{\sqrt{2}\,d^2c^2\left(4\sqrt{2}\,(\cos^6(bx+a))-6\sqrt{2}\,(\cos^4(bx+a))-6\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+\right.}{}$

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40/b*2^(1/2)*d^2*c^2*(4*2^(1/2)*cos(b*x+a)^6-6*2^(1/2)*cos(b*x+a)^4-6*(-c
ot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-c
sc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*co
s(b*x+a)+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)
*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1
/2*2^(1/2))*cos(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b
*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x
+a)+1)^(1/2),1/2*2^(1/2))+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-cs
c(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(
b*x+a)+1)^(1/2),1/2*2^(1/2))-2^(1/2)*cos(b*x+a)^2+3*2^(1/2)*cos(b*x+a))*(c*
sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(-(c^2*d^2*cos(b*x + a)^4 - c^2*d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx=\int (d\,\cos\left(a+b\,x\right))^{5/2}\,(c\,\sin\left(a+b\,x\right))^{5/2}\,dx$$

```
[In] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2), x)
```

3.278 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/3*c*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/d-1/2*c^2*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos
(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2648, 2652, 2719}

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \frac{c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}} - \frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -1/3*(c*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(b*d) + (c^2*Sqrt[d*
Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[
Sin[2*a + 2*b*x]])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{1}{2}c^2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx \\
&= -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} \\
&\quad + \frac{\left(c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}} \\
&= -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$
\int\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}\tan(a+bx)}{7b}
$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/4,
11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(106) = 212$.

Time = 0.26 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.32

method	result
default	$\sqrt{2}\,\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,\Big(2\sqrt{2}\,(\cos^4(bx+a))-6\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(b$

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*(2*2^(1/2)*cos(b*x
+a)^4-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(c
ot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*
2^(1/2))*cos(b*x+a)+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+
a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)
+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b
*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x
+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(co
t(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(
b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-5*2^(1/2)*cos(b*x+a)^2+3*2^(1/2)*co
s(b*x+a))*c^2*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos\left(bx+a\right)}(c\sin\left(bx+a\right))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos\left(bx+a\right)}(c\sin\left(bx+a\right))^{\frac{5}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\,\cos\left(a+b\,x\right)}\,{\left(c\,\sin\left(a+b\,x\right)\right)}^{5/2}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2), x)
```

3.279 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{3c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(1/2)+3*c^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+
a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2646, 2652, 2719}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(b*d*Sqrt[d*Cos[a + b*x]]) - (3*c^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{(3c^2)\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(3c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{3c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int\frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/2}}{7bcd\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(109) = 218$.

Time = 0.22 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.23

method	result
default	$\frac{\sqrt{2}\, c^{2} \left(6 \sqrt{-\cot \left(b x +a \right)+\csc \left(b x +a \right)+1}\, \sqrt{\cot \left(b x +a \right)-\csc \left(b x +a \right)+1}\, \sqrt{\cot \left(b x +a \right)-\csc \left(b x +a \right)}\, E\left(\sqrt{-\cot \left(b x +a \right)+\csc \left(b x +a \right)+1}, \frac{\sqrt{2}}{2}\right) \cos}{}$

[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/b*2^(1/2)*c^2*(6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)^2-3*2^(1/2)*cos(b*x+a)+2*2^(1/2))*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d*csc(b*x+a)

Fricas [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(d^2*cos(b*x + a)^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2), x)
```

3.280 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{6c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2/5*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(5/2)-6/5*c*(c*sin(b*x+a))^(3
/2)/b/d^3/(d*cos(b*x+a))^(1/2)-6/5*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/
4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(
b*x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2646, 2651, 2652, 2719}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{6c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (6*c*(c*Sin[a
 + b*x])^(3/2))/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (6*c^2*Sqrt[d*Cos[a + b*x]
```

```
]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{(3c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{(6c^2)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{5d^4} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} \\
&\quad + \frac{\left(6c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{5d^4\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{6c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\cos^2(a+bx)^{5/4}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+$$
$$7bc^2(d\cos(a+bx))^{7/2}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(5/4)*Cot[a + b*x]*Hypergeometric2F1[7/4, 9/4, 11/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(138) = 276.

Time = 0.23 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.13

method	result
default	$\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\,c^2\left(-6\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{-\cot(bx+a)+\csc(b\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*c^2/d^3/(d*cos(b*x+a))^(1/2)*(-6*(-cot(b
*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b
*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cot(b*
x+a)+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(co
t(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2
^(1/2))*cot(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a
)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+
1)^(1/2),1/2*2^(1/2))*csc(b*x+a)+3*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*
x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+
a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*csc(b*x+a)+3*2^(1/2)*cot(b*x+a)-4*2^(1/
2)*csc(b*x+a)+2^(1/2)*sec(b*x+a)^2*csc(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.59

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{3i\sqrt{i\,cd}c^2\cos{(bx+a)}^3\,E(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)-3i\sqrt{-i\,cd}}{}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1) - 3*I*sqrt(-I*c*d)*c^2*cos(b*x + a)^3*elliptic_e(arcsin(
cos(b*x + a) - I*sin(b*x + a)), -1) - 3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^3*el
liptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 3*I*sqrt(-I*c*d)*c^2*
cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - 2*(3
*c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*
x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin{(bx+a)})^{\frac{5}{2}}}{(d\cos{(bx+a)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2), x)
```

3.281 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{4c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2/9*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(9/2)-2/15*c*(c*sin(b*x+a))^(
3/2)/b/d^3/(d*cos(b*x+a))^(5/2)-4/15*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*cos(b*
x+a))^(1/2)-4/15*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Elliptic
E(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d^
6/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2646, 2651, 2652, 2719}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{4c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*(c*Sin[a
 + b*x])^(3/2))/(15*b*d^3*(d*Cos[a + b*x])^(5/2)) - (4*c*(c*Sin[a + b*x])^(
```

```
3/2))/(15*b*d^5*Sqrt[d*Cos[a + b*x]]) + (4*c^2*Sqrt[d*Cos[a + b*x]]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(15*b*d^6*Sqrt[Sin[2*a + 2*b*x]
])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2 \int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx}{3d^2} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{(2c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{15d^4} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} \\
&\quad - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{(4c^2)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{15d^6}
\end{aligned}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{\left(4c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int\sqrt{\sin(2a+2bx)}\,dx}{15d^6\sqrt{\sin(2a+2bx)}}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{4c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\cos^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{13}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(}{7bc(d\cos(a+bx))^{11/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Cos[a + b*x]^5*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[7/4, 13/4, 11/4,
 Sin[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(7*b*c*(d*Cos[a + b*x])^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(167) = 334$.

Time = 0.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.63

method	result
default	$\frac{\sqrt{2}\,c^2\left(-12\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/b*2^(1/2)*c^2*(-12*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+
a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^5+6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(c
ot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot
(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^5-12*(-cot(b*x+a)+csc(b
*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^4+6*(-
cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-
```

```
csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*c
os(b*x+a)^4+6*2^(1/2)*cos(b*x+a)^5-3*2^(1/2)*cos(b*x+a)^4-8*2^(1/2)*cos(b*x
+a)^2+5*2^(1/2))*(c*sin(b*x+a))^(1/2)/d^5/(d*cos(b*x+a))^(1/2)*sec(b*x+a)^4
*csc(b*x+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx =$$
$$-\frac{2\left(-3i\sqrt{i\,cd}c^2\cos{(bx+a)}^5\,E(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+3i\sqrt{-i\,cd}c^2\cos{(bx+a)}^5\,E(\text{arc}\right.}{}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(-3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^5*elliptic_e(arcsin(cos(b*x + a) +
 I*sin(b*x + a)), -1) + 3*I*sqrt(-I*c*d)*c^2*cos(b*x + a)^5*elliptic_e(arcs
in(cos(b*x + a) - I*sin(b*x + a)), -1) + 3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^5
*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - 3*I*sqrt(-I*c*d)*c
^2*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + (
6*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 5*c^2)*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a))*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{11/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2), x)
```

3.282 $\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 320

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd}$$

```
[Out] -3/8*c^(5/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b
*x+a))^(1/2))/b*2^(1/2)/d^(1/2)+3/8*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin
(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))/b*2^(1/2)/d^(1/2)+3/16*c^(5/2)
*ln(c^(1/2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/
2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)-3/16*c^(5/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c
*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/
2)-1/2*c*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2648, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])])/(4*Sqrt[2]*b*Sqrt[d]) + (3*c^(5/2)*ArcTan[1 + (Sqrt[2]*
Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b
*Sqrt[d]) + (3*c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])
/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (3*c
^(5/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (c*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{1}{4}\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{(3c^3d)\,\mathrm{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2b}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} - \frac{(3c^3)\,\mathrm{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b} + \frac{(3c^3)\,\mathrm{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{(3c^3)\,\mathrm{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8bd} + \frac{(3c^3)\,\mathrm{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8bd} + \frac{(3c^{5/2})\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}+2x}{-\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{(3c^{5/2})\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}-2x}{-\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8\sqrt{2}b\sqrt{d}}$$

$$= \frac{3c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd} + \frac{(3c^{5/2})\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{(3c^{5/2})\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}}$$

$$= -\frac{3c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}b\sqrt{d}} - \frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}\tan(}{7b\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b*Sqrt[d*Cos[a + b*x]])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sqrt{2}\left(4\sin(bx+a)\sqrt{2}\,\cos(bx+a)\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}+4\sin(bx+a)\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}-3\ln\left(-2\sqrt{2}\,\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\right.\right.}{}$

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/16/b*2^(1/2)*(4*sin(b*x+a)*2^(1/2)*cos(b*x+a)*(-sin(b*x+a)*cos(b*x+a)/(1
+cos(b*x+a))^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b
*x+a))^2)^(1/2)-3*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(
1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*c
sc(b*x+a)+2-2*cot(b*x+a))+3*ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2
)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))-6*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a
```

```
)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))-6*arctan
((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*
x+a)-1)/(cos(b*x+a)-1)))*(c*sin(b*x+a))^(1/2)*c^2*cos(b*x+a)/(1+cos(b*x+a))
/(d*cos(b*x+a))^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 1028, normalized size of antiderivative = 3.21

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x + a) + 3*(-
c^10/(b^4*d^2))^(1/4)*b*d*log(27/2*c^8*cos(b*x + a)*sin(b*x + a) + 27/2*((-
c^10/(b^4*d^2))^(1/4)*b*c^5*sin(b*x + a) - (-c^10/(b^4*d^2))^(3/4)*b^3*d*co
s(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 27/4*(2*b^2*c^3*d*c
os(b*x + a)^2 - b^2*c^3*d)*sqrt(-c^10/(b^4*d^2))) - 3*(-c^10/(b^4*d^2))^(1/
4)*b*d*log(27/2*c^8*cos(b*x + a)*sin(b*x + a) - 27/2*((-c^10/(b^4*d^2))^(1/
4)*b*c^5*sin(b*x + a) - (-c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(d*
cos(b*x + a))*sqrt(c*sin(b*x + a)) - 27/4*(2*b^2*c^3*d*cos(b*x + a)^2 - b^2
*c^3*d)*sqrt(-c^10/(b^4*d^2))) - 3*I*(-c^10/(b^4*d^2))^(1/4)*b*d*log(27/2*c
^8*cos(b*x + a)*sin(b*x + a) - 27/2*(I*(-c^10/(b^4*d^2))^(1/4)*b*c^5*sin(b*
x + a) + I*(-c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(d*cos(b*x + a))
*sqrt(c*sin(b*x + a)) + 27/4*(2*b^2*c^3*d*cos(b*x + a)^2 - b^2*c^3*d)*sqrt(
-c^10/(b^4*d^2))) + 3*I*(-c^10/(b^4*d^2))^(1/4)*b*d*log(27/2*c^8*cos(b*x +
a)*sin(b*x + a) - 27/2*(-I*(-c^10/(b^4*d^2))^(1/4)*b*c^5*sin(b*x + a) - I*(
-c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(
b*x + a)) + 27/4*(2*b^2*c^3*d*cos(b*x + a)^2 - b^2*c^3*d)*sqrt(-c^10/(b^4*d
^2))) + 3*(-c^10/(b^4*d^2))^(1/4)*b*d*log(27*c^8 + 54*((-c^10/(b^4*d^2))^(1
/4)*b*c^5*cos(b*x + a) - (-c^10/(b^4*d^2))^(3/4)*b^3*d*sin(b*x + a))*sqrt(d
*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 3*(-c^10/(b^4*d^2))^(1/4)*b*d*log(27
*c^8 - 54*((-c^10/(b^4*d^2))^(1/4)*b*c^5*cos(b*x + a) - (-c^10/(b^4*d^2))^(
3/4)*b^3*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 3*I*(
-c^10/(b^4*d^2))^(1/4)*b*d*log(27*c^8 - 54*(I*(-c^10/(b^4*d^2))^(1/4)*b*c^5
*cos(b*x + a) + I*(-c^10/(b^4*d^2))^(3/4)*b^3*d*sin(b*x + a))*sqrt(d*cos(b*
x + a))*sqrt(c*sin(b*x + a))) + 3*I*(-c^10/(b^4*d^2))^(1/4)*b*d*log(27*c^8
- 54*(-I*(-c^10/(b^4*d^2))^(1/4)*b*c^5*cos(b*x + a) - I*(-c^10/(b^4*d^2))^(
3/4)*b^3*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2), x)
```

3.283 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx &= \frac{c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} \\ &- \frac{c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} \\ &- \frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}} \\ &+ \frac{c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}\end{aligned}$$

```
[Out] 2/3*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(3/2)+1/2*c^(5/2)*arctan(1-2^
(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))/b/d^(5/2)*
2^(1/2)-1/2*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(
d*cos(b*x+a))^(1/2))/b/d^(5/2)*2^(1/2)-1/4*c^(5/2)*ln(c^(1/2)-2^(1/2)*d^(1/
2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b/d^(5/2)*
2^(1/2)+1/4*c^(5/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(
b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2646, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} - \frac{c^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}bd^{5/2}} - \frac{c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{2\sqrt{2}bd^{5/2}} + \frac{c^{5/2}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)+\sqrt{c}\right)}{2\sqrt{2}bd^{5/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*
Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2)) - (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]
*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2))
 - (c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos
[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (c^(5/2)*Log[Sq
rt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sqrt[
c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (2*c*(c*Sin[a + b*x])^(3/2))/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\int\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{(2c^3)\,\text{Subst}\left(\int\frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd}
\end{aligned}$$

$$=\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}+\frac{c^3\text{Subst}\left(\int\frac{c-dx^2}{c^2+d^2x^4}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd^2}-\frac{c^3\text{Subst}\left(\int\frac{c+dx^2}{c^2+d^2x^4}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd^2}$$

$$=\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^3\text{Subst}\left(\int\frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd^3}-\frac{c^3\text{Subst}\left(\int\frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd^3}-\frac{c^{5/2}\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}+2x}{-\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{c}}{\sqrt{d}}-2x}{-\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}-x^2}\,dx,x,\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2\sqrt{2}bd^{5/2}}$$

$$=-\frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}}+\frac{c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}+\frac{c^{5/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}$$

$$=\frac{c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}}-\frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}}+\frac{c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}bd^{5/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(238) = 476.

Time = 0.26 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.56

method	result
default	$\frac{\sqrt{2}\left(6\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\arctan\left(\frac{\sin(bx+a)\sqrt{2}\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}+\cos(bx+a)-1}{\cos(bx+a)-1}\right)\cos(bx+a)+6\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\,\text{arcta}\right.}{}$

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*2^(1/2)*(6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((s
in(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a
)-1)/(cos(b*x+a)-1))*cos(b*x+a)+6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)
^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)
^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))*cos(b*x+a)-3*(-sin(b*x+a)*cos(b*x+a)/(
1+cos(b*x+a))^2)^(1/2)*ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^
2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/
2)*csc(b*x+a)+2-2*cot(b*x+a))*cos(b*x+a)+3*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b
*x+a))^2)^(1/2)*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/
2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc
(b*x+a)+2-2*cot(b*x+a))*cos(b*x+a)-4*2^(1/2)*cos(b*x+a)+4*2^(1/2))*c^2*(c*s
in(b*x+a))^(1/2)*(1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)/d^2*sec(b*x+a)*csc(b*x
+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.70

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(-1/2*c^8*cos(b*x
+ a)*sin(b*x + a) + 1/2*(b^3*d^7*(-c^10/(b^4*d^10))^(3/4)*cos(b*x + a) - b*
c^5*d^2*(-c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*
sin(b*x + a)) + 1/4*(2*b^2*c^3*d^5*cos(b*x + a)^2 - b^2*c^3*d^5)*sqrt(-c^10
/(b^4*d^10))) - 3*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(-1/2*c^
8*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^7*(-c^10/(b^4*d^10))^(3/4)*cos(b*x
 + a) - b*c^5*d^2*(-c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a
))*sqrt(c*sin(b*x + a)) + 1/4*(2*b^2*c^3*d^5*cos(b*x + a)^2 - b^2*c^3*d^5)*
sqrt(-c^10/(b^4*d^10))) - 3*I*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2
*log(-1/2*c^8*cos(b*x + a)*sin(b*x + a) + 1/2*(I*b^3*d^7*(-c^10/(b^4*d^10))
^(3/4)*cos(b*x + a) + I*b*c^5*d^2*(-c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sq
rt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 1/4*(2*b^2*c^3*d^5*cos(b*x + a)^2
 - b^2*c^3*d^5)*sqrt(-c^10/(b^4*d^10))) + 3*I*b*d^3*(-c^10/(b^4*d^10))^(1/4
)*cos(b*x + a)^2*log(-1/2*c^8*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^7*(
-c^10/(b^4*d^10))^(3/4)*cos(b*x + a) - I*b*c^5*d^2*(-c^10/(b^4*d^10))^(1/4)
*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 1/4*(2*b^2*c^3*d
^5*cos(b*x + a)^2 - b^2*c^3*d^5)*sqrt(-c^10/(b^4*d^10))) - 3*b*d^3*(-c^10/(
b^4*d^10))^(1/4)*cos(b*x + a)^2*log(c^8 + 2*(b^3*d^7*(-c^10/(b^4*d^10))^(3/
4)*sin(b*x + a) - b*c^5*d^2*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*c
os(b*x + a))*sqrt(c*sin(b*x + a))) + 3*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b
*x + a)^2*log(c^8 - 2*(b^3*d^7*(-c^10/(b^4*d^10))^(3/4)*sin(b*x + a) - b*c^
5*d^2*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*si
n(b*x + a))) - 3*I*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(c^8 -
2*(I*b^3*d^7*(-c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + I*b*c^5*d^2*(-c^10/(b^
4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 3
*I*b*d^3*(-c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(c^8 - 2*(-I*b^3*d^7*(-
c^10/(b^4*d^10))^(3/4)*sin(b*x + a) - I*b*c^5*d^2*(-c^10/(b^4*d^10))^(1/4)*
cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 16*sqrt(d*cos(b*
x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{5/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2), x)
```

3.284 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(7/2)/b/c/d/(d*cos(b*x+a))^(7/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2643}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
[Out] (2*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\cot(a+bx)(c\sin(a+bx))^{9/2}}{7bc^2(d\cos(a+bx))^{9/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c^2\left(\tan^3(bx+a)\right)}{7b\,d^4\sqrt{d\cos(bx+a)}}$	40

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/7/b*(c*sin(b*x+a))^(1/2)*c^2/d^4/(d*cos(b*x+a))^(1/2)*tan(b*x+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\left(c^2\cos\left(bx+a\right)^2-c^2\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{7\,bd^5\cos\left(bx+a\right)^4}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] -2/7*(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*s
in(b*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\,c^2\,\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(3\,\sin\left(2\,a+2\,b\,x\right)-\sin\left(6\,a+6\,b\,x\right)\right)}{7\,b\,d^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)\right.}$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(9/2),x)
[Out] (2*c^2*(c*sin(a + b*x))^(1/2)*(3*sin(2*a + 2*b*x) - sin(6*a + 6*b*x)))/(7*b
*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*cos(4*a + 4*b*x) + cos
(6*a + 6*b*x) + 10))
```

3.285 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/11*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(11/2)-6/77*c*(c*sin(b*x+a))
^(3/2)/b/d^3/(d*cos(b*x+a))^(7/2)-8/77*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*cos(
b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2646, 2651, 2643}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = -\frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(11*b*d*(d*Cos[a + b*x])^(11/2)) - (6*c*(c*Sin
[a + b*x])^(3/2))/(77*b*d^3*(d*Cos[a + b*x])^(7/2)) - (8*c*(c*Sin[a + b*x])
^(3/2))/(77*b*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{(3c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{11d^2} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{(12c^2)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{77d^4} \\
&= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2c^4(9+2\cos(2(a+bx)))\tan^5(a+bx)}{77bd^6\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c^4*(9 + 2*Cos[2*(a + b*x)])*Tan[a + b*x]^5)/(77*b*d^6*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2c^{2}\sqrt{c\sin(bx+a)}\,\left(4\left(\tan^{3}(bx+a)\right)+7\left(\tan^{3}(bx+a)\right)\left(\sec^{2}(bx+a)\right)\right)}{77b\,d^{6}\sqrt{d\cos(bx+a)}}$	61

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/b*c^2*(c*sin(b*x+a))^(1/2)/d^6/(d*cos(b*x+a))^(1/2)*(4*tan(b*x+a)^3+7*
tan(b*x+a)^3*sec(b*x+a)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = -\frac{2\left(4c^{2}\cos\left(bx+a\right)^{4}+3c^{2}\cos\left(bx+a\right)^{2}-7c^{2}\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{77\,bd^{7}\cos\left(bx+a\right)^{6}}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 7*c^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(13/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.66

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = -\frac{e^{-a\,5i-b\,x\,5i}\sqrt{c\,\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\,\left(\frac{96\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(3\,a+3\,b\,x)}{77\,b\,d^6}+\frac{16\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(5\,a+5\,b\,x)}{77\,b\,d^6}-\frac{368\,c^2\,e^{a\,5i+b\,x\,5i}\,\mathrm{si}}{77\,b\,d^6}\right.}{32\cos(a+b\,x)^5\,\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(13/2),x)
[Out] -(exp(- a*5i - b*x*5i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((96*c^2*exp(a*5i + b*x*5i)*sin(3*a + 3*b*x))/(77*b*d^6) +
(16*c^2*exp(a*5i + b*x*5i)*sin(5*a + 5*b*x))/(77*b*d^6) - (368*c^2*exp(a*5i
 + b*x*5i)*sin(a + b*x))/(77*b*d^6)))/(32*cos(a + b*x)^5*(d*(exp(- a*1i - b
*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

3.286 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/15*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(15/2)-2/55*c*(c*sin(b*x+a))
^(3/2)/b/d^3/(d*cos(b*x+a))^(11/2)-16/385*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*c
os(b*x+a))^(7/2)-64/1155*c*(c*sin(b*x+a))^(3/2)/b/d^7/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2646, 2651, 2643}

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = -\frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}$$

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(15*b*d*(d*Cos[a + b*x])^(15/2)) - (2*c*(c*Sin
[a + b*x])^(3/2))/(55*b*d^3*(d*Cos[a + b*x])^(11/2)) - (16*c*(c*Sin[a + b*x
])^(3/2))/(385*b*d^5*(d*Cos[a + b*x])^(7/2)) - (64*c*(c*Sin[a + b*x])^(3/2)
)/(1155*b*d^7*(d*Cos[a + b*x])^(3/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{c^2\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx}{5d^2}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{(8c^2)\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{55d^4}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}\\
&\quad - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{(32c^2)\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{385d^6}\\
&= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}\\
&\quad - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(117+44\cos(2(a+bx))+4\cos(4(a+bx)))\sec^8(a+bx)(c\sin(}{1155bcd^9}$$

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(117 + 44*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^8*(c*Sin[a + b*x])^(7/2))/(1155*b*c*d^9)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{2c^2(32(\cos^4(bx+a))+56(\cos^2(bx+a))+77)\sqrt{c\sin(bx+a)}\,(\tan^3(bx+a))(\sec^4(bx+a))}{1155b\,d^8\sqrt{d\cos(bx+a)}}$	70

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/b*c^2*(32*cos(b*x+a)^4+56*cos(b*x+a)^2+77)*(c*sin(b*x+a))^(1/2)/d^8/
(d*cos(b*x+a))^(1/2)*tan(b*x+a)^3*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = -\frac{2\left(32\,c^2\cos\left(bx+a\right)^6+24\,c^2\cos\left(bx+a\right)^4+21\,c^2\cos\left(bx+a\right)^2-77\,c^2\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin}{1155\,bd^9\cos\left(bx+a\right)^8}$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*c^2*cos(b*x + a)^6 + 24*c^2*cos(b*x + a)^4 + 21*c^2*cos(b*x + a
)^2 - 77*c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^9
*cos(b*x + a)^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(17/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{17}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{17}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.47

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = -\frac{e^{-a\,7i-b\,x\,7i}\sqrt{c\,\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\,\left(\frac{1216\,c^2\,e^{a\,7i+b\,x\,7i}\,\sin(3\,a+3\,b\,x)}{385\,b\,d^8}+\frac{1024\,c^2\,e^{a\,7i+b\,x\,7i}\,\sin(5\,a+5\,b\,x)}{1155\,b\,d^8}+\frac{128\,c^2\,e^{a\,7i+b\,x}}{115}\right.}{128\cos\left(a+b\,x\right)^7\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(17/2),x)
```

```
[Out] -(exp(- a*7i - b*x*7i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((1216*c^2*exp(a*7i + b*x*7i)*sin(3*a + 3*b*x))/(385*b*d^8)
 + (1024*c^2*exp(a*7i + b*x*7i)*sin(5*a + 5*b*x))/(1155*b*d^8) + (128*c^2*e
xp(a*7i + b*x*7i)*sin(7*a + 7*b*x))/(1155*b*d^8) - (3392*c^2*exp(a*7i + b*x
*7i)*sin(a + b*x))/(231*b*d^8)))/(128*cos(a + b*x)^7*(d*(exp(- a*1i - b*x*1
i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

3.287 $\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*sin(b*x+a)^(5/2)/b/cos(b*x+a)^(5/2)-1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1
/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/2*arctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b
*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x
+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a
)^(1/2))/b*2^(1/2)-2*sin(b*x+a)^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2646, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} - \frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b} + \frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b) - (2*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]]) + (2*
Sin[a + b*x]^(5/2))/(5*b*Cos[a + b*x]^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \int \frac{\sin^{\frac{3}{2}}(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx \\ &= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx\end{aligned}$$

$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$

$$= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},\sin^2(a+bx)\right)\sin^{\frac{9}{2}}(a+bx)}{9b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, Sin[a + b*x]^2]
*Sin[a + b*x]^(9/2))/(9*b*Sqrt[Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(180) = 360.

Time = 2.29 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	892

```
[In] int(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/b*2^(1/2)*(5*ln(-2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(
1/2)*cot(b*x+a)-2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cs
c(b*x+a)+2+2*cot(b*x+a))*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cos
(b*x+a)^3-5*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*ln(2*2^(1/2)*(si
n(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(sin(b*x+a
)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2+2*cot(b*x+a))*cos(b*x+a)^
3-10*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((sin(b*x+a)*(sin
(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2)-cos(b*x+a)+1)/(cos(b*x+a
)-1))*cos(b*x+a)^3-10*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan
((sin(b*x+a)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2)+cos(b*x
+a)-1)/(cos(b*x+a)-1))*cos(b*x+a)^3+5*ln(-2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/
(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(
b*x+a))^2)^(1/2)*csc(b*x+a)+2+2*cot(b*x+a))*(sin(b*x+a)*cos(b*x+a)/(1+cos(b
*x+a))^2)^(1/2)*cos(b*x+a)^2-5*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/
2)*ln(2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2
*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2+2*cot(
b*x+a))*cos(b*x+a)^2-10*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arct
an((sin(b*x+a)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2)-cos(b
```

```
*x+a)+1)/(cos(b*x+a)-1))*cos(b*x+a)^2-10*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+
a))^2)^(1/2)*arctan((sin(b*x+a)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1
/2)*2^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))*cos(b*x+a)^2+24*2^(1/2)*cos(b*x+a
)^2*sin(b*x+a)-4*2^(1/2)*sin(b*x+a))/sin(b*x+a)^(1/2)/cos(b*x+a)^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.39

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40*(5*b*(-1/b^4)^(1/4)*cos(b*x + a)^3*log(2*b^2*sqrt(-1/b^4)*cos(b*x + a)
*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) + b*(
-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) - 5*
b*(-1/b^4)^(1/4)*cos(b*x + a)^3*log(2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(b*x
 + a) - 2*cos(b*x + a)^2 - 2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) + b*(-1/b^4)^
(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 5*I*b*(-1/
b^4)^(1/4)*cos(b*x + a)^3*log(-2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(b*x + a)
 - 2*cos(b*x + a)^2 - 2*(I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) - I*b*(-1/b^4)^(
1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) - 5*I*b*(-1/b
^4)^(1/4)*cos(b*x + a)^3*log(-2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(b*x + a)
- 2*cos(b*x + a)^2 - 2*(-I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(-1/b^4)^(
1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 5*b*(-1/b^4
)^(1/4)*cos(b*x + a)^3*log(2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^
(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) - 5*b*(-1/b^
4)^(1/4)*cos(b*x + a)^3*log(-2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4
)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) + 5*I*b*(-
1/b^4)^(1/4)*cos(b*x + a)^3*log(-2*(I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b
*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) -
5*I*b*(-1/b^4)^(1/4)*cos(b*x + a)^3*log(-2*(-I*b^3*(-1/b^4)^(3/4)*cos(b*x +
 a) - I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)
) - 1) - 16*(6*cos(b*x + a)^2 - 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))/(
b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**(7/2)/cos(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{7}{2}}}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{7}{2}}}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.19

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sin(a+b\,x)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{5}{4};\ -\frac{1}{4};\ \cos(a+b\,x)^2\right)}{5\,b\cos(a+b\,x)^{5/2}\,\left(\sin(a+b\,x)^2\right)^{9/4}}$$

```
[In] int(sin(a + b*x)^(7/2)/cos(a + b*x)^(7/2),x)
[Out] (2*sin(a + b*x)^(9/2)*hypergeom([-5/4, -5/4], -1/4, cos(a + b*x)^2))/(5*b*c
os(a + b*x)^(5/2)*(sin(a + b*x)^2)^(9/4))
```

3.288 $\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

```
[Out] 2/5*sin(x)^(5/2)/cos(x)^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2643}

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

```
[In] Int[Sin[x]^(3/2)/Cos[x]^(7/2),x]
[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

```
[In] Integrate[Sin[x]^(3/2)/Cos[x]^(7/2),x]
[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5\cos(x)^{\frac{5}{2}}}$	11

```
[In] int(sin(x)^(3/2)/cos(x)^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5*sin(x)^(5/2)/cos(x)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = -\frac{2\left(\cos(x)^2-1\right)\sqrt{\sin(x)}}{5\cos(x)^{\frac{5}{2}}}$$

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="fricas")
[Out] -2/5*(cos(x)^2 - 1)*sqrt(sin(x))/cos(x)^(5/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**(3/2)/cos(x)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \int \frac{\sin\left(x\right)^{\frac{3}{2}}}{\cos\left(x\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="maxima")
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \int \frac{\sin\left(x\right)^{\frac{3}{2}}}{\cos\left(x\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="giac")
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = -\frac{8\sqrt{2}\tan\left(\frac{x}{2}\right)^{5/2}\sqrt{1-\tan\left(\frac{x}{2}\right)^2}}{\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2\left(5\tan\left(\frac{x}{2}\right)^2-15\right)+15\right)-5}$$

```
[In] int(sin(x)^(3/2)/cos(x)^(7/2),x)
[Out] -(8*2^(1/2)*tan(x/2)^(5/2)*(1 - tan(x/2)^2)^(1/2))/(tan(x/2)^2*(tan(x/2)^2*
(5*tan(x/2)^2 - 15) + 15) - 5)
```

3.289 $\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}}-\frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/
2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+1/4*ln(1-2^(1/2)*sin(x)^(1/2)/cos(x)^
(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2)+tan(x))*2^
(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}$$

```
[In] Int[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1 + (Sq
rt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] + Log[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Co
s[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{2\sqrt{2}}\\
&= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\\
&= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\\
&\quad + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \frac{2\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(x)\right)\sin^{\frac{3}{2}}(x)}{3\cos^{\frac{3}{2}}(x)}$$

```
[In] Integrate[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[x]^(3/2)
)/(3*Cos[x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(87) = 174$.

Time = 2.77 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{2}\,(\sqrt{\cos}(x))\left(\ln\left(2\sqrt{2}\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\cot(x)+2\sqrt{2}\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\csc(x)+2\cot(x)+2\right)+2\arctan\left(\frac{-\sin(x)\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\sqrt{2}+\cos(x)-}{-1+\cos(x)}\right.\right.}{4\sin(x)}$

```
[In] int(sin(x)^(1/2)/cos(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*cos(x)^(1/2)*(ln(2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*c
ot(x)+2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*csc(x)+2*cot(x)+2)+2*arc
tan((-sin(x)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*2^(1/2)+cos(x)-1)/(-1+cos(x
)))-ln(-2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*cot(x)-2*2^(1/2)*(sin(
x)*cos(x)/(cos(x)+1)^2)^(1/2)*csc(x)+2*cot(x)+2)-2*arctan((sin(x)*(sin(x)*c
os(x)/(cos(x)+1)^2)^(1/2)*2^(1/2)+cos(x)-1)/(-1+cos(x))))*(-1+cos(x))/sin(x
)^(3/2)/(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$
$$= \left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\Big(2i\cos(x)^2 + \big((i+1)\sqrt{2}\cos(x)-(i-1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+2\cos(x)\sin(x)-i\Big)$$
$$-\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\Big(2i\cos(x)^2 + \big(-(i+1)\sqrt{2}\cos(x)+(i-1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+2\cos(x)\sin(x)-i\Big)$$
$$-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\Big(-2i\cos(x)^2 + \big(-(i-1)\sqrt{2}\cos(x)+(i+1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+2\cos(x)\sin(x)+i\Big)$$
$$+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\Big(-2i\cos(x)^2 + \big((i-1)\sqrt{2}\cos(x)-(i+1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+2\cos(x)\sin(x)+i\Big)$$
$$-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\Big(\big((i+1)\sqrt{2}\cos(x)-(i-1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+1\Big)$$
$$+\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\Big(\big(-(i-1)\sqrt{2}\cos(x)+(i+1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+1\Big)$$
$$-\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{2}\log\Big(\big((i-1)\sqrt{2}\cos(x)-(i+1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+1\Big)$$
$$+\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2}\log\Big(\big(-(i+1)\sqrt{2}\cos(x)+(i-1)\sqrt{2}\sin(x)\big)\sqrt{\cos(x)}\sqrt{\sin(x)}+1\Big)$$

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] (1/16*I - 1/16)*sqrt(2)*log(2*I*cos(x)^2 + ((I + 1)*sqrt(2)*cos(x) - (I - 1
)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos(x)*sin(x) - I) - (1/16*
I - 1/16)*sqrt(2)*log(2*I*cos(x)^2 + (-(I + 1)*sqrt(2)*cos(x) + (I - 1)*sqr
t(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos(x)*sin(x) - I) - (1/16*I + 1
/16)*sqrt(2)*log(-2*I*cos(x)^2 + (-(I - 1)*sqrt(2)*cos(x) + (I + 1)*sqrt(2)
*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos(x)*sin(x) + I) + (1/16*I + 1/16)
*sqrt(2)*log(-2*I*cos(x)^2 + ((I - 1)*sqrt(2)*cos(x) - (I + 1)*sqrt(2)*sin(
x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos(x)*sin(x) + I) - (1/16*I + 1/16)*sqrt
(2)*log(((I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt
(sin(x)) + 1) + (1/16*I - 1/16)*sqrt(2)*log((-(I - 1)*sqrt(2)*cos(x) + (I +
```

```
1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 1) - (1/16*I - 1/16)*sqrt(2
)*log(((I - 1)*sqrt(2)*cos(x) - (I + 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(s
in(x)) + 1) + (1/16*I + 1/16)*sqrt(2)*log((-(I + 1)*sqrt(2)*cos(x) + (I - 1
)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 1)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

```
[In] integrate(sin(x)**(1/2)/cos(x)**(1/2),x)
[Out] Integral(sqrt(sin(x))/sqrt(cos(x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

Giac [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = -\frac{2\,\sqrt{\cos\left(x\right)}\,{\sin\left(x\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ {\cos\left(x\right)}^2\right)}{{\left({\sin\left(x\right)}^2\right)}^{3/4}}$$

```
[In] int(sin(x)^(1/2)/cos(x)^(1/2),x)
[Out] -(2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x)
^2)^(3/4)
```

3.290 $\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}}-\frac{3\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}}-\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)$$

```
[Out] -3/8*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+3/8*arctan(1+2^(1/
2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+3/16*ln(1-2^(1/2)*sin(x)^(1/2)/cos(x)
^(1/2)+tan(x))*2^(1/2)-3/16*ln(1+2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2)+tan(x))*
2^(1/2)-1/2*sin(x)^(3/2)*cos(x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2648, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{4\sqrt{2}}-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}+\frac{3\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}-\frac{3\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}$$

```
[In] Int[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*ArcTa
n[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*Log[1 - (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (3*Log[1 + (Sqrt[2]*
Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (Sqrt[Cos[x]]*Sin[x]^(3
/2))/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)+\frac{3}{4}\int\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)+\frac{3}{2}\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)-\frac{3}{4}\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&\quad+\frac{3}{4}\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)+\frac{3}{8}\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&\quad+\frac{3}{8}\text{Subst}\left(\int\frac{1}{1+\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)\\
&\quad+\frac{3\text{Subst}\left(\int\frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{8\sqrt{2}}+\frac{3\text{Subst}\left(\int\frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx,x,\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{8\sqrt{2}}\\
&= \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}}-\frac{3\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}}-\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)\\
&\quad+\frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}-\frac{3\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}
\end{aligned}
$$

$$= -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{3\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.27

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \frac{2\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(x)\right)\sin^{\frac{7}{2}}(x)}{7\cos^{\frac{3}{2}}(x)}$$

```
[In] Integrate[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[x]^2]*Sin[x]^(7/2
))/(7*Cos[x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(97) = 194.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.71

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,(\sin^3(x))+6\cos(x)\arctan\left(\frac{\sin(x)\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\sqrt{2}+\cos(x)-1}{-1+\cos(x)}\right)-6\cos(x)\arctan\left(\frac{-\sin(x)\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\sqrt{2}+\ldots}{-1+\cos(x)}\right.\right.}{}$

```
[In] int(sin(x)^(5/2)/cos(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*2^(1/2)*(4*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*sin(x)^3+6*cos(
x)*arctan((sin(x)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*2^(1/2)+cos(x)-1)/(-1+
cos(x)))-6*cos(x)*arctan((-sin(x)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*2^(1/2
)+cos(x)-1)/(-1+cos(x)))-3*cos(x)*ln(2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)
^(1/2)*cot(x)+2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*csc(x)+2*cot(x)+
2)+3*cos(x)*ln(-2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*cot(x)-2*2^(1/
2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*csc(x)+2*cot(x)+2)-6*arctan((sin(x)*(
sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*2^(1/2)+cos(x)-1)/(-1+cos(x)))+6*arctan((
```

```
-sin(x)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*2^(1/2)+cos(x)-1)/(-1+cos(x)))+3
*ln(2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*cot(x)+2*2^(1/2)*(sin(x)*c
os(x)/(cos(x)+1)^2)^(1/2)*csc(x)+2*cot(x)+2)-3*ln(-2*2^(1/2)*(sin(x)*cos(x)
/(cos(x)+1)^2)^(1/2)*cot(x)-2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*cs
c(x)+2*cot(x)+2))*cos(x)^(1/2)/(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)/sin(x)^(3
/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.20

$$\begin{aligned}
&\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx \\
&= -\frac{1}{2}\sqrt{\cos(x)}\sin(x)^{\frac{3}{2}} + \left(\frac{3}{64}i - \frac{3}{64}\right)\sqrt{2}\log\Big(2i\cos(x)^2 \\
&\quad + \left((i+1)\sqrt{2}\cos(x) - (i-1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 2\cos(x)\sin(x) - i\Big) \\
&\quad - \left(\frac{3}{64}i - \frac{3}{64}\right)\sqrt{2}\log\Big(2i\cos(x)^2 \\
&\quad + \left(-(i+1)\sqrt{2}\cos(x) + (i-1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 2\cos(x)\sin(x) - i\Big) \\
&\quad - \left(\frac{3}{64}i + \frac{3}{64}\right)\sqrt{2}\log\Big(-2i\cos(x)^2 \\
&\quad + \left(-(i-1)\sqrt{2}\cos(x) + (i+1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 2\cos(x)\sin(x) + i\Big) \\
&\quad + \left(\frac{3}{64}i + \frac{3}{64}\right)\sqrt{2}\log\Big(-2i\cos(x)^2 \\
&\quad + \left((i-1)\sqrt{2}\cos(x) - (i+1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 2\cos(x)\sin(x) + i\Big) \\
&\quad - \left(\frac{3}{64}i + \frac{3}{64}\right)\sqrt{2}\log\left(\left((i+1)\sqrt{2}\cos(x) - (i-1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 1\right) \\
&\quad + \left(\frac{3}{64}i - \frac{3}{64}\right)\sqrt{2}\log\Big(\left(-(i-1)\sqrt{2}\cos(x) + (i+1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} \\
&\quad + 1\Big) \\
&\quad - \left(\frac{3}{64}i - \frac{3}{64}\right)\sqrt{2}\log\left(\left((i-1)\sqrt{2}\cos(x) - (i+1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 1\right) \\
&\quad + \left(\frac{3}{64}i + \frac{3}{64}\right)\sqrt{2}\log\Big(\left(-(i+1)\sqrt{2}\cos(x) + (i-1)\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} \\
&\quad + 1\Big)
\end{aligned}$$

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(cos(x))*sin(x)^(3/2) + (3/64*I - 3/64)*sqrt(2)*log(2*I*cos(x)^2 +
 ((I + 1)*sqrt(2)*cos(x) - (I - 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)
) + 2*cos(x)*sin(x) - I) - (3/64*I - 3/64)*sqrt(2)*log(2*I*cos(x)^2 + (-(I
+ 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2
*cos(x)*sin(x) - I) - (3/64*I + 3/64)*sqrt(2)*log(-2*I*cos(x)^2 + (-(I - 1)
*sqrt(2)*cos(x) + (I + 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos
(x)*sin(x) + I) + (3/64*I + 3/64)*sqrt(2)*log(-2*I*cos(x)^2 + ((I - 1)*sqrt
(2)*cos(x) - (I + 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 2*cos(x)*s
in(x) + I) - (3/64*I + 3/64)*sqrt(2)*log(((I + 1)*sqrt(2)*cos(x) - (I - 1)*
sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 1) + (3/64*I - 3/64)*sqrt(2)*lo
g((-(I - 1)*sqrt(2)*cos(x) + (I + 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(
x)) + 1) - (3/64*I - 3/64)*sqrt(2)*log(((I - 1)*sqrt(2)*cos(x) - (I + 1)*sq
rt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 1) + (3/64*I + 3/64)*sqrt(2)*log(
(-(I + 1)*sqrt(2)*cos(x) + (I - 1)*sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)
) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(x)**(5/2)/cos(x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \int \frac{\sin\left(x\right)^{\frac{5}{2}}}{\sqrt{\cos\left(x\right)}}\,dx$$

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \int \frac{\sin{(x)}^{\frac{5}{2}}}{\sqrt{\cos{(x)}}}\,dx$$

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.17

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{2\sqrt{\cos{(x)}}\sin{(x)}^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ \cos{(x)}^2\right)}{\left(\sin{(x)}^2\right)^{7/4}}$$

```
[In] int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

```
[Out] -(2*cos(x)^(1/2)*sin(x)^(7/2)*hypergeom([-3/4, 1/4], 5/4, cos(x)^2))/(sin(x
)^2)^(7/4)
```

3.291 $\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{5d^4\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] 1/3*d*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2)/b/c+5/6*d^3*(d*cos(b*x+a))^
(1/2)*(c*sin(b*x+a))^(1/2)/b/c-5/12*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*c
os(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2649, 2653, 2720}

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{5d^4\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} + \frac{5d^3\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6bc} + \frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

```
[In] Int[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (5*d^3*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b*c) + (d*(d*Cos[a + b
*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*c) + (5*d^4*EllipticF[a - Pi/4 + b*x,
```

```
 2]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]
)
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc}+\frac{1}{6}\left(5d^2\right)\int\frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx\\
&= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc}+\frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc}\\
&\quad+\frac{1}{12}\left(5d^4\right)\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx\\
&= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc}+\frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc}\\
&\quad+\frac{\left(5d^4\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{12\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\
&= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc}+\frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc}\\
&\quad+\frac{5d^4\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2(d\cos(a+bx))^{7/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\sec}{bc}$$

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
[Out] (2*(d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 1/
4, 5/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(b*c)
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 1727, normalized size of antiderivative = 13.08

method	result	size
default	Expression too large to display	1727

```
[In] int((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/48/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*d^3/(c*sin(b*x+a))^(1/2)*(-6*I*(-cot(b*
x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/
2))+6*I*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(c
ot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2
+1/2*I,1/2*2^(1/2))+6*I*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x
+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*sec(b*x+a)-6*I*(-cot(b*x+a)+csc(b*x+a)+
1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*sec(b*x+a)+8
*2^(1/2)*cos(b*x+a)^2*sin(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*
x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x
+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+32*(-cot(b*x+a)+csc(b*x+a)+1
)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellip
ticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))-6*(-cot(b*x+a)+csc(b*x+a
)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*(-cot(b*
x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*
x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/
2))*sec(b*x+a)+32*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1
)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^
```

```
(1/2),1/2*2^(1/2))*sec(b*x+a)-6*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a
)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)
+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*sec(b*x+a)-3*(-sin(b*x+a)*cos(b
*x+a)/(1+cos(b*x+a))^2)^(1/2)*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(
b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a)
)^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))+3*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+
a))^2)^(1/2)*ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*c
ot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x
+a)+2-2*cot(b*x+a))+6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*arcta
n((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b
*x+a)+1)/(cos(b*x+a)-1))+6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*
arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+
cos(b*x+a)-1)/(cos(b*x+a)-1))+20*2^(1/2)*sin(b*x+a)-3*(-sin(b*x+a)*cos(b*x+
a)/(1+cos(b*x+a))^2)^(1/2)*ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2
)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))*sec(b*x+a)+3*(-sin(b*x+a)*cos(b*x+a)/(1+
cos(b*x+a))^2)^(1/2)*ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)
^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)
*csc(b*x+a)+2-2*cot(b*x+a))*sec(b*x+a)+6*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x
+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1))*sec(b*x+a)+6*(-sin(b*x+a)*cos(b
*x+a)/(1+cos(b*x+a))^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b
*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))*sec(b*x+a))
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^3*cos(b*x + a)^3/(c*si
n(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

```
[In] int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2), x)
```

3.292 $\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc} + \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] d*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/c-1/2*d^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2649, 2653, 2720}

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} + \frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi
/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a
 + b*x]])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc}+\frac{1}{2}d^2\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx\\ &= \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc}+\frac{\left(d^2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc}+\frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int\frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx=\frac{2d^2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, Sin[a + b*x
]^2]*Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.21

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\cos(bx+a)}\,d\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(bx-\right.\right.}{}$

```
[In] int((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*2^(1/2)*(d*cos(b*x+a))^(1/2)*d/(c*sin(b*x+a))^(1/2)*((-cot(b*x+a)+csc
(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1
/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-cot(b*x+a)+cs
c(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*sec(b*x+a)+2^(
1/2)*sin(b*x+a))
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*cos(b*x + a)/(c*sin(b*
x + a)), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(a+bx))^{\frac{3}{2}}}{\sqrt{c\sin(a+bx)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((d*cos(a + b*x))**(3/2)/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2), x)
```

3.293 $\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),
2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2653, 2720}

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

```
[In] Int[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(b*Sqrt[d*Cos[a + b*x
]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

$$= \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.13

method	result
default	$\frac{\sqrt{2}\,(1+\cos(bx+a))F\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{-\cot(bx+a)+\csc(bx}}{b\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}}$

```
[In] int(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/b*2^(1/2)*(1+cos(b*x+a))*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2
^(1/2))*(cot(b*x+a)-csc(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(-cot
(b*x+a)+csc(b*x+a)+1)^(1/2)/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = -\frac{\sqrt{i\,cd}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\sqrt{-i\,cd}F(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)|-$$
$$bcd$$

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
[Out] -(sqrt(I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt(
-I*c*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/(b*c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin\left(a+bx\right)}\sqrt{d\cos\left(a+bx\right)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(1/(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\,\cos\left(a+b\,x\right)}\,\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)), x)
```

3.294 $\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] 2/3*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(3/2)-2/3*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2651, 2653, 2720}

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} + \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Int[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt
[c*Sin[a + b*x]])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2} \\
&= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{\left(2\sqrt{\sin(2a+2bx)}\right)\int\frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}} \\
&= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int\frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},\sin^2(a+bx)\right)\sqrt{c\sin(}{bcd(d\cos(a+bx))^{3/2}}$$

```
[In] Integrate[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[a + b*x]^2]*
Sqrt[c*Sin[a + b*x]])/(b*c*d*(d*Cos[a + b*x])^(3/2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.13

method	result
default	$\frac{\sqrt{2}\left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\cos}{3b\sqrt{c\sin}}$

```
[In] int(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*2^(1/2)/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^2*(2*(-cot(b*x+a)
+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+
2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x
+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2
))+2^(1/2)*tan(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = -\frac{2\left(\sqrt{i\,cd}\cos\left(bx+a\right)^2 F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\sqrt{-i\,cd}\cos\left(bx+a\right)^2 F(\arcsin\left(\cos\left(\right.\right.}{3\,bcd^3\cos\left(bx+a\right)^2}$$

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(sqrt(I*c*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-I*c*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) -
I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))/(b*c*d^3*
cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*cos(b*x+a))**(5/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\,\cos(a+b\,x))^{5/2}\,\sqrt{c\,\sin(a+b\,x)}}\,dx$$

```
[In] int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)), x)
```

3.295 $\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{7bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(7/2)+4/7*(c*sin(b*x+a))^(1/2
)/b/c/d^3/(d*cos(b*x+a))^(3/2)-4/7*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi
+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^4/(d*co
s(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2651, 2653, 2720}

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{4\sqrt{\sin(2a+2bx)}\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{7bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[In] Int[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (4*Sqrt[c*Sin[a
 + b*x]])/(7*b*c*d^3*(d*Cos[a + b*x])^(3/2)) + (4*EllipticF[a - Pi/4 + b*x,
```

```
2]*Sqrt[Sin[2*a + 2*b*x]])/(7*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*
x]])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{6\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx}{7d^2}\\
&= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{7d^4}\\
&= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{\left(4\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{7d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\
&= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}+\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}\\
&\quad+\frac{4\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{7bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\frac{5}{4},\sin^2(a$$

$$bc(d\cos(a+bx))^{9/2}$$

```
[In] Integrate[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
[Out] (2*Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 11/4, 5/4,
Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*c*(d*Cos[a + b*x])^(9/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{2}\left(4\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)+1}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\cos\right.}{}$

```
[In] int(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/7/b*2^(1/2)/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^4*(4*(-cot(b*x+a)
+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)+
4*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(cot(b*x+a)-csc(b*x+a)+1)^(1/2)*(cot(b*x
+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2
))+2*2^(1/2)*tan(b*x+a)+2^(1/2)*tan(b*x+a)*sec(b*x+a)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx =$$

$$-\frac{2\left(2\sqrt{i\,cd}\cos(bx+a)^4F(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+2\sqrt{-i\,cd}\cos(bx+a)^4F(\arcsin(c\right.}{7\,bcd^5\cos(bx+a}$$

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] -2/7*(2*sqrt(I*c*d)*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1) + 2*sqrt(-I*c*d)*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a
) - I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*(2*cos(b*x + a)^2 + 1)*sqrt
(c*sin(b*x + a)))/(b*c*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*cos(b*x+a))**(9/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\,\cos\left(a+b\,x\right))^{9/2}\,\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)), x)
```

3.296 $\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 280

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^
(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)-1/2*arctan(1+2^(1/2)*c^(1/2)*(d*cos(b*x+a)
)^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)-1/4*ln(d^(1
/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^
(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)+1/4*ln(d^(1/2)+cot(b*x+a)*d^(1/2)+2^(1/2)*
c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2
)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+bx)+\sqrt{d}\right)}{2\sqrt{2}b\sqrt{c}}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*
Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c]) - (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[c]
*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c])
 - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d*Co
s[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c]) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sqrt[
c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2cd)\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b} - \frac{d\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}\end{aligned}$$

$$= -\frac{\sqrt{d}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2x}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{d\text{Subst}\left(\int \frac{1}{\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2bc} - \frac{d\text{Subst}\left(\int \frac{1}{\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}+x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2bc}$$

$$= -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}$$

$$= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{c\sin(a+bx)}}$$

[In] Integrate[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]

[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[a + b*x]^2]*Tan[a + b*x])/(b*Sqrt[c*Sin[a + b*x]])

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{2}\,\sin(bx+a)\left(\ln\left(-2\sqrt{2}\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\cot(bx+a)-2\sqrt{2}\sqrt{-\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\csc(bx+a)+2-2\cot(bx+a)\right)-2\arctan\left(\dots\right.\right.}{}$

[In] int((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/b*2^(1/2)*sin(b*x+a)*(ln(-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))-2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)+cos(b*x+a)-1)/(cos(b*x+a)-1))-ln(2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)+2*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2-2*cot(b*x+a))-2*arctan((sin(b*x+a)*2^(1/2)*(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1)))*(d*cos(b*x+a))^(1/2)/(1+cos(b*x+a))/(c*sin(b*x+a))^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \text{Too large to display}$$

[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")

[Out] 1/8*(-d^2/(b^4*c^2))^(1/4)*log(2*b^2*c*d*sqrt(-d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) - 2*d^2*cos(b*x + a)^2 + 2*(b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + b*d*(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + d^2) - 1/8*(-d^2/(b^4*c^2))^(1/4)*log(2*b^2*c*d*sqrt(-d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) - 2*d^2*cos(b*x + a)^2 - 2*(b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + b*d*(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + d^2) + 1/8*I*(-d^2/(b^4*c^2))^(1/4)*log(-2*b^2*c*d*sqrt(-d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) - 2*d

```
^2*cos(b*x + a)^2 - 2*(I*b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) - I*b*d*
(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a)) + d^2) - 1/8*I*(-d^2/(b^4*c^2))^(1/4)*log(-2*b^2*c*d*sqrt(-d^2/(b^4*c^2
))*cos(b*x + a)*sin(b*x + a) - 2*d^2*cos(b*x + a)^2 - 2*(-I*b^3*c*(-d^2/(b^
4*c^2))^(3/4)*cos(b*x + a) + I*b*d*(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + d^2) + 1/8*(-d^2/(b^4*c^2))^(1/4)*
log(2*(b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) - b*d*(-d^2/(b^4*c^2))^(1/
4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - d^2) - 1/8*(-d
^2/(b^4*c^2))^(1/4)*log(-2*(b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) - b*d
*(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
 a)) - d^2) + 1/8*I*(-d^2/(b^4*c^2))^(1/4)*log(-2*(I*b^3*c*(-d^2/(b^4*c^2))
^(3/4)*cos(b*x + a) + I*b*d*(-d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos
(b*x + a))*sqrt(c*sin(b*x + a)) - d^2) - 1/8*I*(-d^2/(b^4*c^2))^(1/4)*log(-
2*(-I*b^3*c*(-d^2/(b^4*c^2))^(3/4)*cos(b*x + a) - I*b*d*(-d^2/(b^4*c^2))^(1
/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - d^2)
```

Sympy [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos\left(a+bx\right)}}{\sqrt{c\sin\left(a+bx\right)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*cos(a + b*x))/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos\left(bx+a\right)}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos(bx+a)}}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\,\cos(a+b\,x)}}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2), x)
```

3.297 $\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2643}

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

```
[In] Int[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
[Out] (2*Sqrt[c*Sin[a + b*x]])/(b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sin(2(a+bx))}{b(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
[Out] Sin[2*(a + b*x)]/(b*(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sin(bx+a)}{bd\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}}$	35

```
[In] int(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*sin(b*x+a)/d/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{bcd^2\cos(bx+a)}$$

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
[Out] 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*c*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(a+bx)}}\,(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(1/(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos{(bx+a)})^{\frac{3}{2}}\sqrt{c\sin{(bx+a)}}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos{(bx+a)})^{\frac{3}{2}}\sqrt{c\sin{(bx+a)}}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\,\sin{(a+b\,x)}}}{b\,c\,d\,\sqrt{d\,\cos{(a+b\,x)}}}$$

```
[In] int(1/((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] (2*(c*sin(a + b*x))^(1/2))/(b*c*d*(d*cos(a + b*x))^(1/2))
```

3.298 $\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/5*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(5/2)+8/5*(c*sin(b*x+a))^(1/2
)/b/c/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2651, 2643}

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}} + \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[In] Int[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a
 + b*x]])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{4\int\frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{5d^2}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int\frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2(3+2\cos(2(a+bx)))\tan(a+bx)}{5bd^2(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
[Out] (2*(3 + 2*Cos[2*(a + b*x)])*Tan[a + b*x])/(5*b*d^2*(d*Cos[a + b*x])^(3/2)*S
qrt[c*Sin[a + b*x]])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\frac{8\sin(bx+a)}{5}+\frac{2\tan(bx+a)\sec(bx+a)}{5}}{b\,d^3\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}}$	51

```
[In] int(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b/d^3/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)*(4*sin(b*x+a)+tan(b*x+a
)*sec(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(4\cos(bx+a)^2+1\right)\sqrt{c\sin(bx+a)}}{5\,bcd^4\cos(bx+a)^3}$$

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
[Out] 2/5*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))/(b*c*d
^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{8\sqrt{c\,\sin(a+b\,x)}\,(5\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+4)}{5\,b\,c\,d^3\,\sqrt{d\,\cos(a+b\,x)}\,(4\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3)}$$

```
[In] int(1/((d*cos(a + b*x))^(7/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] (8*(c*sin(a + b*x))^(1/2)*(5*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 4))/(5*b
*c*d^3*(d*cos(a + b*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

3.299 $\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}} + \frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}} + \frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/9*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(9/2)+16/45*(c*sin(b*x+a))^(1
/2)/b/c/d^3/(d*cos(b*x+a))^(5/2)+64/45*(c*sin(b*x+a))^(1/2)/b/c/d^5/(d*cos(
b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2651, 2643}

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}} + \frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}} + \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

```
[In] Int[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (16*Sqrt[c*Sin[
a + b*x]])/(45*b*c*d^3*(d*Cos[a + b*x])^(5/2)) + (64*Sqrt[c*Sin[a + b*x]])/
(45*b*c*d^5*Sqrt[d*Cos[a + b*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{8\int\frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{32\int\frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{45d^4}\\ &= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

$$\int\frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{d\cos(a+bx)}(21+20\cos(2(a+bx))+4\cos(4(a+bx)))\sec^5($$
$$45bcd^6$$

```
[In] Integrate[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(21 + 20*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(45*b*c*d^6)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(32\left(\cos^{4}\left(bx+a\right)\right)+8\left(\cos^{2}\left(bx+a\right)\right)+5\right)\tan\left(bx+a\right)\left(\sec^{3}\left(bx+a\right)\right)}{45 b\, d^{5}\sqrt{c\sin\left(bx+a\right)}\,\sqrt{d\cos\left(bx+a\right)}}$	65

```
[In] int(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/b*(32*cos(b*x+a)^4+8*cos(b*x+a)^2+5)/d^5/(c*sin(b*x+a))^(1/2)/(d*cos(b
*x+a))^(1/2)*tan(b*x+a)*sec(b*x+a)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\left(32\cos\left(bx+a\right)^{4}+8\cos\left(bx+a\right)^{2}+5\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(b\right.}}{45\,bcd^{6}\cos\left(bx+a\right)^{5}}$$

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(32*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 5)*sqrt(d*cos(b*x + a))*sqrt(c
*sin(b*x + a))/(b*c*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*cos(b*x+a))**(11/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{32\sqrt{c\,\sin(a+b\,x)}\,(162\,\cos(2\,a+2\,b\,x)+73\,\cos(4\,a+4\,b\,x)+\cdots}{45\,b\,c\,d^5\,\sqrt{d\,\cos(a+b\,x)}\,(56\,\cos(2\,a+2\,b\,x)+28\,\cos(4\,a+4\,b\cdots}$$

```
[In] int(1/((d*cos(a + b*x))^(11/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] (32*(c*sin(a + b*x))^(1/2)*(162*cos(2*a + 2*b*x) + 73*cos(4*a + 4*b*x) + 18
*cos(6*a + 6*b*x) + 2*cos(8*a + 8*b*x) + 105))/(45*b*c*d^5*(d*cos(a + b*x))
^(1/2)*(56*cos(2*a + 2*b*x) + 28*cos(4*a + 4*b*x) + 8*cos(6*a + 6*b*x) + co
s(8*a + 8*b*x) + 35))
```

3.300 $\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/2*arc
tan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x
+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a
)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{2655, 303, 1176, 631, 210, 1179, 642\}$

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b} - \frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b} + \frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}\\
&= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}\\
&= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}\\
&\quad - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

```
[In] Integrate[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[a + b*x]^2]*
Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(138) = 276.

Time = 0.32 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.97

method	result
default	$\frac{\sqrt{2}\,(\cos(bx+a)-1)\left(\sqrt{\cos}(bx+a)\right)\left(\ln\left(-2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\cot(bx+a)-2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(1+\cos(bx+a))^2}}\csc(bx+a)+2+2\cot(bx+\right.\right.}{}$

```
[In] int(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*2^(1/2)*(cos(b*x+a)-1)*cos(b*x+a)^(1/2)*(ln(-2*2^(1/2)*(sin(b*x+a)*co
s(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*x+a)-2*2^(1/2)*(sin(b*x+a)*cos(b*x+a
)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2+2*cot(b*x+a))-2*arctan((sin(b*x+a)*(
sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2)+cos(b*x+a)-1)/(cos(b*
x+a)-1))-ln(2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*cot(b*
x+a)+2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)*csc(b*x+a)+2+
2*cot(b*x+a))-2*arctan((sin(b*x+a)*(sin(b*x+a)*cos(b*x+a)/(1+cos(b*x+a))^2)
^(1/2)*2^(1/2)-cos(b*x+a)+1)/(cos(b*x+a)-1)))/sin(b*x+a)^(3/2)/(sin(b*x+a)*
cos(b*x+a)/(1+cos(b*x+a))^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.74

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-1/b^4)^(1/4)*log(2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(b*x + a) - 2*cos
(b*x + a)^2 + 2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) + b*(-1/b^4)^(1/4)*sin(b*x
 + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) - 1/8*(-1/b^4)^(1/4)*log(
2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(b^3*(-
1/b^4)^(3/4)*cos(b*x + a) + b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a
))*sqrt(sin(b*x + a)) + 1) + 1/8*I*(-1/b^4)^(1/4)*log(-2*b^2*sqrt(-1/b^4)*c
os(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b^3*(-1/b^4)^(3/4)*cos(b
*x + a) - I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x
+ a)) + 1) - 1/8*I*(-1/b^4)^(1/4)*log(-2*b^2*sqrt(-1/b^4)*cos(b*x + a)*sin(
b*x + a) - 2*cos(b*x + a)^2 - 2*(-I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(
-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 1/
8*(-1/b^4)^(1/4)*log(2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^(1/4)*
sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) - 1/8*(-1/b^4)^(1/
4)*log(-2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^(1/4)*sin(b*x + a))
*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) + 1/8*I*(-1/b^4)^(1/4)*log(-2*(
I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(c
os(b*x + a))*sqrt(sin(b*x + a)) - 1) - 1/8*I*(-1/b^4)^(1/4)*log(-2*(-I*b^3*
(-1/b^4)^(3/4)*cos(b*x + a) - I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x
 + a))*sqrt(sin(b*x + a)) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos{(a+bx)}}}{\sqrt{\sin{(a+bx)}}}\,dx$$

```
[In] integrate(cos(b*x+a)**(1/2)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x))/sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\cos\left(a+b\,x\right)^{3/2}\,\sqrt{\sin\left(a+b\,x\right)}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{4};\ \frac{7}{4};\ \cos\left(a+b\,x\right)^2\right)}{3\,b\,{\left(\sin\left(a+b\,x\right)^2\right)}^{1/4}}$$

```
[In] int(cos(a + b*x)^(1/2)/sin(a + b*x)^(1/2),x)
[Out] -(2*cos(a + b*x)^(3/2)*sin(a + b*x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(a
+ b*x)^2))/(3*b*(sin(a + b*x)^2)^(1/4))
```

3.301 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))/b*2^(1/2)-1/2*arcta
n(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1-2^(1/2)*s
in(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan(b*x+a))/b*2^(1/2)+1/4*ln(1+2^(1/2)*sin
(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan(b*x+a))/b*2^(1/2)-2*cos(b*x+a)^(1/2)/b/s
in(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

$= \{2647, 2654, 303, 1176, 631, 210, 1179, 642\}$

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b} + \frac{\log\left(\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2*Sq
rt[2]*b) + Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a
+ b*x]]/(2*Sqrt[2]*b) - (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx \\
&= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} \\
&= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}
\end{aligned}$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2\sqrt{2}b}$$

$$= -\frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\sin^2(a+bx)\right)}{b\cos^{\frac{3}{2}}(a+bx)\sqrt{\sin(a+bx)}}$$

```
[In] Integrate[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(3/2)*Sqrt[Sin[a + b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.27

method	result
default	$\frac{\sqrt{2}\left(-\frac{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)-1}{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)+1}\right)^{\frac{3}{2}}\left(\ln\left(-\frac{(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{-(1-\cos(bx+a))\left((1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)-1\right)\csc(bx+a)}}{1-\cos(bx+a)}\right.\right.}{}$

```
[In] int(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*2^(1/2)/(1/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(csc(b*x+a)-cot(b*x+a)))
^(3/2)*(-((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/((1-cos(b*x+a))^2*csc(b*x+a)^2+1
))^(3/2)*(ln(-1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(-(1-cos(b*x+
a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)-2+2*cos(
b*x+a)-sin(b*x+a)))*(csc(b*x+a)-cot(b*x+a))-2*arctan(1/(1-cos(b*x+a))*((-(1
-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)
+cos(b*x+a)-1))*(csc(b*x+a)-cot(b*x+a))-ln(1/(1-cos(b*x+a))*(-(1-cos(b*x+a)
)^2*csc(b*x+a)+2*(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x
+a))^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)+sin(b*x+a)))*(csc(b*x+a)-cot(b*x+a))-2
*arctan(1/(1-cos(b*x+a))*((-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1
)*csc(b*x+a))^(1/2)*sin(b*x+a)+1-cos(b*x+a)))*(csc(b*x+a)-cot(b*x+a))+4*(-(
1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2))/((1-cos(
b*x+a))^2*csc(b*x+a)^2-1)/(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1
)*csc(b*x+a))^(1/2)*(csc(b*x+a)-cot(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 766, normalized size of antiderivative = 3.85

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*(-1/b^4)^(1/4)*log(1/2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)
^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1/2*cos(b*x +
a)*sin(b*x + a) + 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^4))*sin(b*x +
a) - b*(-1/b^4)^(1/4)*log(-1/2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4
)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1/2*cos(b*x +
 a)*sin(b*x + a) + 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^4))*sin(b*x +
```

```
 a) - I*b*(-1/b^4)^(1/4)*log(1/2*(I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(
-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1/2*cos
(b*x + a)*sin(b*x + a) - 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^4))*sin
(b*x + a) + I*b*(-1/b^4)^(1/4)*log(1/2*(-I*b^3*(-1/b^4)^(3/4)*cos(b*x + a)
- I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) -
1/2*cos(b*x + a)*sin(b*x + a) - 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^
4))*sin(b*x + a) - b*(-1/b^4)^(1/4)*log(2*(b^3*(-1/b^4)^(3/4)*sin(b*x + a)
- b*(-1/b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)
*sin(b*x + a) + b*(-1/b^4)^(1/4)*log(-2*(b^3*(-1/b^4)^(3/4)*sin(b*x + a) -
b*(-1/b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*s
in(b*x + a) - I*b*(-1/b^4)^(1/4)*log(-2*(I*b^3*(-1/b^4)^(3/4)*sin(b*x + a)
+ I*b*(-1/b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) +
1)*sin(b*x + a) + I*b*(-1/b^4)^(1/4)*log(-2*(-I*b^3*(-1/b^4)^(3/4)*sin(b*x
+ a) - I*b*(-1/b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a
)) + 1)*sin(b*x + a) - 16*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))/(b*sin(b*x
 + a))
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(cos(b*x+a)**(3/2)/sin(b*x+a)**(3/2),x)
[Out] Integral(cos(a + b*x)**(3/2)/sin(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{\cos(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2\cos(a+b\,x)^{5/2}\,\left(\sin(a+b\,x)^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{5}{4};\ \frac{9}{4};\ \cos(a+b\,x)^2\right)}{5\,b\,\sqrt{\sin(a+b\,x)}}$$

```
[In] int(cos(a + b*x)^(3/2)/sin(a + b*x)^(3/2),x)
```

```
[Out] -(2*cos(a + b*x)^(5/2)*(sin(a + b*x)^2)^(1/4)*hypergeom([5/4, 5/4], 9/4, co
s(a + b*x)^2))/(5*b*sin(a + b*x)^(1/2))
```

3.302 $\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)} dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}+\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}-\frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*cos(b*x+a)^(3/2)/b/sin(b*x+a)^(3/2)+1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(
1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(
b*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*
x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x+a)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+
a)^(1/2))/b*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used

= {2647, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b} - \frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b} - \frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2
*Sqrt[2]*b) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[
a + b*x]]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\int\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx\\
&= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}+\frac{2\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}\\
&= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}+\frac{\text{Subst}\left(\int\frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}
\end{aligned}$$

$$= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$
$$+ \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$

$$= \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$
$$- \frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}$$
$$+ \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b}$$
$$- \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{3}{4},\frac{1}{4},\sin^2(a+bx)\right)}{3b\sqrt{\cos(a+bx)}\sin^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
[Out] (-2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, Sin[a + b*x]^
2])/(3*b*Sqrt[Cos[a + b*x]]*Sin[a + b*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(159) = 318$.

Time = 4.13 (sec) , antiderivative size = 737, normalized size of antiderivative = 3.67

method	result
default	$\frac{\sqrt{2}\,(1-\cos(bx+a))\left(-\frac{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)-1}{(1-\cos(bx+a))^2\left(\csc^2(bx+a)\right)+1}\right)^{\frac{5}{2}}\left(2\sqrt{-(1-\cos(bx+a))\left((1-\cos(bx+a))^2(\csc^2(bx+a))-1\right)\csc(bx+a)}\,(1-\cos\right.}{}$

```
[In] int(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*2^(1/2)/(1/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(csc(b*x+a)-cot(b*x+a))
)^(5/2)*(1-cos(b*x+a))*(-((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/((1-cos(b*x+a))^
2*csc(b*x+a)^2+1))^(5/2)*(2*(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2
-1)*csc(b*x+a))^(1/2)*(1-cos(b*x+a))^2*csc(b*x+a)^2-3*ln(1/(1-cos(b*x+a))*(
-(1-cos(b*x+a))^2*csc(b*x+a)+2*(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a
)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)+sin(b*x+a)))*(1-cos(b*x+
a))^2*csc(b*x+a)^2+6*arctan(1/(1-cos(b*x+a))*((-(1-cos(b*x+a))*((1-cos(b*x+
a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+1-cos(b*x+a)))*(1-cos(b*
x+a))^2*csc(b*x+a)^2+3*ln(-1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*
(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*
x+a)-2+2*cos(b*x+a)-sin(b*x+a)))*(1-cos(b*x+a))^2*csc(b*x+a)^2+6*arctan(1/(
1-cos(b*x+a))*((-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a
))^(1/2)*sin(b*x+a)+cos(b*x+a)-1))*(1-cos(b*x+a))^2*csc(b*x+a)^2-2*(-(1-cos
(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2))/((1-cos(b*x+a
))^2*csc(b*x+a)^2-1)^2/(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*c
sc(b*x+a))^(1/2)*csc(b*x+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 803, normalized size of antiderivative = 4.00

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(-I*b*cos(b*x + a)^2 + I*b)*(-1/b^4)^(1/4)*log(2*b^2*sqrt(-1/b^4)*
cos(b*x + a)*sin(b*x + a) + 2*cos(b*x + a)^2 - 2*(I*b^3*(-1/b^4)^(3/4)*cos(
b*x + a) - I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x
 + a)) - 1) + 3*(I*b*cos(b*x + a)^2 - I*b)*(-1/b^4)^(1/4)*log(2*b^2*sqrt(-1
```

```
/b^4)*cos(b*x + a)*sin(b*x + a) + 2*cos(b*x + a)^2 - 2*(-I*b^3*(-1/b^4)^(3/
4)*cos(b*x + a) + I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(
sin(b*x + a)) - 1) - 3*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(-2*b^2*sqr
t(-1/b^4)*cos(b*x + a)*sin(b*x + a) + 2*cos(b*x + a)^2 + 2*(b^3*(-1/b^4)^(3
/4)*cos(b*x + a) + b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(s
in(b*x + a)) - 1) + 3*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(-2*b^2*sqrt
(-1/b^4)*cos(b*x + a)*sin(b*x + a) + 2*cos(b*x + a)^2 - 2*(b^3*(-1/b^4)^(3/
4)*cos(b*x + a) + b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(si
n(b*x + a)) - 1) + 3*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(2*(b^3*(-1/b
^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*
sqrt(sin(b*x + a)) - 1) - 3*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(-2*(b
^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*
x + a))*sqrt(sin(b*x + a)) - 1) + 3*(I*b*cos(b*x + a)^2 - I*b)*(-1/b^4)^(1/
4)*log(-2*(I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(-1/b^4)^(1/4)*sin(b*x +
 a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) + 3*(-I*b*cos(b*x + a)^2 +
I*b)*(-1/b^4)^(1/4)*log(-2*(-I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) - I*b*(-1/b^
4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 1) - 16*cos(
b*x + a)^(3/2)*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**(5/2)/sin(b*x+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{5}{2}}}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cos{(bx+a)}^{\frac{5}{2}}}{\sin{(bx+a)}^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cos{(a+b\,x)}^{7/2}\,{\left(\sin{(a+b\,x)}^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{7}{4},\frac{7}{4};\ \frac{11}{4};\ \cos{(a+b\,x)}^2\right)}{7\,b\sin{(a+b\,x)}^{3/2}}$$

```
[In] int(cos(a + b*x)^(5/2)/sin(a + b*x)^(5/2),x)
[Out] -(2*cos(a + b*x)^(7/2)*(sin(a + b*x)^2)^(3/4)*hypergeom([7/4, 7/4], 11/4, c
os(a + b*x)^2))/(7*b*sin(a + b*x)^(3/2))
```

3.303 $\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -2/5*cos(b*x+a)^(5/2)/b/sin(b*x+a)^(5/2)-1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1
/2)/cos(b*x+a)^(1/2))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b
*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+t
an(b*x+a))/b*2^(1/2)-1/4*ln(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan
(b*x+a))/b*2^(1/2)+2*cos(b*x+a)^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2647, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}+\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b}-\frac{\log\left(\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[In] Int[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2
*Sqrt[2]*b) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan
[a + b*x]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(5/2))/(5*b*Sin[a + b*x]^(5/2))
+ (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx \\ &= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx\end{aligned}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2\sqrt{2}b}$$

$$= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}$$

$$= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{5}{4},-\frac{1}{4},\sin^2(a+bx)\right)}{5b\cos^{\frac{3}{2}}(a+bx)\sin^{\frac{5}{2}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, Sin[a + b*x]
^2])/(5*b*Cos[a + b*x]^(3/2)*Sin[a + b*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(181) = 362.

Time = 0.37 (sec) , antiderivative size = 799, normalized size of antiderivative = 3.54

method	result	size
default	Expression too large to display	799

```
[In] int(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/20/b*2^(1/2)/(1/((1-cos(b*x+a))^2*csc(b*x+a)^2+1)*(csc(b*x+a)-cot(b*x+a))
)^(7/2)*(1-cos(b*x+a))*(-((1-cos(b*x+a))^2*csc(b*x+a)^2-1)/((1-cos(b*x+a))^
2*csc(b*x+a)^2+1))^(7/2)*((-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1
)*csc(b*x+a))^(1/2)*(1-cos(b*x+a))^4*csc(b*x+a)^4+5*ln(1/(1-cos(b*x+a))*(-(
1-cos(b*x+a))^2*csc(b*x+a)+2*(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^
2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)+sin(b*x+a)))*(1-cos(b*x+a)
)^3*csc(b*x+a)^3+10*arctan(1/(1-cos(b*x+a))*((-(1-cos(b*x+a))*((1-cos(b*x+a
))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x+a)+1-cos(b*x+a)))*(1-cos(b*x
+a))^3*csc(b*x+a)^3-5*ln(-1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(
-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*sin(b*x
+a)-2+2*cos(b*x+a)-sin(b*x+a)))*(1-cos(b*x+a))^3*csc(b*x+a)^3+10*arctan(1/(
1-cos(b*x+a))*((-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a
))^(1/2)*sin(b*x+a)+cos(b*x+a)-1))*(1-cos(b*x+a))^3*csc(b*x+a)^3-22*(-(1-co
s(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*(1-cos(b*x+a)
)^2*csc(b*x+a)^2+(-(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*csc(b*x
+a))^(1/2))/((1-cos(b*x+a))^2*csc(b*x+a)^2-1)^3/(-(1-cos(b*x+a))*((1-cos(b*
x+a))^2*csc(b*x+a)^2-1)*csc(b*x+a))^(1/2)*csc(b*x+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.99

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40*(5*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(1/2*(b^3*(-1/b^4)^(3/4)*c
os(b*x + a) - b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*
x + a)) + 1/2*cos(b*x + a)*sin(b*x + a) - 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*
sqrt(-1/b^4))*sin(b*x + a) - 5*(b*cos(b*x + a)^2 - b)*(-1/b^4)^(1/4)*log(-1
/2*(b^3*(-1/b^4)^(3/4)*cos(b*x + a) - b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(c
os(b*x + a))*sqrt(sin(b*x + a)) + 1/2*cos(b*x + a)*sin(b*x + a) - 1/4*(2*b^
2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^4))*sin(b*x + a) - 5*(I*b*cos(b*x + a)^2
- I*b)*(-1/b^4)^(1/4)*log(1/2*(I*b^3*(-1/b^4)^(3/4)*cos(b*x + a) + I*b*(-1/
b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1/2*cos(b*
x + a)*sin(b*x + a) + 1/4*(2*b^2*cos(b*x + a)^2 - b^2)*sqrt(-1/b^4))*sin(b*
x + a) - 5*(-I*b*cos(b*x + a)^2 + I*b)*(-1/b^4)^(1/4)*log(1/2*(-I*b^3*(-1/b
^4)^(3/4)*cos(b*x + a) - I*b*(-1/b^4)^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a)
)*sqrt(sin(b*x + a)) + 1/2*cos(b*x + a)*sin(b*x + a) + 1/4*(2*b^2*cos(b*x +
 a)^2 - b^2)*sqrt(-1/b^4))*sin(b*x + a) + 5*(b*cos(b*x + a)^2 - b)*(-1/b^4)
^(1/4)*log(2*(b^3*(-1/b^4)^(3/4)*sin(b*x + a) - b*(-1/b^4)^(1/4)*cos(b*x +
a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sin(b*x + a) - 5*(b*cos(b*x
+ a)^2 - b)*(-1/b^4)^(1/4)*log(-2*(b^3*(-1/b^4)^(3/4)*sin(b*x + a) - b*(-1/
b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sin(b*x
 + a) - 5*(-I*b*cos(b*x + a)^2 + I*b)*(-1/b^4)^(1/4)*log(-2*(I*b^3*(-1/b^4)
^(3/4)*sin(b*x + a) + I*b*(-1/b^4)^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*s
qrt(sin(b*x + a)) + 1)*sin(b*x + a) - 5*(I*b*cos(b*x + a)^2 - I*b)*(-1/b^4)
^(1/4)*log(-2*(-I*b^3*(-1/b^4)^(3/4)*sin(b*x + a) - I*b*(-1/b^4)^(1/4)*cos(
b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sin(b*x + a) + 16*(6*c
os(b*x + a)^2 - 5)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))/((b*cos(b*x + a)^
2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**(7/2)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{7}{2}}}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(7/2)/sin(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{7}{2}}}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(7/2)/sin(b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.19

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos\left(a+b\,x\right)^{9/2}\,{\left(\sin\left(a+b\,x\right)^2\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{9}{4},\frac{9}{4};\ \frac{13}{4};\ \cos\left(a+b\,x\right)^2\right)}{9\,b\sin\left(a+b\,x\right)^{5/2}}$$

```
[In] int(cos(a + b*x)^(7/2)/sin(a + b*x)^(7/2),x)
[Out] -(2*cos(a + b*x)^(9/2)*(sin(a + b*x)^2)^(5/4)*hypergeom([9/4, 9/4], 13/4, c
os(a + b*x)^2))/(9*b*sin(a + b*x)^(5/2))
```

3.304 $\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([-3/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F]

$$\int \left(\cos^4(fx+e)\right)(b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Sympy [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \sqrt[3]{b\sin{(e+fx)}}\cos^4{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**4, x)
```

Maxima [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin{(fx+e)})^{\frac{1}{3}}\cos{(fx+e)}^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin{(fx+e)})^{\frac{1}{3}}\cos{(fx+e)}^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \cos{(e+f\,x)}^4\,(b\,\sin{(e+f\,x)})^{1/3}\,dx$$

```
[In] int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3), x)
```

3.305 $\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([-1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2657}

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F]

$$\int \left(\cos^2(fx+e)\right)(b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Sympy [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \sqrt[3]{b\sin{(e+fx)}}\cos^2{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin{(fx+e)})^{\frac{1}{3}}\cos{(fx+e)}^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin{(fx+e)})^{\frac{1}{3}}\cos{(fx+e)}^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \cos{(e+f\,x)}^2\,(b\,\sin{(e+f\,x)})^{1/3}\,dx$$

```
[In] int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3), x)
```

3.306 $\int \sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

```
[In] Integrate[(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F]

$$\int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int \sqrt[3]{b\sin{(e+fx)}}\,dx$$

```
[In] integrate((b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\,\sin{(e+f\,x)})^{1/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(1/3), x)
```

3.307 $\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

```
[Out] 3/4*hypergeom([2/3, 3/2],[5/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(4/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2657\}$

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)}{4bf}$$

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral
$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$
$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F]

$$\int \left(\sec^2(fx+e)\right)(b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int \sqrt[3]{b\sin{(e+fx)}}\sec^2{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3)*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{1}{3}}\sec{(fx+e)}^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{1}{3}}\sec{(fx+e)}^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int \frac{(b\,\sin{(e+f\,x)})^{1/3}}{\cos{(e+f\,x)}^2}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2,x)
[Out] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2, x)
```

3.308 $\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

```
[Out] 3/4*hypergeom([2/3, 5/2],[5/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(4/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2657}

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)}{4bf}$$

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral
$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$
$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Maple [F]

$$\int \left(\sec^4(fx+e)\right)(b\sin(fx+e))^{\frac{1}{3}}\,dx$$

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int \frac{(b\sin(e+f\,x))^{1/3}}{\cos(e+f\,x)^4}\,dx$$

```
[In] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4,x)
[Out] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4, x)
```

3.309 $\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([-3/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2657\}$

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F]

$$\int \left(\cos^4(fx+e)\right)(b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^4\,dx$$

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \cos(e+f\,x)^4\,(b\,\sin(e+f\,x))^{5/3}\,dx$$

```
[In] int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3), x)
```

3.310 $\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([-1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

<u>Rule 2657</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F]

$$\int \left(\cos^2(fx+e)\right)(b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^2\,dx$$

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{5/3}\,dx$$

```
[In] int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3), x)
```

3.311 $\int (b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\int (b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

```
[In] Integrate[(b*Sin[e + f*x])^(5/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F]

$$\int (b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] int((b*sin(f*x+e))^(5/3),x)
[Out] int((b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*b*sin(f*x + e), x)
```

Sympy [F]

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\sin{(e+fx)})^{\frac{5}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))**(5/3),x)
[Out] Integral((b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\sin{(fx+e)})^{\frac{5}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\sin{(fx+e)})^{\frac{5}{3}}\,dx$$

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\,\sin{(e+f\,x)})^{5/3}\,dx$$

```
[In] int((b*sin(e + f*x))^(5/3),x)
[Out] int((b*sin(e + f*x))^(5/3), x)
```

3.312 $\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

```
[Out] 3/8*hypergeom([4/3, 3/2],[7/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(8/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)}{8bf}$$

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral

$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F]

$$\int \left(\sec^2(fx+e)\right)(b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \frac{(b\sin(e+fx))^{5/3}}{\cos(e+fx)^2}\,dx$$

```
[In] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2,x)
[Out] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2, x)
```

3.313 $\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

```
[Out] 3/8*hypergeom([4/3, 5/2],[7/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(8/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)}{8bf}$$

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Maple [F]

$$\int \left(\sec^4(fx+e)\right)(b\sin(fx+e))^{\frac{5}{3}}\,dx$$

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^4\,dx$$

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \frac{(b\sin(e+f\,x))^{5/3}}{\cos(e+f\,x)^4}\,dx$$

```
[In] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4,x)
[Out] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4, x)
```

3.314 $\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([-3/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2657}

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{\cos^4(fx+e)}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

```
[In] int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3), x)
```

3.315 $\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([-1/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{\cos^2(fx+e)}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos^2{(e+fx)}}{\sqrt[3]{b\sin{(e+fx)}}}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(cos(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos{(e+f\,x)}^2}{(b\,\sin{(e+f\,x)})^{1/3}}\,dx$$

```
[In] int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3), x)
```

3.316 $\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([1/3, 1/2],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[In] Int[(b*Sin[e + f*x])^(-1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(-1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{1}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(1/(b*sin(f*x+e))^(1/3),x)
[Out] int(1/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int(1/(b*sin(e + f*x))^(1/3),x)
[Out] int(1/(b*sin(e + f*x))^(1/3), x)
```

3.317 $\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

```
[Out] 3/2*hypergeom([1/3, 3/2],[4/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(2/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf}$$

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral

$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{\sec^2(fx+e)}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{1/3}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)),x)
[Out] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)), x)
```

3.318 $\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

```
[Out] 3/2*hypergeom([1/3, 5/2],[4/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(2/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf}$$

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral

$$= \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{\sec^4(fx+e)}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec^4\left(e+fx\right)}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(sec(e + f*x)**4/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^4\,\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

```
[In] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)),x)
[Out] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)), x)
```

3.319 $\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-3/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e
))^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{\cos^4(fx+e)}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

```
[In] int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3), x)
```

3.320 $\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}} \, dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-1/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e
))^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}} \, dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{\cos^2(fx+e)}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos^2{(e+fx)}}{(b\sin{(e+fx)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
[Out] Integral(cos(e + f*x)**2/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos{(e+f\,x)}^2}{{(b\,\sin{(e+f\,x)})}^{5/3}}\,dx$$

```
[In] int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3), x)
```

3.321 $\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-1/3, 1/2],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e)
)^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

```
[In] Int[(b*Sin[e + f*x])^(-5/3),x]
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2])/(2*b*f*
Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

```
[In] Integrate[(b*Sin[e + f*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int(1/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(1/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)/(b^2*cos(f*x + e)^2 - b^2), x)
```

Sympy [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(e+fx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(-5/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\,\sin(e+f\,x))^{5/3}}\,dx$$

```
[In] int(1/(b*sin(e + f*x))^(5/3),x)
[Out] int(1/(b*sin(e + f*x))^(5/3), x)
```

3.322 $\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*hypergeom([-1/3, 3/2],[2/3],sin(f*x+e)^2)*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)/b/f/(b*sin(f*x+e))^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{\sec^2(fx+e)}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
[Out] Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{5/3}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)),x)
[Out] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)), x)
```

3.323 $\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}} dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*hypergeom([-1/3, 5/2],[2/3],sin(f*x+e)^2)*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)/b/f/(b*sin(f*x+e))^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2657}

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{\sec^4(fx+e)}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
[Out] Integral(sec(e + f*x)**4/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{\cos(e+f\,x)^4\,(b\,\sin(e+f\,x))^{5/3}}\,dx$$

```
[In] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)),x)
[Out] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)), x)
```

3.324 $\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

```
[Out] -1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b+1/4*ln(1-sin(b*x+a)^(2/3)/co
s(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b-1/2*arctan(1/3*(1-2*sin
(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2654, 281, 298, 31, 648, 632, 210, 642}

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b} + \frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3
]])/b - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) + Log[1 - Sin[
a + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/
(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{1}{3}}(bx+a)}{\cos(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

```
[Out] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)+\log\left(\frac{\cos(bx+a)^{2}\cdots}{\cdots}\right)}{4b}$$

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^(
1/3)*sin(b*x + a)^(2/3))/cos(b*x + a)) - 2*log((cos(b*x + a)^(1/3)*sin(b*x
+ a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + log((cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*
x + a)^2))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sqrt[3]{\sin{(a+bx)}}}{\sqrt[3]{\cos{(a+bx)}}}\,dx$$

```
[In] integrate(sin(b*x+a)**(1/3)/cos(b*x+a)**(1/3),x)
[Out] Integral(sin(a + b*x)**(1/3)/cos(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sin{(bx+a)}^{\frac{1}{3}}}{\cos{(bx+a)}^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sin{(bx+a)}^{\frac{1}{3}}}{\cos{(bx+a)}^{\frac{1}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos{(a+b\,x)}^{2/3}\sin{(a+b\,x)}^{4/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \cos{(a+b\,x)}^2\right)}{2\,b\,{\left(\sin{(a+b\,x)}^2\right)}^{2/3}}$$

```
[In] int(sin(a + b*x)^(1/3)/cos(a + b*x)^(1/3),x)
[Out] -(3*cos(a + b*x)^(2/3)*sin(a + b*x)^(4/3)*hypergeom([1/3, 1/3], 4/3, cos(a
+ b*x)^2))/(2*b*(sin(a + b*x)^2)^(2/3))
```

3.325 $\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\begin{aligned}\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = & -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} \\ & +\frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}+\frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} \\ & +\frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} \\ & -\frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}\end{aligned}$$

```
[Out] arctan(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b+1/2*arctan(2*sin(b*x+a)^(1/3)/c
os(b*x+a)^(1/3)-3^(1/2))/b+1/2*arctan(2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3)+3
^(1/2))/b+1/4*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)-sin(b*x+a)^(1/3)*3^(1/
2)/cos(b*x+a)^(1/3))*3^(1/2)/b-1/4*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)+s
in(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(1/3))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2654, 301, 648, 632, 210, 642, 209}

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\arctan\left(\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\sqrt{3}\right)}{2b} + \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b} - \frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}$$

```
[In] Int[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/b + ArcTan
[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan[Sin[a
+ b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]
^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
- (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a
+ b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
```

```
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}\\ &\quad + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}\end{aligned}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$+ \frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$- \frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$
$$- \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}$$

$$= -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}$$
$$+ \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$
$$- \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\cos^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)\sin^{\frac{5}{3}}(a+bx)}{5b\cos^{\frac{5}{3}}(a+bx)}$$

```
[In] Integrate[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
[Out] (3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(5/3))/(5*b*Cos[a + b*x]^(5/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{2}{3}}(bx+a)}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
[Out] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(178) = 356.

Time = 0.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.74

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\log\left(-\frac{2\left(\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\sin(bx+a)+\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}\right)}{\sin(bx+a)}\right)-\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\log\left(\cdots\right.}{}$$

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="fricas")
[Out] -1/2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*log(-2*(sqrt(1/2
)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*sin(b*x + a) + cos(b*x + a)^(1
/3)*sin(b*x + a)^(2/3))/sin(b*x + a)) - sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(
-1/b^4) + 1)/b^2)*log(2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^
```

```
2)*sin(b*x + a) - cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a)) + sq
rt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*log(-2*(sqrt(1/2)*b*sqr
t(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*sin(b*x + a) + cos(b*x + a)^(1/3)*si
n(b*x + a)^(2/3))/sin(b*x + a)) - sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^
4) - 1)/b^2)*log(2*(sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*s
in(b*x + a) - cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a)) + 2*arct
an(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3)))/b
```

Sympy [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

```
[In] integrate(sin(b*x+a)**(2/3)/cos(b*x+a)**(2/3),x)
[Out] Integral(sin(a + b*x)**(2/3)/cos(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\sin\left(bx+a\right)^{\frac{2}{3}}}{\cos\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\sin\left(bx+a\right)^{\frac{2}{3}}}{\cos\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.20

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{1/3}\sin(a+b\,x)^{5/3}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{6};\ \frac{7}{6};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{5/6}}$$

```
[In] int(sin(a + b*x)^(2/3)/cos(a + b*x)^(2/3),x)
```

```
[Out] -(3*cos(a + b*x)^(1/3)*sin(a + b*x)^(5/3)*hypergeom([1/6, 1/6], 7/6, cos(a
+ b*x)^2))/(b*(sin(a + b*x)^2)^(5/6))
```

3.326 $\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 249

$$\begin{aligned}\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = & -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} \\ & +\frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}+\frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} \\ & +\frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} \\ & -\frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}+\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}\end{aligned}$$

```
[Out] arctan(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b+1/2*arctan(2*cos(b*x+a)^(1/3)/s
in(b*x+a)^(1/3)-3^(1/2))/b+1/2*arctan(2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3)+3
^(1/2))/b+3*sin(b*x+a)^(1/3)/b/cos(b*x+a)^(1/3)+1/4*ln(1+cos(b*x+a)^(2/3)/s
in(b*x+a)^(2/3)-cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1/3))*3^(1/2)/b-1/4*ln
(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)+cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1
/3))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2646, 2655, 301, 648, 632, 210, 642, 209}

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}+\frac{\arctan\left(\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\sqrt{3}\right)}{2b}+\frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}+\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}$$

```
[In] Int[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/b + ArcTan
[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan[Cos[a
+ b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Si
n[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
- (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
 b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) + (3*Sin[a + b*x]^(1/3))/(b*Cos[a +
 b*x]^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
```

```
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} - \int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{3\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}$$

$$= \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}$$

$$= -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}+\frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}+\frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}+\frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}-\frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}+\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)\sin^{\frac{7}{3}}(a+bx)}{7b\sqrt[3]{\cos(a+bx)}}$$

```
[In] Integrate[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(7/3))/(7*b*Cos[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{4}{3}}(bx+a)}{\cos(bx+a)^{\frac{4}{3}}}dx$$

```
[In] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
[Out] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(199) = 398.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.78

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = -\frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\cos(bx+a)\log\left(\frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\cos(bx+a)+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}}{\cos(bx+a)}\right)-\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}}{}$$

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*cos(b*x + a)*log
((sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*cos(b*x + a) + cos(b
*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a)) - sqrt(1/2)*b*sqrt((sqrt(3)
*b^2*sqrt(-1/b^4) + 1)/b^2)*cos(b*x + a)*log(-(sqrt(1/2)*b*sqrt((sqrt(3)*b^
2*sqrt(-1/b^4) + 1)/b^2)*cos(b*x + a) - cos(b*x + a)^(2/3)*sin(b*x + a)^(1/
3))/cos(b*x + a)) + sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*c
os(b*x + a)*log((sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*cos(
b*x + a) + cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a)) - sqrt(1/2)
*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*cos(b*x + a)*log(-(sqrt(1/2)*b
*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*cos(b*x + a) - cos(b*x + a)^(2/3
)*sin(b*x + a)^(1/3))/cos(b*x + a)) + 2*arctan(sin(b*x + a)^(1/3)/cos(b*x +
 a)^(1/3))*cos(b*x + a) - 6*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/(b*cos(b
*x + a))
```

Sympy [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$$

```
[In] integrate(sin(b*x+a)**(4/3)/cos(b*x+a)**(4/3),x)
```

```
[Out] Integral(sin(a + b*x)**(4/3)/cos(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{4}{3}}}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{4}{3}}}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sin\left(a+b\,x\right)^{7/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},-\frac{1}{6};\ \frac{5}{6};\ \cos\left(a+b\,x\right)^2\right)}{b\cos\left(a+b\,x\right)^{1/3}\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{7/6}}$$

```
[In] int(sin(a + b*x)^(4/3)/cos(a + b*x)^(4/3),x)
[Out] (3*sin(a + b*x)^(7/3)*hypergeom([-1/6, -1/6], 5/6, cos(a + b*x)^2))/(b*cos(
a + b*x)^(1/3)*(sin(a + b*x)^2)^(7/6))
```

3.327 $\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}+\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b}-\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

```
[Out] 1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3
))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b+3/2*sin(b*x+a)^(2/3)/b/c
os(b*x+a)^(2/3)-1/2*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{2646, 2655, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} - \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + 1\right)}{4b} - \frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + 1\right)}{2b}$$

```
[In] Int[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3
]])/b + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) + (3*Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$
\begin{aligned}
&= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} \\
&= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&\quad + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} \\
&\quad - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$
\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{4}{3},\frac{7}{3},\sin^2(a+bx)\right)\sin^{\frac{8}{3}}(a+bx)}{8b\cos^{\frac{2}{3}}(a+bx)}
$$

```
[In] Integrate[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(8/3))/(8*b*Cos[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{5}{3}}(bx+a)}{\cos(bx+a)^{\frac{5}{3}}}dx$$

```
[In] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

```
[Out] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.27

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)\cos(bx+a)+\cos(bx+a)\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(b\ldots}{\cos(b\ldots}\right.}{4b\ldots}$$

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
- sqrt(3)*sin(b*x + a))/sin(b*x + a))*cos(b*x + a) + cos(b*x + a)*log(4*(co
s(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*s
in(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*cos(b*x + a)*log(-2*(cos(b
*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x + a))/sin(b*x + a)) + 6*cos(b*x
+ a)^(1/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(sin(b*x+a)**(5/3)/cos(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sin(a+b\,x)^{8/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ \cos(a+b\,x)^2\right)}{2\,b\cos(a+b\,x)^{2/3}\,\left(\sin(a+b\,x)^2\right)^{4/3}}$$

```
[In] int(sin(a + b*x)^(5/3)/cos(a + b*x)^(5/3),x)
[Out] (3*sin(a + b*x)^(8/3)*hypergeom([-1/3, -1/3], 2/3, cos(a + b*x)^2))/(2*b*co
s(a + b*x)^(2/3)*(sin(a + b*x)^2)^(4/3))
```

3.328 $\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

```
[Out] 1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/cos
(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b+3/4*sin(b*x+a)^(4/3)/b/c
os(b*x+a)^(4/3)+1/2*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{2646, 2654, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b} - \frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)} - \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) + (3*Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx \\
&= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} \\
&= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}$$

$$=\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}-\frac{3\text{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$=\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}+\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}+\frac{3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$=\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}+\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}+\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int\frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx=\frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{5}{3},\frac{8}{3},\sin^2(a+bx)\right)\sin^{\frac{10}{3}}(a+bx)}{10b\cos^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(10/3))/(10*b*Cos[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{7}{3}}(bx+a)}{\cos(bx+a)^{\frac{7}{3}}}dx$$

[In] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)

[Out] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.26

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = -\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\cos(bx+a)^2 - 2\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a}{\cos(bx+a}\right.}{4b}$$

[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="fricas")

[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*cos(b*x + a)^2 - 2*cos(b*x + a)^2*log((cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + cos(b*x + a)^2*log((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2) - 3*cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/(b*cos(b*x + a)^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

[In] integrate(sin(b*x+a)**(7/3)/cos(b*x+a)**(7/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sin{(bx+a)}^{\frac{7}{3}}}{\cos{(bx+a)}^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sin{(bx+a)}^{\frac{7}{3}}}{\cos{(bx+a)}^{\frac{7}{3}}}\,dx$$

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\sin{(a+b\,x)}^{10/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{2}{3};\ \frac{1}{3};\ \cos{(a+b\,x)}^2\right)}{4\,b\cos{(a+b\,x)}^{4/3}\,{\left(\sin{(a+b\,x)}^2\right)}^{5/3}}$$

```
[In] int(sin(a + b*x)^(7/3)/cos(a + b*x)^(7/3),x)
[Out] (3*sin(a + b*x)^(10/3)*hypergeom([-2/3, -2/3], 1/3, cos(a + b*x)^2))/(4*b*c
os(a + b*x)^(4/3)*(sin(a + b*x)^2)^(5/3))
```

3.329 $\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

```
[Out] -1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/
3))/b+1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b+1/2*arctan(1/3*(1-2*cos
(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2655, 281, 298, 31, 648, 632, 210, 642}

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b} + \frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}$$

```
[In] Int[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) - Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/S
in[a + b*x]^(2/3)]/(4*b) + Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/(
2*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},\sin^2(a+bx)\right)\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{1}{3}}(bx+a)}{\sin(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

```
[Out] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx-}{\cos(bx+a)^2-1}\right.}{4b}$$

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
 - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + log(4*(cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(c
os(b*x + a)^2 - 1)) - 2*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin
(b*x + a))/sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

```
[In] integrate(cos(b*x+a)**(1/3)/sin(b*x+a)**(1/3),x)
[Out] Integral(cos(a + b*x)**(1/3)/sin(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\cos(bx+a)^{\frac{1}{3}}}{\sin(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\cos(bx+a)^{\frac{1}{3}}}{\sin(bx+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = -\frac{3\cos(a+b\,x)^{4/3}\sin(a+b\,x)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{2}{3};\ \frac{5}{3};\ \cos(a+b\,x)^2\right)}{4\,b\left(\sin(a+b\,x)^2\right)^{1/3}}$$

```
[In] int(cos(a + b*x)^(1/3)/sin(a + b*x)^(1/3),x)
[Out] -(3*cos(a + b*x)^(4/3)*sin(a + b*x)^(2/3)*hypergeom([2/3, 2/3], 5/3, cos(a
+ b*x)^2))/(4*b*(sin(a + b*x)^2)^(1/3))
```

3.330 $\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}$$

```
[Out] -arctan(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b-1/2*arctan(2*cos(b*x+a)^(1/3)/
sin(b*x+a)^(1/3)-3^(1/2))/b-1/2*arctan(2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3)+
3^(1/2))/b-1/4*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)-cos(b*x+a)^(1/3)*3^(1
/2)/sin(b*x+a)^(1/3))*3^(1/2)/b+1/4*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)+
cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1/3))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2655, 301, 648, 632, 210, 642, 209}

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\sqrt{3}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b} + \frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}$$

```
[In] Int[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[Cos[a +
 b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin
[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) +
 (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
```

```
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}$$

$$= -\frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}$$

$$= \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.24

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},\sin^2(a+bx)\right)\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

```
[In] Integrate[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, Sin[a + b*x]^2]*
Sin[a + b*x]^(1/3))/(b*Cos[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{2}{3}}(bx+a)}{\sin(bx+a)^{\frac{2}{3}}}dx$$

```
[In] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

```
[Out] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(179) = 358.

Time = 0.32 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.72

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\log\left(\frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\cos(bx+a)+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}}{\cos(bx+a)}\right)-\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}b\sqrt{}\right.}{}$$

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*log((sqrt(1/2)*b*
sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*cos(b*x + a) + cos(b*x + a)^(2/3)*
sin(b*x + a)^(1/3))/cos(b*x + a)) - sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b
^4) + 1)/b^2)*log(-(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*co
s(b*x + a) - cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a)) + sqrt(1/
```

```
2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*log((sqrt(1/2)*b*sqrt(-(sqrt
(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*cos(b*x + a) + cos(b*x + a)^(2/3)*sin(b*x +
a)^(1/3))/cos(b*x + a)) - sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/
b^2)*log(-(sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*cos(b*x +
a) - cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a)) + 2*arctan(sin(b*
x + a)^(1/3)/cos(b*x + a)^(1/3)))/b
```

Sympy [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

```
[In] integrate(cos(b*x+a)**(2/3)/sin(b*x+a)**(2/3),x)
[Out] Integral(cos(a + b*x)**(2/3)/sin(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.20

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\cos\left(a+b\,x\right)^{5/3}\sin\left(a+b\,x\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{5}{6},\frac{5}{6};\ \frac{11}{6};\ \cos\left(a+b\,x\right)^2\right)}{5\,b\,\left(\sin\left(a+b\,x\right)^2\right)^{1/6}}$$

```
[In] int(cos(a + b*x)^(2/3)/sin(a + b*x)^(2/3),x)
```

```
[Out] -(3*cos(a + b*x)^(5/3)*sin(a + b*x)^(1/3)*hypergeom([5/6, 5/6], 11/6, cos(a
 + b*x)^2))/(5*b*(sin(a + b*x)^2)^(1/6))
```

3.331 $\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

```
[Out] -arctan(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b-1/2*arctan(2*sin(b*x+a)^(1/3)/
cos(b*x+a)^(1/3)-3^(1/2))/b-1/2*arctan(2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3)+
3^(1/2))/b-3*cos(b*x+a)^(1/3)/b/sin(b*x+a)^(1/3)-1/4*ln(1+sin(b*x+a)^(2/3)/
cos(b*x+a)^(2/3)-sin(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(1/3))*3^(1/2)/b+1/4*l
n(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)+sin(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(
1/3))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2647, 2654, 301, 648, 632, 210, 642, 209}

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\sqrt{3}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b} + \frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}$$

```
[In] Int[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[Sin[a +
 b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]^
(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) +
 (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
 b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) - (3*Cos[a + b*x]^(1/3))/(b*Sin[a +
b*x]^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
```

```
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

$$=-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{3\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$=-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$
$$-\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$
$$-\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$=-\frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$
$$-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$-\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$-\frac{\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$
$$+\frac{\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx,x,\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b}$$

$$=-\frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}-\frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$
$$+\frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$
$$+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}$$
$$+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}$$

$$= \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cos^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},-\frac{1}{6},\frac{5}{6},\sin^2(a+bx)\right)}{b\cos^{\frac{5}{3}}(a+bx)\sqrt[3]{\sin(a+bx)}}$$

```
[In] Integrate[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
[Out] (-3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(5/3)*Sin[a + b*x]^(1/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{4}{3}}(bx+a)}{\sin(bx+a)^{\frac{4}{3}}}dx$$

```
[In] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
[Out] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(200) = 400.

Time = 0.35 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.78

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\log\left(\frac{2\left(\sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+1}{b^2}}\sin(bx+a)+\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}\right)}{\sin(bx+a)}\right)\sin(bx+a) - \sqrt{\frac{1}{2}}b\sqrt{\frac{\sqrt{3}b^2\sqrt{-\frac{1}{b^4}}+}{b^2}}}{}$$

[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*log(2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*sin(b*x + a) + cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a))*sin(b*x + a) - sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*log(-2*(sqrt(1/2)*b*sqrt((sqrt(3)*b^2*sqrt(-1/b^4) + 1)/b^2)*sin(b*x + a) - cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a))*sin(b*x + a) + sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*log(2*(sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*sin(b*x + a) + cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a))*sin(b*x + a) - sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*log(-2*(sqrt(1/2)*b*sqrt(-(sqrt(3)*b^2*sqrt(-1/b^4) - 1)/b^2)*sin(b*x + a) - cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a))*sin(b*x + a) + 2*arctan(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3))*sin(b*x + a) - 6*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/(b*sin(b*x + a))

Sympy [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos^{\frac{4}{3}}\left(a+bx\right)}{\sin^{\frac{4}{3}}\left(a+bx\right)}\,dx$$

[In] integrate(cos(b*x+a)**(4/3)/sin(b*x+a)**(4/3),x)

[Out] Integral(cos(a + b*x)**(4/3)/sin(a + b*x)**(4/3), x)

Maxima [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{4}{3}}}{\sin\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)

Giac [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{4}{3}}}{\sin(bx+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{7/3}\,\left(\sin(a+b\,x)^2\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{7}{6},\frac{7}{6};\ \frac{13}{6};\ \cos(a+b\,x)^2\right)}{7\,b\sin(a+b\,x)^{1/3}}$$

```
[In] int(cos(a + b*x)^(4/3)/sin(a + b*x)^(4/3),x)
[Out] -(3*cos(a + b*x)^(7/3)*(sin(a + b*x)^2)^(1/6)*hypergeom([7/6, 7/6], 13/6, c
os(a + b*x)^2))/(7*b*sin(a + b*x)^(1/3))
```

3.332 $\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}+\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

```
[Out] 1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/cos
(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b-3/2*cos(b*x+a)^(2/3)/b/s
in(b*x+a)^(2/3)+1/2*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{2647, 2654, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b} - \frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}$$

```
[In] Int[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) - (3*Cos[a + b*x]^(2/3))/(2*b*Sin[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx \\
&= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} \\
&= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}
$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{3},\frac{2}{3},\sin^2(a+bx)\right)}{2b\cos^{\frac{4}{3}}(a+bx)\sin^{\frac{2}{3}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
[Out] (-3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, Sin[a + b*x]^
2])/(2*b*Cos[a + b*x]^(4/3)*Sin[a + b*x]^(2/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{5}{3}}(bx+a)}{\sin(bx+a)^{\frac{5}{3}}}dx$$

[In] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)

[Out] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\sin(bx+a)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)}{4\,b}$$

[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="fricas")

[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*sin(b*x + a) - 2*log((cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a))*sin(b*x + a) + log((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2)*sin(b*x + a) + 6*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/(b*sin(b*x + a))

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

[In] integrate(cos(b*x+a)**(5/3)/sin(b*x+a)**(5/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{8/3}\left(\sin(a+b\,x)^2\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{4}{3};\ \frac{7}{3};\ \cos(a+b\,x)^2\right)}{8\,b\sin(a+b\,x)^{2/3}}$$

[In] int(cos(a + b*x)^(5/3)/sin(a + b*x)^(5/3),x)

[Out] -(3*cos(a + b*x)^(8/3)*(sin(a + b*x)^2)^(1/3)*hypergeom([4/3, 4/3], 7/3, cos(a + b*x)^2))/(8*b*sin(a + b*x)^(2/3))

3.333 $\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}+\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b}-\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

```
[Out] 1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3
))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b-3/4*cos(b*x+a)^(4/3)/b/s
in(b*x+a)^(4/3)-1/2*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

$= \{2647, 2655, 281, 298, 31, 648, 632, 210, 642\}$

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)} - \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + 1\right)}{4b} - \frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} + 1\right)}{2b}$$

```
[In] Int[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3
]])/b + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) - (3*Cos[a + b*x]^(4/3))/(4*b*Sin[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx \\
&= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} \\
&= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} \\
&= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}
\end{aligned}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},\sin^2(a+bx)\right)}{4b\cos^{\frac{2}{3}}(a+bx)\sin^{\frac{4}{3}}(a+bx)}$$

```
[In] Integrate[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, Sin[a + b*x]^
2])/(4*b*Cos[a + b*x]^(2/3)*Sin[a + b*x]^(4/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{7}{3}}(bx+a)}{\sin(bx+a)^{\frac{7}{3}}}dx$$

```
[In] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

```
[Out] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.41

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \frac{2\left(\sqrt{3}\cos(bx+a)^2-\sqrt{3}\right)\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\left(\cos(bx+a)^2-1\right)\log\left(\frac{4\left(\cos(bx+a\right.}{}\right.}{}$$

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cos(b*x + a)^2 - sqrt(3))*arctan(1/3*(2*sqrt(3)*cos(b*x + a
)^(2/3)*sin(b*x + a)^(1/3) - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + (cos(b*x
 + a)^2 - 1)*log(4*(cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3)
+ cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*(cos
(b*x + a)^2 - 1)*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x +
a))/sin(b*x + a)) + 3*cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a
)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**(7/3)/sin(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{10/3}\,\left(\sin(a+b\,x)^2\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{5}{3},\frac{5}{3};\ \frac{8}{3};\ \cos(a+b\,x)^2\right)}{10\,b\sin(a+b\,x)^{4/3}}$$

```
[In] int(cos(a + b*x)^(7/3)/sin(a + b*x)^(7/3),x)
[Out] -(3*cos(a + b*x)^(10/3)*(sin(a + b*x)^2)^(2/3)*hypergeom([5/3, 5/3], 8/3, c
os(a + b*x)^2))/(10*b*sin(a + b*x)^(4/3))
```

3.334 $\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

```
[Out] -3/5*cos(x)^(5/3)/sin(x)^(5/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2643}

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

```
[In] Int[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

```
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

```
[In] Integrate[Cos[x]^(2/3)/Sin[x]^(8/3),x]
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Maple [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin(x)^{\frac{8}{3}}}dx$$

```
[In] int(cos(x)^(2/3)/sin(x)^(8/3),x)
[Out] int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \frac{3\,\cos(x)^{\frac{5}{3}}\sin(x)^{\frac{1}{3}}}{5\,(\cos(x)^2-1)}$$

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="fricas")
[Out] 3/5*cos(x)^(5/3)*sin(x)^(1/3)/(cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(x)**(2/3)/sin(x)**(8/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="maxima")
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="giac")
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos(x)^{5/3}}{5\sin(x)^{5/3}}$$

```
[In] int(cos(x)^(2/3)/sin(x)^(8/3),x)
[Out] -(3*cos(x)^(5/3))/(5*sin(x)^(5/3))
```

3.335 $\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

```
[Out] 3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2643}

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

```
[In] Int[Sin[x]^(2/3)/Cos[x]^(8/3),x]
[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx=\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

```
[In] Integrate[Sin[x]^(2/3)/Cos[x]^(8/3),x]
[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Maple [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos(x)^{\frac{8}{3}}}dx$$

```
[In] int(sin(x)^(2/3)/cos(x)^(8/3),x)
[Out] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx=\frac{3\,\sin(x)^{\frac{5}{3}}}{5\,\cos(x)^{\frac{5}{3}}}$$

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="fricas")
[Out] 3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx=\text{Timed out}$$

```
[In] integrate(sin(x)**(2/3)/cos(x)**(8/3),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="maxima")
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="giac")
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.88

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \frac{6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{5/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}+6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{11/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}}{5\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^2\left(10\tan\left(\frac{x}{2}\right)^2-5\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)+10\right)+5}$$

```
[In] int(sin(x)^(2/3)/cos(x)^(8/3),x)
[Out] (6*2^(2/3)*tan(x/2)^(5/3)*(1 - tan(x/2)^2)^(1/3) + 6*2^(2/3)*tan(x/2)^(11/3
)*(1 - tan(x/2)^2)^(1/3))/(5*tan(x/2)^2 - tan(x/2)^2*(10*tan(x/2)^2 - 5*tan
(x/2)^2*(tan(x/2)^2 + 1) + 10) + 5)
```

3.336 $\int \cos^n(e+fx)\sin^m(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1+n)}$$

```
[Out] -cos(f*x+e)^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x+e)^2
)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2656}

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)}$$

```
[In] Int[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 +
 n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral =

$$-\frac{\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1+n)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \frac{\cos^{-1+n}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin^{1+m}(e+fx)}{f(1+m)}$$

```
[In] Integrate[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*(1 + m
))
```

Maple [F]

$$\int (\cos^n(fx+e))(\sin^m(fx+e))\,dx$$

```
[In] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
[Out] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \cos(fx+e)^n\sin(fx+e)^m\,dx$$

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
[Out] integral(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \sin^m(e+fx)\cos^n(e+fx)\,dx$$

```
[In] integrate(cos(f*x+e)**n*sin(f*x+e)**m,x)
[Out] Integral(sin(e + f*x)**m*cos(e + f*x)**n, x)
```

Maxima [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \cos(fx+e)^n \sin(fx+e)^m\,dx$$

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \cos(fx+e)^n \sin(fx+e)^m\,dx$$

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\cos(e+f\,x)^{n+1}\sin(e+f\,x)^{m+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2}-\frac{m}{2},\frac{n}{2}+\frac{1}{2};\ \frac{n}{2}+\frac{3}{2};\ \cos(e+f\,x)^2\right)}{f\,\left(n+1\right)\,\left(\sin(e+f\,x)^2\right)^{\frac{m}{2}+\frac{1}{2}}}$$

```
[In] int(cos(e + f*x)^n*sin(e + f*x)^m,x)
[Out] -(cos(e + f*x)^(n + 1)*sin(e + f*x)^(m + 1)*hypergeom([1/2 - m/2, n/2 + 1/2
], n/2 + 3/2, cos(e + f*x)^2))/(f*(n + 1)*(sin(e + f*x)^2)^(m/2 + 1/2))
```

3.337 $\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = -\frac{(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{df(1+n)}$$

```
[Out] -(d*cos(f*x+e))^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x+
e)^2)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = -\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[In] Int[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -(((d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)
/2, Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(d*
f*(1 + n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral =
$$-\frac{(d\cos(e+fx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{df(1+n)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int(d\cos(e+fx))^n\sin^m(e+fx)\,dx$$
$$=\frac{d(d\cos(e+fx))^{-1+n}\cos^2(e+fx)^{\frac{1-n}{2}}\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin^{1+m}(e+fx)}{f(1+m)}$$

```
[In] Integrate[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
[Out] (d*(d*Cos[e + f*x])^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*
(1 + m))
```

Maple [F]

$$\int(d\cos(fx+e))^n(\sin^m(fx+e))\,dx$$

```
[In] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
[Out] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int(d\cos(e+fx))^n\sin^m(e+fx)\,dx=\int(d\cos(fx+e))^n\sin(fx+e)^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int (d\cos{(e+fx)})^n \sin^m{(e+fx)}\,dx$$

[In] integrate((d*cos(f*x+e))**n*sin(f*x+e)**m,x)

[Out] Integral((d*cos(e + f*x))**n*sin(e + f*x)**m, x)

Maxima [F]

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int (d\cos{(fx+e)})^n \sin{(fx+e)}^m\,dx$$

[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")

[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)

Giac [F]

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int (d\cos{(fx+e)})^n \sin{(fx+e)}^m\,dx$$

[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")

[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int \sin{(e+f\,x)}^m\,(d\,\cos{(e+f\,x)})^n\,dx$$

[In] int(sin(e + f*x)^m*(d*cos(e + f*x))^n,x)

[Out] int(sin(e + f*x)^m*(d*cos(e + f*x))^n, x)

3.338 $\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = -\frac{b\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1-}{2}}}{f(1+n)}$$

```
[Out] -b*cos(f*x+e)^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x+e)
^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = -\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)}$$

```
[In] Int[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2
, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(
f*(1 + n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral =

$$-\frac{b\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1+n)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int\cos^n(e+fx)(b\sin(e+fx))^m\,dx = \frac{\cos^{-1+n}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\sin(e+fx))^m}{f(1+m)}$$

```
[In] Integrate[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Sin[e + f*x])^m
)/(f*(1 + m))
```

Maple [F]

$$\int(\cos^n(fx+e))\,(b\sin(fx+e))^m\,dx$$

```
[In] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
[Out] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int\cos^n(e+fx)(b\sin(e+fx))^m\,dx = \int(b\sin(fx+e))^m\cos(fx+e)^n\,dx$$

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Sympy [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=\int (b\sin{(e+fx)})^m\cos^n{(e+fx)}\,dx$$

```
[In] integrate(cos(f*x+e)**n*(b*sin(f*x+e))**m,x)
[Out] Integral((b*sin(e + f*x))**m*cos(e + f*x)**n, x)
```

Maxima [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=\int (b\sin{(fx+e)})^m\cos{(fx+e)}^n\,dx$$

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Giac [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=\int (b\sin{(fx+e)})^m\cos{(fx+e)}^n\,dx$$

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=\int {\cos\left(e+f\,x\right)}^n\,{\left(b\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

```
[In] int(cos(e + f*x)^n*(b*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^n*(b*sin(e + f*x))^m, x)
```

3.339 $\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = -\frac{b(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}}{df(1+n)}$$

```
[Out] -b*(d*cos(f*x+e))^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*
x+e)^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2656}

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = -\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[In] Int[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*(d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 +
n)/2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2
))/(d*f*(1 + n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral =
$$-\frac{b(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+fx}{df(1+n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx$$
$$=\frac{(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)(b\sin(e+fx))^m\tan(}{f(1+m)}$$

```
[In] Integrate[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
[Out] ((d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 + m)/
2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*(b*Sin[e + f*x])^m*Tan[e + f*x])/(
f*(1 + m))
```

Maple [F]

$$\int (d\cos(fx+e))^n\,(b\sin(fx+e))^m\,dx$$

```
[In] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
[Out] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (d\cos(fx+e))^n\,(b\sin(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (b\sin{(e+fx)})^m\,(d\cos{(e+fx)})^n\,\,dx$$

```
[In] integrate((d*cos(f*x+e))**n*(b*sin(f*x+e))**m,x)
[Out] Integral((b*sin(e + f*x))**m*(d*cos(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (d\cos{(fx+e)})^n\,(b\sin{(fx+e)})^m\,\,dx$$

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (d\cos{(fx+e)})^n\,(b\sin{(fx+e)})^m\,\,dx$$

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int {\left(d\,\cos\left(e+f\,x\right)\right)}^n\,{\left(b\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

```
[In] int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m,x)
[Out] int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m, x)
```

3.340 $\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx=\frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}-\frac{2(c\sin(a+bx))^{3+m}}{bc^3(3+m)}+\frac{(c\sin(a+bx))^{5+m}}{bc^5(5+m)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)-2*(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)+(c*sin(b*
x+a))^(5+m)/b/c^5/(5+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2644, 276}

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx=\frac{(c\sin(a+bx))^{m+5}}{bc^5(m+5)}-\frac{2(c\sin(a+bx))^{m+3}}{bc^3(m+3)}+\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[In] Int[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (2*(c*Sin[a + b*x])^(3 + m))/(b*c^
3*(3 + m)) + (c*Sin[a + b*x])^(5 + m)/(b*c^5*(5 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)^2\,dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{\text{Subst}\left(\int\left(x^m-\frac{2x^{2+m}}{c^2}+\frac{x^{4+m}}{c^4}\right)\,dx,x,c\sin(a+bx)\right)}{bc}\\ &= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}-\frac{2(c\sin(a+bx))^{3+m}}{bc^3(3+m)}+\frac{(c\sin(a+bx))^{5+m}}{bc^5(5+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int\cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sin(a+bx)(c\sin(a+bx))^m\left(\frac{1}{1+m}-\frac{2\sin^2(a+bx)}{3+m}+\frac{\sin^4(a+bx)}{5+m}\right)}{b}$$

```
[In] Integrate[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m*((1 + m)^(-1) - (2*Sin[a + b*x]^2)/(3 + m)
 + Sin[a + b*x]^4/(5 + m)))/b
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

method	result	size
parallelrisch	$\frac{(c\sin(bx+a))^m\left(\left(\frac{3}{2}m^2+14m+\frac{25}{2}\right)\sin(3bx+3a)+\left(\frac{1}{2}m^2+2m+\frac{3}{2}\right)\sin(5bx+5a)+\sin(bx+a)(m^2+12m+75)\right)}{8b(m^3+9m^2+23m+15)}$	87
derivativedivides	$\frac{\sin(bx+a)e^{m\ln(c\sin(bx+a))}}{b(1+m)}+\frac{(\sin^5(bx+a))e^{m\ln(c\sin(bx+a))}}{b(5+m)}-\frac{2(\sin^3(bx+a))e^{m\ln(c\sin(bx+a))}}{b(3+m)}$	88
default	$\frac{\sin(bx+a)e^{m\ln(c\sin(bx+a))}}{b(1+m)}+\frac{(\sin^5(bx+a))e^{m\ln(c\sin(bx+a))}}{b(5+m)}-\frac{2(\sin^3(bx+a))e^{m\ln(c\sin(bx+a))}}{b(3+m)}$	88

```
[In] int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*sin(b*x+a))^m*((3/2*m^2+14*m+25/2)*sin(3*b*x+3*a)+(1/2*m^2+2*m+3/2)*
sin(5*b*x+5*a)+sin(b*x+a)*(m^2+12*m+75))/b/(m^3+9*m^2+23*m+15)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\left((m^2+4\,m+3)\cos\left(bx+a\right)^4+4\,(m+1)\cos\left(bx+a\right)^2+8\right)\left(c\sin\left(bx+a\right)\right)^m\sin\left(bx+a\right)}{bm^3+9\,bm^2+23\,bm+15\,b}$$

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 + 4*m + 3)*cos(b*x + a)^4 + 4*(m + 1)*cos(b*x + a)^2 + 8)*(c*sin(b*x
+ a))^m*sin(b*x + a)/(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2040 vs. 2(60) = 120.

Time = 4.41 (sec) , antiderivative size = 2040, normalized size of antiderivative = 27.57

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \text{Too large to display}$$

```
[In] integrate(cos(b*x+a)**5*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**5, Eq(b, 0)), ((log(sin(a + b*x))/b + co
s(a + b*x)**2/(2*b*sin(a + b*x)**2) - cos(a + b*x)**4/(4*b*sin(a + b*x)**4)
)/c**5, Eq(m, -5)), ((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(
8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**
2) + 32*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan
```

```
(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(
a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*t
an(a/2 + b*x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan
(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*
tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*t
an(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
18*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*
x/2)**4 + 8*b*tan(a/2 + b*x/2)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*x/
2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/
2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*
log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b
*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b
*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 +
b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 +
 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 +
 b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**4/(b*tan(a/
2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan
(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*
tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b
))/c, Eq(m, -1)), (m**2*(c*sin(a + b*x))**m*sin(a + b*x)*cos(a + b*x)**4/(b
*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 4*m*(c*sin(a + b*x))**m*sin(a + b*x)**3
*cos(a + b*x)**2/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*m*(c*sin(a + b*x))
**m*sin(a + b*x)*cos(a + b*x)**4/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*(c
*sin(a + b*x))**m*sin(a + b*x)**5/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 20*
(c*sin(a + b*x))**m*sin(a + b*x)**3*cos(a + b*x)**2/(b*m**3 + 9*b*m**2 + 23
*b*m + 15*b) + 15*(c*sin(a + b*x))**m*sin(a + b*x)*cos(a + b*x)**4/(b*m**3
+ 9*b*m**2 + 23*b*m + 15*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\frac{c^m \sin(bx+a)^m \sin(bx+a)^5}{m+5} - \frac{2\,c^m \sin(bx+a)^m \sin(bx+a)^3}{m+3} + \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] (c^m*sin(b*x + a)^m*sin(b*x + a)^5/(m + 5) - 2*c^m*sin(b*x + a)^m*sin(b*x +
 a)^3/(m + 3) + (c*sin(b*x + a))^(m + 1)/(c*(m + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(74) = 148.

Time = 0.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.35

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(bx+a))^m c^5m^2 \sin(bx+a)^5 + 4\,(c\sin(bx+a))^m c^5m \sin(bx+a)^5 - 2\,(c\sin(bx+a))^m c^5m^2 \sin(b$$

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] ((c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^5 + 4*(c*sin(b*x + a))^m*c^5*m*sin
(b*x + a)^5 - 2*(c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^3 + 3*(c*sin(b*x +
a))^m*c^5*sin(b*x + a)^5 - 12*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a)^3 + (c*
sin(b*x + a))^m*c^5*m^2*sin(b*x + a) - 10*(c*sin(b*x + a))^m*c^5*sin(b*x +
a)^3 + 8*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a) + 15*(c*sin(b*x + a))^m*c^5*
sin(b*x + a))/((c^4*m^3 + 9*c^4*m^2 + 23*c^4*m + 15*c^4)*b*c)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\,\sin(a+b\,x))^m\,(150\,\sin(a+b\,x) + 25\,\sin(3\,a+3\,b\,x) + 3\,\sin(5\,a+5\,b\,x) + 24\,m\,\sin(a+b\,x) + 28\,m}{16\,b\,(m^3+9\,m}$$

```
[In] int(cos(a + b*x)^5*(c*sin(a + b*x))^m,x)
```

```
[Out] ((c*sin(a + b*x))^m*(150*sin(a + b*x) + 25*sin(3*a + 3*b*x) + 3*sin(5*a + 5
*b*x) + 24*m*sin(a + b*x) + 28*m*sin(3*a + 3*b*x) + 4*m*sin(5*a + 5*b*x) +
2*m^2*sin(a + b*x) + 3*m^2*sin(3*a + 3*b*x) + m^2*sin(5*a + 5*b*x)))/(16*b*
(23*m + 9*m^2 + m^3 + 15))
```

3.341 $\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)} - \frac{(c\sin(a+bx))^{3+m}}{bc^3(3+m)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)-(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2644, 14}

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{m+1}}{bc(m+1)} - \frac{(c\sin(a+bx))^{m+3}}{bc^3(m+3)}$$

```
[In] Int[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (c*Sin[a + b*x])^(3 + m)/(b*c^3*(3
 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)\,dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{\text{Subst}\left(\int \left(x^m-\frac{x^{2+m}}{c^2}\right)\,dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)} - \frac{(c\sin(a+bx))^{3+m}}{bc^3(3+m)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{(5+m+(1+m)\cos(2(a+bx)))\sin(a+bx)(c\sin(a+bx))^m}{2b(1+m)(3+m)}$$

```
[In] Integrate[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
[Out] ((5 + m + (1 + m)*Cos[2*(a + b*x)])*Sin[a + b*x]*(c*Sin[a + b*x])^m)/(2*b*(
1 + m)*(3 + m))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{((1+m)\sin(3bx+3a)+\sin(bx+a)(m+9))(c\sin(bx+a))^m}{4b(m^2+4m+3)}$	50
derivativedivides	$\frac{\sin(bx+a)e^{m\ln(c\sin(bx+a))}}{b(1+m)} - \frac{(\sin^3(bx+a))e^{m\ln(c\sin(bx+a))}}{b(3+m)}$	59
default	$\frac{\sin(bx+a)e^{m\ln(c\sin(bx+a))}}{b(1+m)} - \frac{(\sin^3(bx+a))e^{m\ln(c\sin(bx+a))}}{b(3+m)}$	59

```
[In] int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
[Out] 1/4*((1+m)*sin(3*b*x+3*a)+sin(b*x+a)*(m+9))*(c*sin(b*x+a))^m/b/(m^2+4*m+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\left((m+1)\cos\left(bx+a\right)^2+2\right)\left(c\sin\left(bx+a\right)\right)^m\sin\left(bx+a\right)}{bm^2+4\,bm+3\,b}$$

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] ((m + 1)*cos(b*x + a)^2 + 2)*(c*sin(b*x + a))^m*sin(b*x + a)/(b*m^2 + 4*b*m
 + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(37) = 74.

Time = 1.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 10.50

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$$

$$= \begin{cases} x(c\sin(a))^m\cos^3(a) \\ \frac{-\frac{\log(\sin(a+bx))}{b}-\frac{\cos^2(a+bx)}{2b\sin^2(a+bx)}}{c^3} \\ \frac{-\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{2\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} }{c} \\ \frac{m(c\sin(a+bx))^m\sin(a+bx)\cos^2(a+bx)}{bm^2+4bm+3b}+\frac{2(c\sin(a+bx))^m\sin^3(a+bx)}{bm^2+4bm+3b}+\frac{3(c\sin(a+bx))^m\sin(a+bx)\cos^2(a+bx)}{bm^2+4bm+3b} \end{cases}$$

```
[In] integrate(cos(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**3, Eq(b, 0)), ((-log(sin(a + b*x))/b - c
os(a + b*x)**2/(2*b*sin(a + b*x)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*
x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x
/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b
) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*
b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**
4 + 2*b*tan(a/2 + b*x/2)**2 + b))/c, Eq(m, -1)), (m*(c*sin(a + b*x))**m*sin
(a + b*x)*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b) + 2*(c*sin(a + b*x))**m*si
n(a + b*x)**3/(b*m**2 + 4*b*m + 3*b) + 3*(c*sin(a + b*x))**m*sin(a + b*x)*c
os(a + b*x)**2/(b*m**2 + 4*b*m + 3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = -\frac{\frac{c^m\sin(bx+a)^m\sin(bx+a)^3}{m+3} - \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] -(c^m*sin(b*x + a)^m*sin(b*x + a)^3/(m + 3) - (c*sin(b*x + a))^(m + 1)/(c*(
m + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(50) = 100.

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.36

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = -\frac{(c\sin(bx+a))^m c^3 m\sin(bx+a)^3 + (c\sin(bx+a))^m c^3\sin(bx+a)^3 - (c\sin(bx+a))^m c^3 m\sin(bx+a}{(c^2m^2+4c^2m+3c^2)bc}$$

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] -((c*sin(b*x + a))^m*c^3*m*sin(b*x + a)^3 + (c*sin(b*x + a))^m*c^3*sin(b*x
+ a)^3 - (c*sin(b*x + a))^m*c^3*m*sin(b*x + a) - 3*(c*sin(b*x + a))^m*c^3*s
in(b*x + a))/((c^2*m^2 + 4*c^2*m + 3*c^2)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^m(9\sin(a+bx)+\sin(3a+3bx)+m\sin(a+bx)+m\sin(3a+3bx))}{4b(m^2+4m+3)}$$

```
[In] int(cos(a + b*x)^3*(c*sin(a + b*x))^m,x)
```

```
[Out] ((c*sin(a + b*x))^m*(9*sin(a + b*x) + sin(3*a + 3*b*x) + m*sin(a + b*x) + m
*sin(3*a + 3*b*x)))/(4*b*(4*m + m^2 + 3))
```

3.342 $\int \cos(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 30}

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[In] Int[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^m\,dx, x, c\sin(a+bx)\right)}{bc} \\
&= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$
\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}
$$

```
[In] Integrate[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$
default	$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$
parallelrisch	$\frac{(c\sin(bx+a))^m\sin(bx+a)}{b(1+m)}$
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\mathrm{e}^{m\ln\left(\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}}{b(1+m)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
risch	$\left(\mathrm{e}^{i(bx+a)}\right)^{-m}(\sin(bx)\cos(a)+\cos(bx)\sin(a))\left(\mathrm{e}^{2i(bx+a)}-1\right)^m\left(\frac{1}{2}\right)^m c^m \mathrm{e}^{-\frac{i\pi m\left(-\operatorname{csgn}(\sin(bx)\cos(a)+\cos(bx)\sin(a))\operatorname{csgn}\left(i\mathrm{e}^{-i(}\right.\right.}{}}$

```
[In] int(cos(b*x+a)*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin{(bx+a)})^m \sin{(bx+a)}}{bm+b}$$

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] (c*sin(b*x + a))^m*sin(b*x + a)/(b*m + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \begin{cases} \frac{x\cos{(a)}}{c\sin{(a)}} & \text{for}\ b=0 \wedge m=-1 \\ x(c\sin{(a)})^m\cos{(a)} & \text{for}\ b=0 \\ \frac{\log{(\sin{(a+bx)})}}{bc} & \text{for}\ m=-1 \\ \frac{(c\sin{(a+bx)})^m\sin{(a+bx)}}{bm+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*cos(a)/(c*sin(a)), Eq(b, 0) & Eq(m, -1)), (x*(c*sin(a))**m*cos
(a), Eq(b, 0)), (log(sin(a + b*x))/(b*c), Eq(m, -1)), ((c*sin(a + b*x))**m*
sin(a + b*x)/(b*m + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin{(bx+a)})^{m+1}}{bc(m+1)}$$

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] (c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] (c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sin(a+b\,x)\,\left(c\,\sin(a+b\,x)\right)^m}{b\,(m+1)}$$

```
[In] int(cos(a + b*x)*(c*sin(a + b*x))^m,x)
[Out] (sin(a + b*x)*(c*sin(a + b*x))^m)/(b*(m + 1))
```

3.343 $\int \sec(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

```
[Out] hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/c
/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 371}

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[In] Int[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{c^2}}\,dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, 1+\frac{1+m}{2}, \sin^2(a+bx)\right)\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}$$

```
[In] Integrate[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
[Out] (Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [F]

$$\int \sec(bx+a)(c\sin(bx+a))^m\,dx$$

```
[In] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
[Out] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Sympy [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x), x)
```

Maxima [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Giac [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx=\int \frac{(c\,\sin\left(a+b\,x\right))^m}{\cos\left(a+b\,x\right)}\,dx$$

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x),x)
[Out] int((c*sin(a + b*x))^m/cos(a + b*x), x)
```

3.344 $\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

```
[Out] hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/c
/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2644, 371}

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[In] Int[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x^2}{c^2}\right)^2}\,dx, x, c\sin(a+bx)\right)}{bc}\\ &= \frac{\text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, \sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx\\ &= \frac{\text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, 1+\frac{1+m}{2}, \sin^2(a+bx)\right)\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}\end{aligned}$$

```
[In] Integrate[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
[Out] (Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Maple [F]

$$\int \left(\sec^3(bx+a)\right)(c\sin(bx+a))^m\,dx$$

```
[In] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
[Out] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Sympy [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec^3(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**3*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Giac [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx=\int \frac{(c\,\sin\left(a+b\,x\right))^m}{\cos\left(a+b\,x\right)^3}\,dx$$

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^3,x)
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^3, x)
```

3.345 $\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2657}

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

```
[In] Integrate[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int \left(\cos^4(bx+a)\right)(c\sin(bx+a))^m\,dx$$

```
[In] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
[Out] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**4*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Giac [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^4\,dx$$

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int \cos(a+b\,x)^4\,(c\,\sin(a+b\,x))^m\,dx$$

```
[In] int(cos(a + b*x)^4*(c*sin(a + b*x))^m,x)
[Out] int(cos(a + b*x)^4*(c*sin(a + b*x))^m, x)
```

3.346 $\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2657}

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

```
[In] Integrate[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int \left(\cos^2(bx+a)\right)(c\sin(bx+a))^m\,dx$$

```
[In] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
[Out] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \text{Timed out}$$

```
[In] integrate(cos(b*x+a)**2*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^2\,dx$$

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int \cos(a+b\,x)^2\,(c\,\sin(a+b\,x))^m\,dx$$

```
[In] int(cos(a + b*x)^2*(c*sin(a + b*x))^m,x)
[Out] int(cos(a + b*x)^2*(c*sin(a + b*x))^m, x)
```

3.347 $\int (c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+
a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2722}

$$\int (c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int (c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

```
[In] Integrate[(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int (c\sin(bx+a))^m\,dx$$

```
[In] int((c*sin(b*x+a))^m,x)
[Out] int((c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\,dx$$

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m\,dx$$

```
[In] integrate((c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m, x)
```

Maxima [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\,dx$$

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\,dx$$

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\,dx$$

```
[In] int((c*sin(a + b*x))^m,x)
[Out] int((c*sin(a + b*x))^m, x)
```

3.348 $\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

```
[Out] hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*(c*sin(b*x+
a))^(1+m)*(cos(b*x+a)^2)^(1/2)/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2657}

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[In] Int[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral

$$= \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

```
[In] Integrate[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int \left(\sec^2(bx+a)\right)(c\sin(bx+a))^m\,dx$$

```
[In] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
[Out] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\sec(bx+a)^2\,dx$$

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Sympy [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m\sec^2(a+bx)\,dx$$

```
[In] integrate(sec(b*x+a)**2*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x)**2, x)
```

Maxima [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\sec(bx+a)^2\,dx$$

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Giac [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\sec(bx+a)^2\,dx$$

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int \frac{(c\sin(a+b\,x))^m}{\cos(a+b\,x)^2}\,dx$$

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^2,x)
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^2, x)
```

3.349 $\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

```
[Out] hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*(c*sin(b*x+
a))^(1+m)*(cos(b*x+a)^2)^(1/2)/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2657}

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[In] Int[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

integral

$$= \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

```
[In] Integrate[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int \left(\sec^4(bx+a)\right)(c\sin(bx+a))^m\,dx$$

```
[In] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
[Out] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\sec(bx+a)^4\,dx$$

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Sympy [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin{(a+bx)})^m \sec^4{(a+bx)}\,dx$$

```
[In] integrate(sec(b*x+a)**4*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x)**4, x)
```

Maxima [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin{(bx+a)})^m \sec{(bx+a)}^4\,dx$$

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Giac [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin{(bx+a)})^m \sec{(bx+a)}^4\,dx$$

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int \frac{(c\,\sin{(a+b\,x)})^m}{\cos{(a+b\,x)}^4}\,dx$$

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^4,x)
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^4, x)
```

3.350 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt{d\cos(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx}{bc(1+m)\sqrt[4]{\cos^2(a+bx)}}$$

```
[Out] d*hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m
)*(d*cos(b*x+a))^(1/2)/b/c/(1+m)/(cos(b*x+a)^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{2657\}$

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(Cos[a + b*x]^2)^(1/4))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
```

```
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{d\sqrt{d\cos(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt[4]{\cos^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d^2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+$$

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
[Out] (d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, S
in[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b
*x]])
```

Maple [F]

$$\int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
[Out] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m*d*cos(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m\,(d\cos(a+bx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\,\cos(a+b\,x))^{3/2}\,(c\,\sin(a+b\,x))^m\,dx$$

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m,x)
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m, x)
```

3.351 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{d\cos(a+bx)}}$$

```
[Out] d*(cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2657}

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \frac{\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx}{b(1+m)}$$

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Maple [F]

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^m\,dx$$

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
[Out] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int (c\sin{(a+bx)})^m\sqrt{d\cos{(a+bx)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\cos{(bx+a)}}(c\sin{(bx+a)})^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\cos{(bx+a)}}(c\sin{(bx+a)})^m\,dx$$

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\,\cos{(a+b\,x)}}\,(c\,\sin{(a+b\,x)})^m\,dx$$

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m,x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m, x)
```

3.352 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)(d\cos(a+bx))^{3/2}}$$

```
[Out] d*(cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2657}

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}dx$$

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d*cos(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin{(a+bx)})^m}{\sqrt{d\cos{(a+bx)}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(1/2),x)
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{\sqrt{d\cos{(bx+a)}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{\sqrt{d\cos{(bx+a)}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin{(a+b\,x)})^m}{\sqrt{d\,\cos{(a+b\,x)}}}\,dx$$

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2), x)
```

3.353 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx=\frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt{d\cos(a+bx)}}$$

```
[Out] (cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2657}

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx=\frac{\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{bd^2(1+m)}$$

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 +
 m))
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}dx$$

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^2*cos(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(a+bx)})^m}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin{(a+b\,x)})^m}{(d\,\cos{(a+b\,x)})^{3/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2), x)
```

3.354 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)(d\cos(a+bx))^{3/2}}$$

```
[Out] (cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2657}

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m \text{t}}{bd^2(1+m)\sqrt{d\cos(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 + m)*Sqrt[d*Cos[a + b*
x]])
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}dx$$

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^3*cos(b*x + a)^3), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin{(a+bx)})^m}{(d\cos{(a+bx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(5/2),x)
[Out] Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\cos{(bx+a)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\,\sin{(a+b\,x)})^m}{(d\,\cos{(a+b\,x)})^{5/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2), x)
```

3.355 $\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)} + \frac{2(d\cos(a+bx))^{3+n}}{bd^3(3+n)} - \frac{(d\cos(a+bx))^{5+n}}{bd^5(5+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)+2*(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)-(d*cos(b
*x+a))^(5+n)/b/d^5/(5+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 276}

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n+5}}{bd^5(n+5)} + \frac{2(d\cos(a+bx))^{n+3}}{bd^3(n+3)} - \frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (2*(d*Cos[a + b*x])^(3 + n))/(b
*d^3*(3 + n)) - (d*Cos[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)^2\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int\left(x^n-\frac{2x^{2+n}}{d^2}+\frac{x^{4+n}}{d^4}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\cos(a+bx))^{3+n}}{bd^3(3+n)}-\frac{(d\cos(a+bx))^{5+n}}{bd^5(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int (d\cos(a+bx))^n\sin^5(a+bx)\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\left(89+28n+3n^2-4(7+8n+n^2)\cos(2(a+bx))+(3+4n+n^2)\cos(4(a+\right.}{8b(1+n)(3+n)(5+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -1/8*(Cos[a + b*x]*(d*Cos[a + b*x])^n*(89 + 28*n + 3*n^2 - 4*(7 + 8*n + n^2
)*Cos[2*(a + b*x)] + (3 + 4*n + n^2)*Cos[4*(a + b*x)]))/(b*(1 + n)*(3 + n)*
(5 + n))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$-\frac{\left(\left(-\frac{3}{2}n^2-14n-\frac{25}{2}\right)\cos(3bx+3a)+\left(\frac{1}{2}n^2+2n+\frac{3}{2}\right)\cos(5bx+5a)+\cos(bx+a)\left(n^2+12n+75\right)\right)(d\cos(bx+a))^n}{8(n^3+9n^2+23n+15)b}$	87
derivativedivides	$-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}+\frac{2(\cos^3(bx+a))e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{(\cos^5(bx+a))e^{n\ln(d\cos(bx+a))}}{b(5+n)}$	90
default	$-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}+\frac{2(\cos^3(bx+a))e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{(\cos^5(bx+a))e^{n\ln(d\cos(bx+a))}}{b(5+n)}$	90

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((-3/2*n^2-14*n-25/2)*cos(3*b*x+3*a)+(1/2*n^2+2*n+3/2)*cos(5*b*x+5*a)+
cos(b*x+a)*(n^2+12*n+75))*(d*cos(b*x+a))^n/(n^3+9*n^2+23*n+15)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx =$$
$$-\frac{\left((n^2+4n+3)\cos{(bx+a)}^5-2(n^2+6n+5)\cos{(bx+a)}^3+(n^2+8n+15)\cos{(bx+a)}\right)(d\cos{(bx+a)})^n}{bn^3+9bn^2+23bn+15b}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -((n^2 + 4*n + 3)*cos(b*x + a)^5 - 2*(n^2 + 6*n + 5)*cos(b*x + a)^3 + (n^2
+ 8*n + 15)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^3 + 9*b*n^2 + 23*b*n + 15
*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(60) = 120$.

Time = 4.42 (sec) , antiderivative size = 2451, normalized size of antiderivative = 32.25

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**5, Eq(b, 0)), ((-log(cos(a + b*x))/b + s
in(a + b*x)**4/(4*b*cos(a + b*x)**4) - sin(a + b*x)**2/(2*b*cos(a + b*x)**2
))/d**5, Eq(n, -5)), ((2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**8/(b*t
an(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2)
- 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 +
 b) + 2*log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*
```

x/2)**4 + b) + 2*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 2*log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 8*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4

```
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b))/d, Eq(n, -1)), (-n**2*(d*cos(a + b*x))**n*sin(a + b*x)**4*cos(a + b*x
)/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 8*n*(d*cos(a + b*x))**n*sin(a + b*x
)**4*cos(a + b*x)/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*n*(d*cos(a + b*x)
)**n*sin(a + b*x)**2*cos(a + b*x)**3/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) -
15*(d*cos(a + b*x))**n*sin(a + b*x)**4*cos(a + b*x)/(b*n**3 + 9*b*n**2 + 23
*b*n + 15*b) - 20*(d*cos(a + b*x))**n*sin(a + b*x)**2*cos(a + b*x)**3/(b*n*
*3 + 9*b*n**2 + 23*b*n + 15*b) - 8*(d*cos(a + b*x))**n*cos(a + b*x)**5/(b*n
**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{\frac{d^n\cos(bx+a)^n\cos(bx+a)^5}{n+5} - \frac{2\,d^n\cos(bx+a)^n\cos(bx+a)^3}{n+3} + \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -(d^n*cos(b*x + a)^n*cos(b*x + a)^5/(n + 5) - 2*d^n*cos(b*x + a)^n*cos(b*x
+ a)^3/(n + 3) + (d*cos(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.28

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{(d\cos(bx+a))^n d^5 n^2 \cos(bx+a)^5 + 4\,(d\cos(bx+a))^n d^5 n \cos(bx+a)^5 - 2\,(d\cos(bx+a))^n d^5 n^2 \cos}{}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -((d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^5 + 4*(d*cos(b*x + a))^n*d^5*n*co
s(b*x + a)^5 - 2*(d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^3 + 3*(d*cos(b*x +
 a))^n*d^5*cos(b*x + a)^5 - 12*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a)^3 + (d
*cos(b*x + a))^n*d^5*n^2*cos(b*x + a) - 10*(d*cos(b*x + a))^n*d^5*cos(b*x +
 a)^3 + 8*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a) + 15*(d*cos(b*x + a))^n*d^5
*cos(b*x + a))/((d^4*n^3 + 9*d^4*n^2 + 23*d^4*n + 15*d^4)*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx =$$
$$-\frac{(d\cos(a+b\,x))^n\,(150\cos(a+b\,x)-25\cos(3\,a+3\,b\,x)+3\cos(5\,a+5\,b\,x)+24\,n\cos(a+b\,x)-28\,n}{16\,b\,(n^3+9\,n}$$

```
[In] int(sin(a + b*x)^5*(d*cos(a + b*x))^n,x)
```

```
[Out] -((d*cos(a + b*x))^n*(150*cos(a + b*x) - 25*cos(3*a + 3*b*x) + 3*cos(5*a +
5*b*x) + 24*n*cos(a + b*x) - 28*n*cos(3*a + 3*b*x) + 4*n*cos(5*a + 5*b*x) +
 2*n^2*cos(a + b*x) - 3*n^2*cos(3*a + 3*b*x) + n^2*cos(5*a + 5*b*x)))/(16*b
*(23*n + 9*n^2 + n^3 + 15))
```

3.356 $\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)} + \frac{(d\cos(a+bx))^{3+n}}{bd^3(3+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)+(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 14}

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{(d\cos(a+bx))^{n+3}}{bd^3(n+3)} - \frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (d*Cos[a + b*x])^(3 + n)/(b*d^3
*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{\text{Subst}\left(\int \left(x^n-\frac{x^{2+n}}{d^2}\right)\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\cos(a+bx))^{3+n}}{bd^3(3+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n\sin^3(a+bx)\,dx = \frac{\cos(a+bx)(d\cos(a+bx))^n(-5-n+(1+n)\cos(2(a+bx)))}{2b(1+n)(3+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
[Out] (Cos[a + b*x]*(d*Cos[a + b*x])^n*(-5 - n + (1 + n)*Cos[2*(a + b*x)]))/(2*b*
(1 + n)*(3 + n))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{\left((-n-1)\cos(3bx+3a)+\cos(bx+a)(n+9)\right)(d\cos(bx+a))^n}{4b(3+n)(1+n)}$	52
derivativedivides	$\frac{\left(\cos^3(bx+a)\right)e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}$	59
default	$\frac{\left(\cos^3(bx+a)\right)e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}$	59

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*((-n-1)*cos(3*b*x+3*a)+cos(b*x+a)*(n+9))*(d*cos(b*x+a))^n/b/(3+n)/(1+n
)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{\left((n+1)\cos\left(bx+a\right)^3-(n+3)\cos\left(bx+a\right)\right)\left(d\cos\left(bx+a\right)\right)^n}{bn^2+4\,bn+3\,b}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((n + 1)*cos(b*x + a)^3 - (n + 3)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^2 +
 4*b*n + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(37) = 74.

Time = 1.17 (sec) , antiderivative size = 688, normalized size of antiderivative = 13.76

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**3, Eq(b, 0)), ((log(cos(a + b*x))/b + si
n(a + b*x)**2/(2*b*cos(a + b*x)**2))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x
/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)*
*2 + b) - 2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*
log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*
tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*
x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/
2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 +
2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b))/d, Eq(n, -1)), (-n*(d*cos(a + b*x))**n*sin
(a + b*x)**2*cos(a + b*x)/(b*n**2 + 4*b*n + 3*b) - 3*(d*cos(a + b*x))**n*si
n(a + b*x)**2*cos(a + b*x)/(b*n**2 + 4*b*n + 3*b) - 2*(d*cos(a + b*x))**n*c
os(a + b*x)**3/(b*n**2 + 4*b*n + 3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{\frac{d^n\cos(bx+a)^n\cos(bx+a)^3}{n+3} - \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (d^n*cos(b*x + a)^n*cos(b*x + a)^3/(n + 3) - (d*cos(b*x + a))^(n + 1)/(d*(n
 + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(50) = 100.

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.34

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$$
$$= \frac{(d\cos(bx+a))^n d^3 n\cos(bx+a)^3 + (d\cos(bx+a))^n d^3\cos(bx+a)^3 - (d\cos(bx+a))^n d^3 n\cos(bx+a) -}{(d^2n^2+4\,d^2n+3\,d^2)bd}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((d*cos(b*x + a))^n*d^3*n*cos(b*x + a)^3 + (d*cos(b*x + a))^n*d^3*cos(b*x +
 a)^3 - (d*cos(b*x + a))^n*d^3*n*cos(b*x + a) - 3*(d*cos(b*x + a))^n*d^3*co
s(b*x + a))/((d^2*n^2 + 4*d^2*n + 3*d^2)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx =$$
$$-\frac{(d\,\cos(a+b\,x))^n\,(9\,\cos(a+b\,x)-\cos(3\,a+3\,b\,x)+n\,\cos(a+b\,x)-n\,\cos(3\,a+3\,b\,x))}{4\,b\,(n^2+4\,n+3)}$$

```
[In] int(sin(a + b*x)^3*(d*cos(a + b*x))^n,x)
```

```
[Out] -((d*cos(a + b*x))^n*(9*cos(a + b*x) - cos(3*a + 3*b*x) + n*cos(a + b*x) -
n*cos(3*a + 3*b*x)))/(4*b*(4*n + n^2 + 3))
```

3.357 $\int (d\cos(a+bx))^n \sin(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2645, 30}

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^n\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n\sin(a+bx)\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n}{b(1+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n)/(b*(1 + n)))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$
default	$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$
parallelrisch	$-\frac{(d\cos(bx+a))^n\cos(bx+a)}{b(1+n)}$
norman	$\frac{\frac{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)e^{n\ln\left(\frac{d\left(1-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}}{b(1+n)}-\frac{e^{n\ln\left(\frac{d\left(1-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}}{b(1+n)}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
risch	$\left(e^{i(bx+a)}\right)^{-n}(\sin(bx)\sin(a)-\cos(bx)\cos(a))\left(e^{2i(bx+a)}+1\right)^n\left(\frac{1}{2}\right)^n d^n e^{-\frac{i\pi n\left(\text{csgn}(id(\cos(bx)\cos(a)-\sin(bx)\sin(a)))\,\text{csgn}(id)\,\text{csg}\right.}{}}$

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(bx+a))^n\cos(bx+a)}{bn+b}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="fricas")
[Out] -(d*cos(b*x + a))^n*cos(b*x + a)/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(19) = 38.

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = \begin{cases} \frac{x\sin(a)}{d\cos(a)} & \text{for } b=0 \wedge n=-1 \\ x(d\cos(a))^n\sin(a) & \text{for } b=0 \\ -\frac{\log(\cos(a+bx))}{bd} & \text{for } n=-1 \\ -\frac{(d\cos(a+bx))^n\cos(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a),x)
[Out] Piecewise((x*sin(a)/(d*cos(a)), Eq(b, 0) & Eq(n, -1)), (x*(d*cos(a))**n*sin
(a), Eq(b, 0)), (-log(cos(a + b*x))/(b*d), Eq(n, -1)), (-(d*cos(a + b*x))**
n*cos(a + b*x)/(b*n + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="maxima")
[Out] -(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin(a+bx)\, dx = -\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="giac")
[Out] -(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n \sin(a+bx)\, dx = -\frac{\cos(a+b\,x)\ (d\ \cos(a+b\,x))^n}{b\ (n+1)}$$

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^n,x)
[Out] -(cos(a + b*x)*(d*cos(a + b*x))^n)/(b*(n + 1))
```

3.358 $\int (d\cos(a+bx))^n \csc(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2645, 371}

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^n}{1-\frac{x^2}{d^2}} \, dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n} \text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int (d\cos(a+bx))^n \csc(a+bx) \, dx$$
$$= -\frac{\cos(a+bx)(d\cos(a+bx))^n \text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, 1+\frac{1+n}{2}, \cos^2(a+bx)\right)}{b(1+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Cos[a + b*x]^2])/(b*(1 + n)))
```

Maple [F]

$$\int (d\cos(bx+a))^n \csc(bx+a) \, dx$$

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a),x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(a+bx))^n \csc(a+bx)\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a),x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int \frac{(d\ \cos(a+b\,x))^n}{\sin(a+b\,x)}\,dx$$

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x), x)
```

3.359 $\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 371}

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(2, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. $2(49) = 98$.

Time = 0.90 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.14

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx =$$
$$-\frac{2^{-3-n}\cos(a+bx)(d\cos(a+bx))^n \left(2^{1+n}\,\text{Hypergeometric2F1}(1, 1+n, 2+n, \cos(a+bx)) + 2^{1+n}\,\text{Hyperg}\right.}{}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] -((2^(-3 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(2^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, Cos[a + b*x]] + 2^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 +
n, Cos[a + b*x]] + (Hypergeometric2F1[n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)/2] + Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec
[(a + b*x)/2]^2)/2])*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

Maple [F]

$$\int (d\cos(bx+a))^n \left(\csc^3(bx+a)\right) dx$$

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**3,x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**3, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^n}{\sin(a+b\,x)^3}\,dx$$

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^3,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^3, x)
```

3.360 $\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2645, 371}

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^3} \, dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n} \text{Hypergeometric2F1}\left(3, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(49) = 98$.

Time = 1.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.98

$$\int (d\cos(a+bx))^n \csc^5(a+bx) \, dx =$$
$$-\frac{2^{-5-n}\cos(a+bx)(d\cos(a+bx))^n \left(3\ 2^{1+n} \text{Hypergeometric2F1}(1, 1+n, 2+n, \cos(a+bx)) + 3\ 2^{1+n} \text{Hy}\right.}{}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] -((2^(-5 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(3*2^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, Cos[a + b*x]] + 3*2^(1 + n)*Hypergeometric2F1[2, 1 + n,
2 + n, Cos[a + b*x]] + 2^(2 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, Cos[a +
 b*x]] + 2*Hypergeometric2F1[-1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a + b
*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n) + 3*Hypergeometric2F1[n, 1 + n, 2
 + n, (Cos[a + b*x]*Sec[(a + b*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n) + 3
*Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a + b*x)/2]^2)/2
]*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

Maple [F]

$$\int (d\cos(bx+a))^n \left(\csc^5(bx+a)\right) dx$$

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\, dx = \int (d\cos(bx+a))^n \csc(bx+a)^5\, dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\, dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\, dx = \int (d\cos(bx+a))^n \csc(bx+a)^5\, dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^5\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^n}{\sin(a+b\,x)^5}\,dx$$

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^5,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^5, x)
```

3.361 $\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
```

```
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(2(a+bx))}{2b(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
[Out] -1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Maple [F]

$$\int (d\cos(bx+a))^n\left(\sin^4(bx+a)\right)dx$$

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^n\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="fricas")
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^n\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^n\sin(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^n\,dx$$

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^n,x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^n, x)
```

3.362 $\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([-1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = -\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
```

```
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(2(a+bx))}{2b(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
[Out] -1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Maple [F]

$$\int (d\cos(bx+a))^n\left(\sin^2(bx+a)\right)dx$$

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^n\sin(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx=\int (d\cos{(a+bx)})^n\sin^2{(a+bx)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**2,x)
[Out] Integral((d*cos(a + b*x))**n*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx=\int (d\cos{(bx+a)})^n\sin{(bx+a)}^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx=\int (d\cos{(bx+a)})^n\sin{(bx+a)}^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\sin^2(a+bx)\,dx=\int \sin{(a+b\,x)}^2\,(d\,\cos{(a+b\,x)})^n\,dx$$

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^n,x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^n, x)
```

3.363 $\int (d\cos(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int (d\cos(a+bx))^n\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*
sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2722}

$$\int (d\cos(a+bx))^n\,dx = -\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int (d\cos(a+bx))^n\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n,x]
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F]

$$\int (d\cos(bx+a))^n\,dx$$

```
[In] int((d*cos(b*x+a))^n,x)
[Out] int((d*cos(b*x+a))^n,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos(bx+a))^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos{(a+bx)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))**n,x)
[Out] Integral((d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos{(bx+a)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos{(bx+a)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\,dx = \int (d\,\cos{(a+b\,x)})^n\,dx$$

```
[In] int((d*cos(a + b*x))^n,x)
[Out] int((d*cos(a + b*x))^n, x)
```

3.364 $\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([3/2, 1/2+1/2*n],[3/2+1/2*n],cos
(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = -\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[3/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
```

```
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int (d\cos(a+bx))^n\csc^2(a+bx)\,dx = \frac{d(d\cos(a+bx))^{-1+n}\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},1-\frac{n}{2},2-\frac{n}{2},\csc^2(a+bx\right.}{b(-2+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]*Hyp
ergeometric2F1[(1 - n)/2, 1 - n/2, 2 - n/2, Csc[a + b*x]^2])/(b*(-2 + n))
```

Maple [F]

$$\int (d\cos(bx+a))^n\left(\csc^2(bx+a)\right)dx$$

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^n\csc(bx+a)^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(a+bx)})^n \csc^2{(a+bx)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**2,x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**2, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^2\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int \frac{(d\,\cos{(a+b\,x)})^n}{\sin{(a+b\,x)}^2}\,dx$$

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^2, x)
```

3.365 $\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([5/2, 1/2+1/2*n],[3/2+1/2*n],cos
(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2656}

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = -\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[5/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
```

```
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral =
$$-\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int (d\cos(a+bx))^n\csc^4(a+bx)\,dx = \frac{d(d\cos(a+bx))^{-1+n}\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},2-\frac{n}{2},3-\frac{n}{2},\csc^2(a+bx)\right)}{b(-4+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^3*H
ypergeometric2F1[(1 - n)/2, 2 - n/2, 3 - n/2, Csc[a + b*x]^2])/(b*(-4 + n))
```

Maple [F]

$$\int (d\cos(bx+a))^n\left(\csc^4(bx+a)\right)dx$$

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\csc^4(a+bx)\,dx = \int (d\cos(bx+a))^n\csc(bx+a)^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="fricas")
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(a+bx)})^n \csc^4{(a+bx)}\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**4,x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**4, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^4\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int \frac{(d\,\cos{(a+b\,x)})^n}{\sin{(a+b\,x)}^4}\,dx$$

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^4,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^4, x)
```

3.366 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{bd(1+n)\sin^2(a+bx)^{3/4}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([-3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(c*sin(b*x+a))^(3/2)/b/d/(1+n)/(sin(b*x+a)^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2656}

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = -\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*d*(1 + n)*(Sin[a + b*x]^2)^(3/4
)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

integral

$$= -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{bd(1+n)\sin^2(a+bx)^{3/4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(76) = 152$.

Time = 0.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.08

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = \frac{(d\cos(a+bx))^n\cot(a+bx)\left(-\left((3+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\right)-\right.}{}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] ((d*Cos[a + b*x])^n*Cot[a + b*x]*(-((3 + n)*Hypergeometric2F1[-3/4, (1 + n)
/2, (3 + n)/2, Cos[a + b*x]^2]) - (3 + n)*Hypergeometric2F1[1/4, (1 + n)/2,
 (3 + n)/2, Cos[a + b*x]^2] + (1 + n)*Cos[a + b*x]^2*Hypergeometric2F1[1/4,
 (3 + n)/2, (5 + n)/2, Cos[a + b*x]^2])*(c*Sin[a + b*x])^(5/2))/(2*b*(1 + n
)*(3 + n)*(Sin[a + b*x]^2)^(3/4))
```

Maple [F]

$$\int (d\cos(bx+a))^n (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^
n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (d\,\cos(a+b\,x))^n\,(c\,\sin(a+b\,x))^{5/2}\,dx$$

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2), x)
```

3.367 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}}{bd(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([-1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(c*sin(b*x+a))^(1/2)/b/d/(1+n)/(sin(b*x+a)^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2656}

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*d*(1 + n)*(Sin[a + b*x]^2)^(1/4))
)
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}}{bd(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{b(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[-1/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*(1 + n)*(Sin[a + b*x]^2)^(
1/4)))
```

Maple [F]

$$\int (d\cos(bx+a))^n\,(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2} \, dx = \int (c\sin(bx+a))^{\frac{3}{2}} (d\cos(bx+a))^n \, dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2} \, dx = \int (c\sin(a+bx))^{\frac{3}{2}} (d\cos(a+bx))^n \, dx$$

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**(3/2)*(d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2} \, dx = \int (c\sin(bx+a))^{\frac{3}{2}} (d\cos(bx+a))^n \, dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2} \, dx = \int (c\sin(bx+a))^{\frac{3}{2}} (d\cos(bx+a))^n \, dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (d\,\cos\left(a+b\,x\right))^n\,(c\,\sin\left(a+b\,x\right))^{3/2}\,dx$$

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2), x)
```

3.368 $\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bd(1+n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*(sin(b*x+a)^2)^(1/4)/b/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2656}

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin(a+bx)}}{b(1+n)\sin^2(a+bx)^{3/4}}$$

```
[In] Integrate[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]])/(b*(1 + n)*(Sin[a +
 b*x]^2)^(3/4)))
```

Maple [F]

$$\int (d\cos(bx+a))^n\sqrt{c\sin(bx+a)}dx$$

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin(bx+a)}(d\cos(bx+a))^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(a+bx)}}(d\cos{(a+bx)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}(d\cos{(bx+a)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}(d\cos{(bx+a)})^n\,dx$$

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int {\left(d\,\cos\left(a+b\,x\right)\right)}^n\,\sqrt{c\,\sin\left(a+b\,x\right)}\,dx$$

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2), x)
```

3.369 $\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{3/4}}{bd(1+n)(c\sin(a+bx))^{3/2}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*(sin(b*x+a)^2)^(3/4)/b/d/(1+n)/(c*sin(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2656}

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

```
[In] Int[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(3/4))/(b*d*(1 + n)*(c*Sin[a + b*x])^(3/2)
))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{3/4}}{bd(1+n)(c\sin(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int\frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{c\sin(a+bx)}\sqrt[4]{\sin^2(a+bx)}}$$

```
[In] Integrate[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[c*Sin[a + b*x]]*(Sin[a +
 b*x]^2)^(1/4)))
```

Maple [F]

$$\int\frac{(d\cos(bx+a))^n}{\sqrt{c\sin(bx+a)}}dx$$

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int\frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int\frac{(d\cos(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(a+bx)})^n}{\sqrt{c\sin{(a+bx)}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
[Out] Integral((d*cos(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos{(a+b\,x)})^n}{\sqrt{c\,\sin{(a+b\,x)}}}\,dx$$

```
[In] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

3.370 $\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bcd(1+n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([5/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*
(sin(b*x+a)^2)^(1/4)/b/c/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2656}

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[In] Int[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*c*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\text{integral} = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bcd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}\sqrt[4]{\sin^2(a+bx)}}{bc^2(1+n)}$$

```
[In] Integrate[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*(Sin[a + b*x]^2)^(1/4))/(b*c^2*(1
 + n)))
```

Maple [F]

$$\int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}dx$$

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
[Out] Integral((d*cos(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\,\cos(a+b\,x))^n}{(c\,\sin(a+b\,x))^{3/2}}\,dx$$

```
[In] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

3.371 $\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2b^7}{13f(b\sec(e+fx))^{13/2}} - \frac{2b^5}{3f(b\sec(e+fx))^{9/2}} + \frac{6b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/13*b^7/f/(b*sec(f*x+e))^(13/2)-2/3*b^5/f/(b*sec(f*x+e))^(9/2)+6/5*b^3/f/(
b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2b^7}{13f(b\sec(e+fx))^{13/2}} - \frac{2b^5}{3f(b\sec(e+fx))^{9/2}} + \frac{6b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(13*f*(b*Sec[e + f*x])^(13/2)) - (2*b^5)/(3*f*(b*Sec[e + f*x])^(9/2
)) + (6*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^7\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{15/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^7\text{Subst}\left(\int \left(-\frac{1}{x^{15/2}}+\frac{3}{b^2x^{11/2}}-\frac{3}{b^4x^{7/2}}+\frac{1}{b^6x^{3/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$$
$$= \frac{(-8939\cos(e+fx)+887\cos(3(e+fx))-155\cos(5(e+fx))+15\cos(7(e+fx)))\sqrt{b\sec(e+fx)}}{6240f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
[Out] ((-8939*Cos[e + f*x] + 887*Cos[3*(e + f*x)] - 155*Cos[5*(e + f*x)] + 15*Cos
[7*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(6240*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(71) = 142.

Time = 1.01 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.24

method	result
default	$\frac{\left(60\left(\cos^7(fx+e)\right)-260\left(\cos^5(fx+e)\right)+468\left(\cos^3(fx+e)\right)-195\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right.\right.}{}$

[In] int(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/390/f*(60*cos(f*x+e)^7-260*cos(f*x+e)^5+468*cos(f*x+e)^3-195*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+195*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-195*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+195*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-780*cos(f*x+e))*(b*sec(f*x+e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2\left(15\cos(fx+e)^7-65\cos(fx+e)^5+117\cos(fx+e)^3-195\cos(fx+e)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{195\,f}$$

[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 2/195*(15*cos(f*x + e)^7 - 65*cos(f*x + e)^5 + 117*cos(f*x + e)^3 - 195*cos(f*x + e))*sqrt(b/cos(f*x + e))/f

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \text{Timed out}$$

[In] integrate(sin(f*x+e)**7*(b*sec(f*x+e))**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2\left(15\,b^6-\frac{65\,b^6}{\cos(fx+e)^2}+\frac{117\,b^6}{\cos(fx+e)^4}-\frac{195\,b^6}{\cos(fx+e)^6}\right)b}{195\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{13}{2}}}$$

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/195*(15*b^6 - 65*b^6/cos(f*x + e)^2 + 117*b^6/cos(f*x + e)^4 - 195*b^6/co
s(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(13/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$$
$$= \frac{2\left(15\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^6-65\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^4+117\sqrt{b\cos(fx+e)}b^6\cos(fx\right.}{195\,b^6 f}$$

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^6 - 65*sqrt(b*cos(f*x + e))
*b^6*cos(f*x + e)^4 + 117*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^2 - 195*sqr
t(b*cos(f*x + e))*b^6)*sgn(cos(f*x + e))/(b^6*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \int \sin(e+f\,x)^7\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2), x)
```

3.372 $\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}} + \frac{4b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/9*b^5/f/(b*sec(f*x+e))^(9/2)+4/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}} + \frac{4b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(9*f*(b*Sec[e + f*x])^(9/2)) + (4*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)
) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^5\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{x^{11/2}}-\frac{2}{b^2x^{7/2}}+\frac{1}{b^4x^{3/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}\\
&= -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\begin{aligned}
&\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx\\
&= -\frac{(554\cos(e+fx)-47\cos(3(e+fx))+5\cos(5(e+fx)))\sqrt{b\sec(e+fx)}}{360f}
\end{aligned}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
[Out] -1/360*((554*Cos[e + f*x] - 47*Cos[3*(e + f*x)] + 5*Cos[5*(e + f*x)])*Sqrt[
b*Sec[e + f*x]])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(53) = 106$.

Time = 0.21 (sec) , antiderivative size = 435, normalized size of antiderivative = 6.90

method	result
default	$-\frac{\left(20\left(\cos^{5}(fx+e)\right)+45\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}\cos(fx+e)-45\,\mathrm{l}}{}$

```
[In] int(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/90/f*(20*cos(f*x+e)^5+45*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-45*ln(2*(2*cos(f*x+e)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-
72*cos(f*x+e)^3+45*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-45*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+
e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+180*cos(f*x+e))*(b*sec(f*x+e))^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2\left(5\,\cos\left(fx+e\right)^5-18\,\cos\left(fx+e\right)^3+45\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45\,f}$$

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*cos(f*x + e)^5 - 18*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(b/cos(f
*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2\left(5\,b^4 - \frac{18\,b^4}{\cos(fx+e)^2} + \frac{45\,b^4}{\cos(fx+e)^4}\right)b}{45\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}$$

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -2/45*(5*b^4 - 18*b^4/cos(f*x + e)^2 + 45*b^4/cos(f*x + e)^4)*b/(f*(b/cos(f
*x + e))^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2\left(5\,\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^4 - 18\,\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^2 + 45\,\sqrt{b\cos(fx+e)}b^4\right)\mathrm{sgn}(\cos(fx+e))}{45\,b^4 f}$$

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] -2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 18*sqrt(b*cos(f*x + e))*
b^4*cos(f*x + e)^2 + 45*sqrt(b*cos(f*x + e))*b^4)*sgn(cos(f*x + e))/(b^4*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = \int \sin(e+f\,x)^5\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2), x)
```

3.373 $\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 14}

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
[Out] (2*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{7/2}}+\frac{1}{b^2x^{3/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{(-17\cos(e+fx)+\cos(3(e+fx)))\sqrt{b\sec(e+fx)}}{10f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
[Out] ((-17*Cos[e + f*x] + Cos[3*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(10*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 425, normalized size of antiderivative = 10.37

method	result
default	$-\frac{\left(5\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)-5\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos}{(\cos}}}{}\right.\right.}{}$

```
[In] int(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/10/f*(5*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-5*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*
x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-4*cos(f*x+e)^3+5*l
n((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*
```

```
x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-5*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)+20*cos(f*x+e))*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(\cos\left(fx+e\right)^3-5\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{5\,f}$$

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/5*(cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(b^2-\frac{5\,b^2}{\cos(fx+e)^2}\right)b}{5\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}$$

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 2/5*(b^2 - 5*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(\sqrt{b\cos\left(fx+e\right)}b^2\cos\left(fx+e\right)^2-5\sqrt{b\cos\left(fx+e\right)}b^2\right)\operatorname{sgn}(\cos\left(fx+e\right))}{5\,b^2f}$$

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 5*sqrt(b*cos(f*x + e))*b^2)*
sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \int \sin\left(e+f\,x\right)^3\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2), x)
```

3.374 $\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx=-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 30}

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx=-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2b}{f\sqrt{b\sec(fx+e)}}$	17
default	$-\frac{2b}{f\sqrt{b\sec(fx+e)}}$	17
risch	$-\frac{2\sqrt{2}\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\cos(fx+e)}{f}$	41

```
[In] int(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*b/f/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{f}$$

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = \int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{f}$$

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{b\cos(fx+e)}\operatorname{sgn}(\cos(fx+e))}{f}$$

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(b*cos(f*x + e))*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\cos(e+fx)\sqrt{\frac{b}{\cos(e+fx)}}}{f}$$

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(1/2),x)
[Out] -(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/f
```

3.375 $\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

```
[Out] arctan((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f-arctanh((b*sec(f*x+e))^(1/2)
/b^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2702, 335, 304, 209, 212}

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

```
[In] Int[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (Sqrt[b]*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf} \\
&= \frac{2\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf} \\
&= -\frac{b\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} \\
&= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$=\frac{\left(2\arctan\left(\sqrt{\sec(e+fx)}\right)+\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(1+\sqrt{\sec(e+fx)}\right)\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt
[Sec[e + f*x]]])*Sqrt[b*Sec[e + f*x]])/(2*f*Sqrt[Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(46) = 92.

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.60

method	result
default	$\frac{\sqrt{b\sec(fx+e)}\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\right)\cos(fx+e}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(b*sec(f*x+e))^(1/2)*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
)-ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*cos(f*x+e)/(cos(f*x+e)
+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.

Time = 0.32 (sec) , antiderivative size = 247, normalized size of antiderivative = 4.26

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6\,b\,c}{\cos(fx+e)^2+2\,\cos(fx+e)+1}\right.}{4\,f}\right.$$
$$-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)-\sqrt{b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}+6\,b\cos(fx+e)-}{\cos(fx+e)^2-2\,\cos(fx+e)+1}\right.}{4\,f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1
)/b) + sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)))/f, -1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f
*x + e) - 1)/sqrt(b)) - sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
 cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*
x + e)^2 - 2*cos(f*x + e) + 1)))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\csc{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b\left(\frac{2\,\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}\right)}{2\,f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + log(-(sqrt(b) - sqr
t(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b^2\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}\right)\operatorname{sgn}(\cos(fx+e))}{f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] b^2*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b) - arctan(sqrt(b*cos
(f*x + e))/sqrt(b))/b^(3/2))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin(e+f\,x)}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x),x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x), x)
```

3.376 $\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}$$

```
[Out] -1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/f+3/4*arctan((b*sec(f*x+e))^(1/2)/
b^(1/2))*b^(1/2)/f-3/4*arctanh((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2702, 294, 335, 304, 209, 212}

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}$$

```
[In] Int[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (3*Sqrt[b]*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3
/2))/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} - \frac{(3b)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} + \frac{(3b)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f}$$

$$= \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{\left(-6\arctan\left(\sqrt{\sec(e+fx)}\right) - 3\log\left(1-\sqrt{\sec(e+fx)}\right) + 3\log\left(1+\sqrt{\sec(e+fx)}\right) + \frac{4\csc^2(e+fx)}{\sqrt{\sec(e+fx)}}\right)\sqrt{}}{8f\sqrt{\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
[Out] -1/8*((-6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Lo
g[1 + Sqrt[Sec[e + f*x]]] + (4*Csc[e + f*x]^2)/Sqrt[Sec[e + f*x]])*Sqrt[b*S
ec[e + f*x]])/(f*Sqrt[Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 461, normalized size of antiderivative = 4.96

method	result
default	$-\frac{\sqrt{-\frac{b\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1\right)}{(1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)-1}}\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)-1\right)\left(-(1-\cos(fx+e))^4\sqrt{(1-\cos(fx+e))^4(\csc^4(fx+e))-}\right.}{}$

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(-b*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(-(1-cos(f*x+e))^4*((1-cos(f*x
+e))^4*csc(f*x+e)^4-1)^(1/2)*csc(f*x+e)^4+((1-cos(f*x+e))^4*csc(f*x+e)^4-1)
^(3/2)-(1-cos(f*x+e))^2*((1-cos(f*x+e))^4*csc(f*x+e)^4-1)^(1/2)*csc(f*x+e)^
2+ln((1-cos(f*x+e))^2*csc(f*x+e)^2+((1-cos(f*x+e))^4*csc(f*x+e)^4-1)^(1/2))
*(1-cos(f*x+e))^2*csc(f*x+e)^2+3*arctan(1/((1-cos(f*x+e))^4*csc(f*x+e)^4-1)
^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2-4*ln(2*(1-cos(f*x+e))^2*csc(f*x+e)^2+
2*((1-cos(f*x+e))^4*csc(f*x+e)^4-1)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2)/(
((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/
(1-cos(f*x+e))^2*sin(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(73) = 146$.

Time = 0.32 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.81

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \left[\frac{6\left(\cos(fx+e)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+3\left(\cos(fx+e)^2-1\right)\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2\cdots}{\cdots}\right)\cdots}{16\left(f\cos(fx+e)^2-f\right)}\cdots, -\frac{6\left(\cos(fx+e)^2-1\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)-3\left(\cos(fx+e)^2-1\right)\sqrt{b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos\cdots\right.}{\cdots}\right)\cdots}{16\left(f\cos(fx+e)^2-f\right)}\cdots\right]$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
 + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*sqrt(b/co
s(f*x + e))*cos(f*x + e))/(f*cos(f*x + e)^2 - f), -1/16*(6*(cos(f*x + e)^2
- 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) -
3*(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 - 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(f*co
s(f*x + e)^2 - f)]
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\csc^3{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b\left(\frac{4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} + \frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}} + \frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}\right)}{8f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 6*arctan(sqrt(
b/cos(f*x + e))/sqrt(b))/sqrt(b) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/
(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b^4\left(\frac{2\sqrt{b\cos(fx+e)}}{\left(b^2\cos(fx+e)^2-b^2\right)b^2} + \frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3} - \frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}\right)\operatorname{sgn}(\cos{(fx+e)})}{4f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] 1/4*b^4*(2*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^2) + 3*arctan
(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(sqrt(b*cos(f*x +
e))/sqrt(b))/b^(7/2))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^3}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3,x)
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

3.377 $\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{21\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}$$

```
[Out] -7/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))
^(7/2)/b^3/f+21/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f-21/32*arc
tanh((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{2702, 294, 335, 304, 209, 212\}$

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{21\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f} - \frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf}$$

```
[In] Int[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (21*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (21*Sqrt[b]*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (7*Cot[e + f*x]^2*(b*Sec[e + f*
x])^(3/2))/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(7/2))/(4*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{7\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{32bf}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{16bf}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} - \frac{(21b)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f} + \frac{(21b)\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f}$$

$$= \frac{21\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b\left(-28\csc^2(e+fx) - 16\csc^4(e+fx) + 42\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)} + 21\left(\log\left(1-\sqrt{\sec(e-\right.\right.\right.}{64f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-28*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*
Sqrt[Sec[e + f*x]] + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e +
 f*x]]])*Sqrt[Sec[e + f*x]]))/(64*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(99) = 198.

Time = 0.23 (sec) , antiderivative size = 509, normalized size of antiderivative = 4.14

method	result
default	$-\frac{\sqrt{-\frac{b\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1\right)}{(1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)-1}}\left((1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)-1\right)\left(-11(1-\cos(fx+e))^6\sqrt{(1-\cos(fx+e))^4(\csc^4(fx+e))-}\right.}{}$

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(-b*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(-11*(1-cos(f*x+e))^6*((1-cos
(f*x+e))^4*csc(f*x+e)^4-1)^(1/2)*csc(f*x+e)^6+10*((1-cos(f*x+e))^4*csc(f*x+
e)^4-1)^(3/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-11*(1-cos(f*x+e))^4*((1-cos(f*x
+e))^4*csc(f*x+e)^4-1)^(1/2)*csc(f*x+e)^4+11*ln((1-cos(f*x+e))^2*csc(f*x+e)
^2+((1-cos(f*x+e))^4*csc(f*x+e)^4-1)^(1/2))*(1-cos(f*x+e))^4*csc(f*x+e)^4+2
1*arctan(1/((1-cos(f*x+e))^4*csc(f*x+e)^4-1)^(1/2))*(1-cos(f*x+e))^4*csc(f*
x+e)^4-32*ln(2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*((1-cos(f*x+e))^4*csc(f*x+e)
^4-1)^(1/2))*(1-cos(f*x+e))^4*csc(f*x+e)^4+((1-cos(f*x+e))^4*csc(f*x+e)^4-1
```

```
)^(3/2))/(((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*((1-cos(f*x+e))^2*csc(f*x+e)^2-
1))^(1/2)/(1-cos(f*x+e))^4*sin(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(99) = 198$.

Time = 0.35 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.56

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \left[\frac{42\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+21\left(\cos\left(fx+e\right)^4-2\right.}{128}\right.$$
$$-\frac{42\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)-21\left(\cos\left(fx+e\right)^4-2\cos\right.}{128\,(f}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(42*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(cos(f*x + e)^4 - 2*co
s(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)) + 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b
/cos(f*x + e)))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f), -1/128*(42*(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e)
)*(cos(f*x + e) - 1)/sqrt(b)) - 21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*
sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*s
qrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e
) + 1)) - 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(f*c
os(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)]
```

Sympy [F]

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(e+fx\right)}\csc^5\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{b\left(\frac{42\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}} + \frac{21\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}} + \frac{4\left(7\,b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} - 11\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}\right)}{b^4 - \frac{2\,b^4}{\cos(fx+e)^2} + \frac{b^4}{\cos(fx+e)^4}}\right)}{64\,f}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(42*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + 21*log(-(sqrt(b)
- sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b) + 4*(7*b^
2*(b/cos(f*x + e))^(3/2) - 11*(b/cos(f*x + e))^(7/2))/(b^4 - 2*b^4/cos(f*x
+ e)^2 + b^4/cos(f*x + e)^4))/f
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{b^6\left(\frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5} - \frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}} + \frac{2\left(7\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2 - 11\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2 - b^2\right)^2 b^4}\right)\mathrm{sgn}(\cos(fx+e))}{32\,f}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^6*(21*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arct
an(sqrt(b*cos(f*x + e))/sqrt(b))/b^(11/2) + 2*(7*sqrt(b*cos(f*x + e))*b^2*c
os(f*x + e)^2 - 11*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2)^2*
b^4))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^5}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5,x)
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5, x)
```

3.378 $\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \frac{80\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77f} - \frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

```
[Out] -40/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-20/77*b*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(1/2)-2/11*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)+80/77*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos
(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 3856, 2720}

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = -\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}} - \frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} + \frac{80\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77f}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

```
[Out] (80*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*
f) - (40*b*Sin[e + f*x])/(77*f*Sqrt[b*Sec[e + f*x]]) - (20*b*Sin[e + f*x]^3
)/(77*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^5)/(11*f*Sqrt[b*Sec[e + f
*x]])
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{10}{11}\int\sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx\\
&= -\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{60}{77}\int\sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx\\
&= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{40}{77}\int\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{77}\left(40\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\
&= \frac{80\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77f}\\
&\quad-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx$$
$$=\frac{\sqrt{b\sec(e+fx)}\left(1280\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-435\sin(2(e+fx))+68\sin(4(e+fx))-7\sin\right.}{1232f}$$

[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]

[Out] (Sqrt[b*Sec[e + f*x]]*(1280*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 435*Sin[2*(e + f*x)] + 68*Sin[4*(e + f*x)] - 7*Sin[6*(e + f*x)]))/(1232*f)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

method	result
default	$\frac{2\left(-7(\cos^5(fx+e))\sin(fx+e)+40i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\cot(fx+e)-\csc(fx+e)),i)\cos(fx+e)+40i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos}{\cos(}}\right.}{77f}$

[In] int(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/77/f*(-7*cos(f*x+e)^5*sin(f*x+e)+40*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)+40*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+24*cos(f*x+e)^3*sin(f*x+e)-37*sin(f*x+e)*cos(f*x+e))*(b*sec(f*x+e))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx =$$
$$-\frac{2\left(\left(7\cos(fx+e)^5-24\cos(fx+e)^3+37\cos(fx+e)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)+20i\sqrt{2}\sqrt{b}\text{weierstrass}\right.}{}$$

[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

```
[Out] -2/77*((7*cos(f*x + e)^5 - 24*cos(f*x + e)^3 + 37*cos(f*x + e))*sqrt(b/cos(
f*x + e))*sin(f*x + e) + 20*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, co
s(f*x + e) + I*sin(f*x + e)) - 20*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4,
 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^6\,dx$$

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^6\,dx$$

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sin\left(e+f\,x\right)^6\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2), x)
```

3.379 $\int \sqrt{b \sec(e+fx)} \sin^4(e+fx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sqrt{b \sec(e+fx)} \sin^4(e+fx) \, dx = \frac{8\sqrt{\cos(e+fx)} \operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right) \sqrt{b \sec(e+fx)}}{7f} - \frac{4b \sin(e+fx)}{7f\sqrt{b \sec(e+fx)}} - \frac{2b \sin^3(e+fx)}{7f\sqrt{b \sec(e+fx)}}$$

```
[Out] -4/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-2/7*b*sin(f*x+e)^3/f/(b*sec(f*x+e)
)^(1/2)+8/7*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 3856, 2720}

$$\begin{aligned} &\int \sqrt{b \sec(e+fx)} \sin^4(e+fx) \, dx \\ &= -\frac{2b \sin^3(e+fx)}{7f\sqrt{b \sec(e+fx)}} - \frac{4b \sin(e+fx)}{7f\sqrt{b \sec(e+fx)}} \\ &\quad + \frac{8\sqrt{\cos(e+fx)} \operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right) \sqrt{b \sec(e+fx)}}{7f} \end{aligned}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(7*f)
 - (4*b*Sin[e + f*x])/(7*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(7*
f*Sqrt[b*Sec[e + f*x]])
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{6}{7}\int\sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx\\
&= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{4}{7}\int\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{7}\left(4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\
&= \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{7f}\\
&\quad-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$
$$=\frac{\sqrt{b\sec(e+fx)}\left(32\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-10\sin(2(e+fx))+\sin(4(e+fx))\right)}{28f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(32*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 10
*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(28*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

method	result
default	$\frac{2\left(4i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\cos(fx+e)+4i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx\right.}{7f}$

```
[In] int(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/f*(4*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+cos
(f*x+e)^3*sin(f*x+e)-3*sin(f*x+e)*cos(f*x+e))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$
$$=\frac{2\left(\left(\cos(fx+e)^3-3\cos(fx+e)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)-2i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(fx+\right.}{7f}$$

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*((cos(f*x + e)^3 - 3*cos(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e) -
2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e
```

```
)) + 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2), x)
```

3.380 $\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} - \frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/3*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)+4/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2707, 3856, 2720}

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} - \frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f)
 - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{2}{3}\int\sqrt{b\sec(e+fx)}\,dx \\ &= -\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{3}\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx \\ &= \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int\sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = -\frac{\sqrt{b\sec(e+fx)}\left(-4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sin(2(e+fx))\right)}{3f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
[Out] -1/3*(Sqrt[b*Sec[e + f*x]]*(-4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]
 + Sin[2*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.19

method	result
default	$\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\cot(fx+e)-\csc(fx+e)),i)\cos(fx+e)+2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\cot(fx+e)-\csc(fx}{3f}$

```
[In] int(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/f*(2*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-sin
(f*x+e)*cos(f*x+e))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx =$$
$$-\frac{2\left(\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e\right.}{3f}$$

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2)*sqrt(b)*we
ierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(2)*sqrt(b)
*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)}\sin(fx+e)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)}\sin(fx+e)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2), x)
```

3.381 $\int \sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{b\sec(e+fx)}\,dx=\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2720}

$$\int \sqrt{b\sec(e+fx)}\,dx=\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int\sqrt{b\sec(e+fx)}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]],x]
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

method	result	size
default	$\frac{2i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{b\sec(fx+e)}}{f}$	77

```
[In] int((b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/f*(cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \sqrt{b\sec(e+fx)}\,dx = \frac{-i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))}{f}$$

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(e+fx)}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sqrt{b\sec(e+fx)}\,dx=\frac{2\sqrt{\cos\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,\mathrm{F}\left(\frac{e}{2}+\frac{f\,x}{2}\middle|2\right)}{f}$$

```
[In] int((b/cos(e + f*x))^(1/2),x)
[Out] (2*cos(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)*ellipticF(e/2 + (f*x)/2, 2))/f
```

3.382 $\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x
+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2720}

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f} - \frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\int\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\
&= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$
\int\csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\left(-\cot(e+fx)+\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b\sec(e+fx)}}{f}
$$

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
[Out] ((-Cot[e + f*x] + Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[
e + f*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

method	result
default	$\frac{i\sqrt{b\sec(fx+e)}\left(\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\cos(fx+e)+\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+\right.}{f}$

[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] I/f*(b*sec(f*x+e))^(1/2)*((1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)+(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+I*cot(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}\sqrt{b}\sin(fx+e)\,\text{weier}}{2f\sin(fx+e)}$$

[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/2*(-I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e))/(f*sin(f*x + e))

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(e+fx)}\csc^2(e+fx)\,dx$$

[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**2, x)

Maxima [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin(e+f\,x)^2}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

3.383 $\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[Out] -5/6*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e)
)^(1/2)+5/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2720}

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[In] Int[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*b*Csc[e + f*x])/(6*f*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^3)/(3*f*Sq
rt[b*Sec[e + f*x]]) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
b*Sec[e + f*x]])/(6*f)
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5}{6}\int\csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5}{12}\int\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{12}\left(5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\
&= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\left(-\cot(e+fx)\left(5+2\csc^2(e+fx)\right)+5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-(Cot[e + f*x]*(5 + 2*Csc[e + f*x]^2)) + 5*Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(6*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.96

method	result
default	$\frac{i\sqrt{b\sec(fx+e)}\left(-5\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\left(\sin^2(fx+e)\right)\cos(fx+e)-5\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx}{\cos(fx+}}}{6f(\cos^2(fx+e)-1)}$

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I/f*(b*sec(f*x+e))^(1/2)/(cos(f*x+e)^2-1)*(-5*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*si
n(f*x+e)^2*cos(f*x+e)-5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*sin(f*x+e)^2+5*I*cos(f*x+e)^
2*cot(f*x+e)-7*I*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{5\sqrt{2}\left(i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5\sqrt{}}{}$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(5*sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassPIn
verse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*cos(f*x + e)^2
+ I)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)) + 2*(5*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(b/cos(f*x + e)))/((f*
cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\csc^4{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**4, x)
```

Maxima [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}\csc{(fx+e)}^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}\csc{(fx+e)}^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin{(e+f\,x)}^4}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4, x)
```

3.384 $\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx &= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}\\&\quad+\frac{3\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{4f}\end{aligned}$$

```
[Out] -3/4*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-3/10*b*csc(f*x+e)^3/f/(b*sec(f*x+e
))^(1/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)+3/4*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e
)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2720}

$$\begin{aligned}\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx &= -\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}\\&\quad+\frac{3\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{4f}\end{aligned}$$

```
[In] Int[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-3*b*Csc[e + f*x])/(4*f*Sqrt[b*Sec[e + f*x]]) - (3*b*Csc[e + f*x]^3)/(10*f
*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^5)/(5*f*Sqrt[b*Sec[e + f*x]]) + (3
*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(4*f)
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{9}{10}\int\csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3}{4}\int\csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3}{8}\int\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{8}\left(3\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\
&= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{3\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{4f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$=\frac{\left(-\cot(e+fx)\left(15+6\csc^2(e+fx)+4\csc^4(e+fx)\right)+15\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b\sec}}{20f}$$

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-(Cot[e + f*x]*(15 + 6*Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)) + 15*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(20*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.67

method	result
default	$\frac{i\sqrt{b\sec(fx+e)}\left(15\left(\sin^5(fx+e)\right)F\left(i(\cot(fx+e)-\csc(fx+e)),i\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\cos(fx+e)+15\left(\sin^5(fx+e)\right)F(i(\cot(fx\right.}{20f(\cos(fx+e)-1)^2(\cos(fx}$

```
[In] int(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*I/f*(b*sec(f*x+e))^(1/2)*(15*sin(f*x+e)^5*EllipticF(I*(cot(f*x+e)-csc(
f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)+15*sin(f*x+e)^5*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+15*I*cos(f*x+e)^5-36*I*cos(f*x
+e)^3+25*I*cos(f*x+e))/(cos(f*x+e)-1)^2/(cos(f*x+e)+1)^2*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx =$$
$$-\frac{15\sqrt{2}\left(i\cos(fx+e)^4-2i\cos(fx+e)^2+i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i}{}$$

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(15*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f
*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*sqrt
(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weier
strassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(15*cos(f*x + e)^5
 - 36*cos(f*x + e)^3 + 25*cos(f*x + e))*sqrt(b/cos(f*x + e)))/((f*cos(f*x +
 e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Giac [F]

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^6\,dx$$

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^6}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6,x)
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6, x)
```

3.385 $\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx=\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2/11*b^7/f/(b*sec(f*x+e))^(11/2)-6/7*b^5/f/(b*sec(f*x+e))^(7/2)+2*b^3/f/(b*
sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx=\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(11*f*(b*Sec[e + f*x])^(11/2)) - (6*b^5)/(7*f*(b*Sec[e + f*x])^(7/2
)) + (2*b^3)/(f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^7\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^7\text{Subst}\left(\int \left(-\frac{1}{x^{13/2}}+\frac{3}{b^2x^{9/2}}-\frac{3}{b^4x^{5/2}}+\frac{1}{b^6\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{b(3370+809\cos(2(e+fx))-90\cos(4(e+fx))+7\cos(6(e+fx)))\sqrt{b\sec(e+fx)}}{1232f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
[Out] (b*(3370 + 809*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 7*Cos[6*(e + f*x)])
*Sqrt[b*Sec[e + f*x]])/(1232*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(71) = 142.

Time = 0.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.08

method	result
default	$-\frac{b\left((77\cos(fx+e)+77)\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+(-77\cos(fx+\right.}{}$

[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)

[Out] -1/154/f*b*((77*cos(f*x+e)+77)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+(-77*cos(f*x+e)-77)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-28*cos(f*x+e)^6+132*cos(f*x+e)^4-308*cos(f*x+e)^2-308)*(b*sec(f*x+e))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\left(7\,b\cos\left(fx+e\right)^6-33\,b\cos\left(fx+e\right)^4+77\,b\cos\left(fx+e\right)^2+77\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{77\,f}$$

[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="fricas")

[Out] 2/77*(7*b*cos(f*x + e)^6 - 33*b*cos(f*x + e)^4 + 77*b*cos(f*x + e)^2 + 77*b)*sqrt(b/cos(f*x + e))/f

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \text{Timed out}$$

[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**7,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\,b\left(\frac{7\,b^6}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}} - \frac{33\,b^4}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}} + \frac{77\,b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}} + 77\sqrt{\frac{b}{\cos(fx+e)}}\right)}{77\,f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 2/77*b*(7*b^6/(b/cos(f*x + e))^(11/2) - 33*b^4/(b/cos(f*x + e))^(7/2) + 77*
b^2/(b/cos(f*x + e))^(3/2) + 77*sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\left(7\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^5 - 33\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^3 + 77\sqrt{b\cos(fx+e)}b^5\,c\right.}{77\,b^4 f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/77*(7*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 33*sqrt(b*cos(f*x + e))*b
^5*cos(f*x + e)^3 + 77*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e) + 77*b^6/sqrt(
b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^4*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \int \sin\left(e+f\,x\right)^7\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2), x)
```

3.386 $\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}} + \frac{4b^3}{3f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -2/7*b^5/f/(b*sec(f*x+e))^(7/2)+4/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f
*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}} + \frac{4b^3}{3f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(7*f*(b*Sec[e + f*x])^(7/2)) + (4*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)
) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b^5\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}-\frac{2}{b^2x^{5/2}}+\frac{1}{b^4\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{b(215+44\cos(2(e+fx))-3\cos(4(e+fx)))\sqrt{b\sec(e+fx)}}{84f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
[Out] (b*(215 + 44*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(
84*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 835, normalized size of antiderivative = 13.25

method	result	size
default	Expression too large to display	835

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/f*b*(b*sec(f*x+e))^(1/2)*(21*cos(f*x+e)^2*ln((2*cos(f*x+e)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-21*ln(2*(2*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(3/2)+63*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)-63*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(co
s(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)+63*ln((2*cos(f
*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-6
3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(3/2)+12*cos(f*x+e)^4+21*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e
)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sec(f*x+e)-21*ln(2*(2*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sec(f*
x+e)-56*cos(f*x+e)^2-84)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\,b\cos\left(fx+e\right)^4-14\,b\cos\left(fx+e\right)^2-21\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{21\,f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -2/21*(3*b*cos(f*x + e)^4 - 14*b*cos(f*x + e)^2 - 21*b)*sqrt(b/cos(f*x + e)
)/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2\,b\left(\frac{3\,b^4}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}}-\frac{14\,b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}-21\,\sqrt{\frac{b}{\cos(fx+e)}}\right)}{21\,f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
[Out] -2/21*b*(3*b^4/(b/cos(f*x + e))^(7/2) - 14*b^2/(b/cos(f*x + e))^(3/2) - 21*
sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\,\sqrt{b\cos\left(fx+e\right)}b^3\cos\left(fx+e\right)^3-14\,\sqrt{b\cos\left(fx+e\right)}b^3\cos\left(fx+e\right)-\frac{21\,b^4}{\sqrt{b\cos(fx+e)}}\right)\mathrm{sgn}(\cos\left(fx+e\right))}{21\,b^2 f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
[Out] -2/21*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e)^3 - 14*sqrt(b*cos(f*x + e))*
b^3*cos(f*x + e) - 21*b^4/sqrt(b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2), x)
```

3.387 $\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx=\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 14}

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx=\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{5/2}}\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{5/2}}+\frac{1}{b^2\sqrt{x}}\right)\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{b(7+\cos(2(e+fx)))\sqrt{b\sec(e+fx)}}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
[Out] (b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(35) = 70.

Time = 0.23 (sec) , antiderivative size = 825, normalized size of antiderivative = 20.12

method	result	size
default	Expression too large to display	825

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/6/f*b*(b*sec(f*x+e))^(1/2)*(3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+
e)+1))*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*cos(f*x+e)^2*ln(
(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)+9*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*cos(f*x+e)-9*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
```

```
+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+
e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)+9*ln(2*(2*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-9*ln((
2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)+3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*sec(f*x+e)-3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e
)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*sec(f*x+e)+4*cos(f*x+e)^2+12)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\left(b\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
[Out] 2/3*(b*cos(f*x + e)^2 + 3*b)*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\,b\left(\frac{b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}+3\,\sqrt{\frac{b}{\cos(fx+e)}}\right)}{3\,f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
[Out] 2/3*b*(b^2/(b/cos(f*x + e))^(3/2) + 3*sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\left(\sqrt{b\cos(fx+e)}b\cos(fx+e)+\frac{3b^2}{\sqrt{b\cos(fx+e)}}\right)\operatorname{sgn}(\cos(fx+e))}{3f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
[Out] 2/3*(sqrt(b*cos(f*x + e))*b*cos(f*x + e) + 3*b^2/sqrt(b*cos(f*x + e)))*sgn(
cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int \sin(e+f\,x)^3\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}dx$$

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2), x)
```

3.388 $\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 30}

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2b\sqrt{b\sec(fx+e)}}{f}$	17
default	$\frac{2b\sqrt{b\sec(fx+e)}}{f}$	17

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 2*b*(b*sec(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2\,b\sqrt{\frac{b}{\cos(fx+e)}}}{f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
[Out] 2*b*sqrt(b/cos(f*x + e))/f
```

Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\int (b\sec{(e+fx)})^{\frac{3}{2}}\sin{(e+fx)}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e),x)
[Out] Integral((b*sec(e + f*x))**(3/2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos{(fx+e)}}{f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
[Out] 2*(b/cos(f*x + e))^(3/2)*cos(f*x + e)/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2\,b^2\mathrm{sgn}(\cos{(fx+e)})}{\sqrt{b\cos{(fx+e)}}f}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
[Out] 2*b^2*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2\,b\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{f}$$

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(3/2),x)
[Out] (2*b*(b/cos(e + f*x))^(1/2))/f
```

3.389 $\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(3/2)*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2702, 327, 335, 218, 212, 209}

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f) - (b^(3/2)*ArcTanh[Sqrt
[b*Sec[e + f*x]]/Sqrt[b]])/f + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{(2b)\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} - \frac{b^2\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} - \frac{b^2\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\left(-2\arctan\left(\sqrt{\sec(e+fx)}\right) + \log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(1+\sqrt{\sec(e+fx)}\right) + 4\sqrt{\sec}\right.}{2f\sec^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] + 4*Sqrt[Sec[e + f*x]])*(b*Sec[e + f*x])^(3/2))/(2*f*Sec[e
 + f*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(63) = 126$.

Time = 0.23 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.86

method	result
default	$-\frac{\sqrt{b\sec(fx+e)}\,b\left((\cos^2(fx+e))\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+(\cos^2(fx+e))\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+}{(\cos(fx+e)}}}{\cos(fx+e)+1}\right.\right.}{2f\left(-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)^{\frac{3}{2}}(\cos(fx+e)+}$

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(b*sec(f*x+e))^(1/2)*b/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/(cos(f*x
+e)+1)^3*(cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+cos
(f*x+e)^2*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.61

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \left[\frac{2\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+\sqrt{-b}b\log\left(\frac{b\cos(fx+e)^2+4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\cdots}}{\cos(fx+e)^2+2\cos(fx+\cdots}\right.}{4f}\right.$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*b*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) +
 1)/b) + sqrt(-b)*b*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) + 8*b*sqrt(b/cos(f*x + e)))/f, 1/4*(2*b^(3/2)*arctan(1
/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) + b^(3/2)*log((b*cos(f*
x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) +
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*b*sqrt(b/
cos(f*x + e)))/f]
```

Sympy [F]

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec{(e+fx)})^{\frac{3}{2}}\csc{(e+fx)}\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(3/2)*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{\left(2\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)-\sqrt{b}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)-4\sqrt{\frac{b}{\cos(fx+e)}}\right)b}{2f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -1/2*(2*sqrt(b)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) - sqrt(b)*log(-(sqrt(b
) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) - 4*sqrt(b/cos(
f*x + e)))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b^4\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2}{\sqrt{b\cos(fx+e)}b^2}\right)\mathrm{sgn}(\cos(fx+e))}{f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] b^4*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + arctan(sqrt(b*c
os(f*x + e))/sqrt(b))/b^(5/2) + 2/(sqrt(b*cos(f*x + e))*b^2))*sgn(cos(f*x +
 e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x),x)
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x), x)
```

3.390 $\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

```
[Out] -5/4*b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-5/4*b^(3/2)*arctanh((b*
sec(f*x+e))^(1/2)/b^(1/2))/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(5/2)/b/f+5/2*
b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2702, 294, 327, 335, 218, 212, 209}

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-5*b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (5*b^(3/2)*ArcTan
h[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (5*b*Sqrt[b*Sec[e + f*x]])/(2*f) -
 (Cot[e + f*x]^2*(b*Sec[e + f*x])^(5/2))/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{5\text{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4f}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2f}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f}$$

$$= -\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{\left(10\arctan\left(\sqrt{\sec(e+fx)}\right)-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(1+\sqrt{\sec(e+fx)}\right)+4(-5+\csc^2(e+\right.}{8f\sec^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/8*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*Lo
g[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2)*Sqrt[Sec[e + f*x]])*(b*
Sec[e + f*x])^(3/2))/(f*Sec[e + f*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(89) = 178.

Time = 0.24 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.32

method	result
default	$\frac{\sqrt{b\sec(fx+e)}\,b\left(4(\cos^4(fx+e))\left(-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)^{\frac{3}{2}}+8(\cos^3(fx+e))\left(-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)^{\frac{3}{2}}+4(\cos^2(fx+e))\left(-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}\right)^{\frac{3}{2}}\right.}{}$

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(b*sec(f*x+e))^(1/2)*b/(cos(f*x+e)-1)/(cos(f*x+e)+1)^3/(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*(4*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+8*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(3/2)-16*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-5*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-4*c
os(f*x+e)^3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-cos(f*x+e)^3*
ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+5*cos(f*x+e)^2*arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*cos(f*x+e)^2*ln(2*(2*cos(f*x+e)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos
(f*x+e)+1)/(cos(f*x+e)+1))+cos(f*x+e)^2*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(co
s(f*x+e)+1))+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(89) = 178.

Time = 0.35 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.43

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \left[\frac{10\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+5\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-b}}{16\left(f\cos\right.}\right.$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(b*cos(f*x + e)^2 - b)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*
x + e))*(cos(f*x + e) + 1)/b) + 5*(b*cos(f*x + e)^2 - b)*sqrt(-b)*log((b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(5*b
*cos(f*x + e)^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f), 1/16*(
10*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x
+ e) - 1)/sqrt(b)) + 5*(b*cos(f*x + e)^2 - b)*sqrt(b)*log((b*cos(f*x + e)^2
 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*(5*b*cos(f*x + e)
^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\left(\frac{4b^2\sqrt{\frac{b}{\cos(fx+e)}}}{b^2-\frac{b^2}{\cos(fx+e)^2}}-10\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)+5\sqrt{b}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)+16\sqrt{\frac{b}{\cos(fx+e)}}\right)b}{8f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^2*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 10*sqrt(b)*arc
tan(sqrt(b/cos(f*x + e))/sqrt(b)) + 5*sqrt(b)*log(-(sqrt(b) - sqrt(b/cos(f*
x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*sqrt(b/cos(f*x + e)))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b^6\left(\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}}+\frac{2\left(5b^2\cos(fx+e)^2-4b^2\right)}{\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-\sqrt{b\cos(fx+e)}b^2\right)b^4}\right)\mathrm{sgn}(\mathrm{co}}{4f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*b^6*(5*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^4) + 5*arctan(
sqrt(b*cos(f*x + e))/sqrt(b))/b^(9/2) + 2*(5*b^2*cos(f*x + e)^2 - 4*b^2)/((
sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - sqrt(b*cos(f*x + e))*b^2)*b^4))*s
gn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3, x)
```

3.391 $\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = -\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f}$$

```
[Out] 8/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+20/9*b^3*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(3/2)-16/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b
*sin(f*x+e)^5*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2704, 2707, 3856, 2719}

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-16*b^2*EllipticE[(e + f*x)/2, 2])/(3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (8*b^3*Sin[e + f*x])/(3*f*(b*Sec[e + f*x])^(3/2)) + (20*b^3*Sin[e
+ f*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e +
f*x]^5)/f
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \left(10b^2\right)\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx \\
&= \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{1}{3}\left(20b^2\right)\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx \\
&= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} \\
&\quad + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{1}{3}\left(8b^2\right)\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx \\
&= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} \\
&\quad + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f} - \frac{\left(8b^2\right)\int \sqrt{\cos(e+fx)}\,dx}{3\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx =$$

$$-\frac{b\sqrt{b\sec(e+fx)}\left(384\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right) - 158\sin(e+fx) - 13\sin(3(e+fx)) + \sin(5(e+fx)\right.}{72f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] -1/72*(b*Sqrt[b*Sec[e + f*x]]*(384*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2
, 2] - 158*Sin[e + f*x] - 13*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.59

method	result
default	$\frac{2\left(i(24\cos(fx+e)+24)\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\cot(fx+e)-\csc(fx+e)),i)+i(-24\cos(fx+e)-24)\sin(fx+e)\sqrt{\frac{}{co}}\right.}{}$

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9/f*(I*(24*cos(f*x+e)+24)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)+I*(-24*cos(f*
x+e)-24)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+cos(f*x+e)^6-5*cos(f*x+e)^4+19*c
os(f*x+e)^2-24*cos(f*x+e)+9)*b*(b*sec(f*x+e))^(1/2)*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx =$$

$$-\frac{2\left(12i\sqrt{2}b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-12i\sqrt{2}b^{\frac{3}{2}}\mathrm{v}\right.}{}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -2/9*(12*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) + I*sin(f*x + e))) - 12*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + (b*cos(f*
x + e)^4 - 4*b*cos(f*x + e)^2 - 9*b)*sqrt(b/cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int \sin(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2), x)
```

3.392 $\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f}$$

```
[Out] 12/5*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-24/5*b^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)
^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)^3*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2704, 2707, 3856, 2719}

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-24*b^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (12*b^3*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*S
ec[e + f*x]]*Sin[e + f*x]^3)/f
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - (6b^2)\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - \frac{1}{5}(12b^2)\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f} - \frac{(12b^2)\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx=\frac{b\sqrt{b\sec(e+fx)}\left(-48\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+21\sin(e+fx)+\sin(3(e+fx))\right)}{10f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-48*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 21*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.98

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\,b\left(i(12\cos(fx+e)+12)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\cot(fx+e)-\csc(fx+e)),i)+i(-12\cos(fx+e)-12)\sqrt{\frac{1}{\cos(fx}}\right.}{}$

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
[Out] 2/5/f*(b*sec(f*x+e))^(1/2)*b*(I*(12*cos(f*x+e)+12)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)+I
*(-12*cos(f*x+e)-12)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-cos(f*x+e)^3*cot(f*x+e)+8*cos(f
*x+e)*cot(f*x+e)-12*cot(f*x+e)+5*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx =$$

$$-\frac{2\left(6i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-6i\sqrt{2}b^{\frac{3}{2}}\text{wei}\right.}{}$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -2/5*(6*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - (b*cos(f*x
+ e)^2 + 5*b)*sqrt(b/cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2), x)
```

3.393 $\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

```
[Out] -4*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2704, 3856, 2719}

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f} - \frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - \left(2b^2\right)\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - \frac{\left(2b^2\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\left(-2\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)\right)}{f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ Sin[e + f*x]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.65

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\, b\left(i(2\cos(fx+e)+2)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, E(i(\cot(fx+e)-\csc(fx+e)),i)+i(-2\cos(fx+e)-2)\sqrt{\frac{1}{\cos(fx+e)+}}\right.}{f}$

[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)

[Out] 2/f*(b*sec(f*x+e))^(1/2)*b*(I*(2*cos(f*x+e)+2)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)+I*(-2
*cos(f*x+e)-2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+cs
c(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx =$$

$$-\frac{2\left(i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-i\sqrt{2}b^{\frac{3}{2}}\text{weier}\right.}{f}$$

[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")

[Out] -2*(I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - b*sqrt(b/cos(f*x
+ e))*sin(f*x + e))/f

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**2,x)

[Out] Timed out

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}dx$$

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2), x)
```

3.394 $\int (b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (b\sec(e+fx))^{3/2}\,dx=-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

```
[Out] -2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2719}

$$\int (b\sec(e+fx))^{3/2}\,dx=\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - b^2\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f} - \frac{b^2\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(e+fx)}\left(-\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)\right)}{f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(3/2),x]
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
+ Sin[e + f*x]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 392, normalized size of antiderivative = 5.94

method	result
default	$2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(-\cot(fx+e)+\csc(fx+e)),i)(\cos^2(fx+e))-i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}E(i(-\cot(fx+e)+c$

```
[In] int((b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)^2-I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f
*x+e)^2+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)-2*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*
cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)+sin(f*x+e))*
(b*sec(f*x+e))^(1/2)*b/(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int (b\sec(e+fx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+i}{}$$

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e))) + I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*b*sqrt(b/cos(f*x
+ e))*sin(f*x + e))/f
```

Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(e+fx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\,dx = \int \left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2),x)
[Out] int((b/cos(e + f*x))^(3/2), x)
```

3.395 $\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

```
[Out] -3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)-b*csc(f*x+e)*(b*s
ec(f*x+e))^(1/2)/f+3*b*sin(f*x+e)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2705, 3853, 3856, 2719}

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) - (b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/f + (3*b*Sqrt[b*Sec[e + f*x]]*S
in[e + f*x])/f
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3}{2}\int (b\sec(e+fx))^{3/2}\,dx\\
&= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}\\
&\quad -\frac{1}{2}\left(3b^2\right)\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}\\
&\quad -\frac{\left(3b^2\right)\int \sqrt{\cos(e+fx)}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b\sqrt{b\sec(e+fx)}\left(-\csc(e+fx)-3\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+3\sin(e+fx)\right)}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-Csc[e + f*x] - 3*Sqrt[Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2] + 3*Sin[e + f*x]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.94

method	result
default	$-\frac{ib\sqrt{b\sec(fx+e)}\left(3\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)-3\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(\right.}{}$

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -I/f*b*(b*sec(f*x+e))^(1/2)*(3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)-3*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e
)+csc(f*x+e)),I)*cos(f*x+e)+3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)-3*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)
),I)-3*I*cot(f*x+e)+2*I*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{-3i\sqrt{2}b^{\frac{3}{2}}\sin(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin}{}$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*b^(3/2)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*b^(3/2)*sin(f*x
 + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) - 2*(3*b*cos(f*x + e)^2 - 2*b)*sqrt(b/cos(f*x + e)))/(f*sin(f*
x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

3.396 $\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7b\sqrt{b\sec(e+fx)}\sin(e+fx)}{2f}$$

```
[Out] -7/2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)-7/6*b*csc(f*x+e
)*(b*sec(f*x+e))^(1/2)/f-1/3*b*csc(f*x+e)^3*(b*sec(f*x+e))^(1/2)/f+7/2*b*si
n(f*x+e)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2705, 3853, 3856, 2719}

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}+\frac{7b\sin(e+fx)\sqrt{b\sec(e+fx)}}{2f}$$

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-7*b^2*EllipticE[(e + f*x)/2, 2])/(2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (7*b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/(6*f) - (b*Csc[e + f*x]^3*S
qrt[b*Sec[e + f*x]])/(3*f) + (7*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(2*f)
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7}{6}\int\csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx\\
&= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7}{4}\int(b\sec(e+fx))^{3/2}\,dx\\
&= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}\\
&\quad+\frac{7b\sqrt{b\sec(e+fx)}\sin(e+fx)}{2f}-\frac{1}{4}\left(7b^2\right)\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}\\
&\quad+\frac{7b\sqrt{b\sec(e+fx)}\sin(e+fx)}{2f}-\frac{\left(7b^2\right)\int\sqrt{\cos(e+fx)}\,dx}{4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{7b^2 E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f} - \frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f} + \frac{7b\sqrt{b\sec(e+fx)}\sin(e+fx)}{2f}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{b\left(-21+7\csc^2(e+fx)+2\csc^4(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(b*(-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 21*Sqrt[Cos[e + f*x]]*
Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/
f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.23

method	result
default	$-\frac{ib\sqrt{b\sec(fx+e)}\left(21\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)-21\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(\right.}{}$

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I/f*b*(b*sec(f*x+e))^(1/2)*(21*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)-21*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot
(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+21*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)-21*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+cs
c(f*x+e)),I)-21*I*cot(f*x+e)+14*I*csc(f*x+e)-2*I*csc(f*x+e)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{21\sqrt{2}(i\,b\cos(fx+e)^2 - i\,b)\sqrt{b}\sin(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+}{}$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(21*sqrt(2)*(I*b*cos(f*x + e)^2 - I*b)*sqrt(b)*sin(f*x + e)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) +
21*sqrt(2)*(-I*b*cos(f*x + e)^2 + I*b)*sqrt(b)*sin(f*x + e)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(21*
b*cos(f*x + e)^4 - 35*b*cos(f*x + e)^2 + 12*b)*sqrt(b/cos(f*x + e)))/((f*co
s(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)^4}\,dx$$

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4, x)
```

3.397 $\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2b^7}{9f(b\sec(e+fx))^{9/2}} - \frac{6b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{6b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/9*b^7/f/(b*sec(f*x+e))^(9/2)-6/5*b^5/f/(b*sec(f*x+e))^(5/2)+2/3*b*(b*sec(
f*x+e))^(3/2)/f+6*b^3/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2b^7}{9f(b\sec(e+fx))^{9/2}} - \frac{6b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{6b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(9*f*(b*Sec[e + f*x])^(9/2)) - (6*b^5)/(5*f*(b*Sec[e + f*x])^(5/2))
 + (6*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^7 \text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{11/2}} \, dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^7 \text{Subst}\left(\int \left(-\frac{1}{x^{11/2}} + \frac{3}{b^2 x^{7/2}} - \frac{3}{b^4 x^{3/2}} + \frac{\sqrt{x}}{b^6}\right) \, dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b^7}{9f(b\sec(e+fx))^{9/2}} - \frac{6b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{6b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int (b\sec(e+fx))^{5/2} \sin^7(e+fx) \, dx = \frac{b(2366 + 1803\cos(2(e+fx)) - 78\cos(4(e+fx)) + 5\cos(6(e+fx)))(b\sec(e+fx))^{3/2}}{720f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
[Out] (b*(2366 + 1803*Cos[2*(e + f*x)] - 78*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)]
)*(b*Sec[e + f*x])^(3/2))/(720*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(71) = 142.

Time = 0.87 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.25

$$\sqrt{b\sec(fx+e)}\, b^2 \left(20(\cos^5(fx+e)) + 135\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}} + 2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}} - \cos(fx+e) + 1}{\cos(fx+e)+1}\right)\sqrt{-\frac{\cdots}{(\cdots}}$$

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x)
```

```
[Out] 1/90/f*(b*sec(f*x+e))^(1/2)*b^2*(20*cos(f*x+e)^5+135*ln((2*cos(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-1
35*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*cos(f*x+e)-108*cos(f*x+e)^3+135*ln((2*cos(f*x+e)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-135*ln(2*(2*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+54
0*cos(f*x+e)+60*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(5\,b^2\cos\left(fx+e\right)^6 - 27\,b^2\cos\left(fx+e\right)^4 + 135\,b^2\cos\left(fx+e\right)^2 + 15\,b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45\,f\cos\left(fx+e\right)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*cos(f*x + e)^6 - 27*b^2*cos(f*x + e)^4 + 135*b^2*cos(f*x + e)^2
 + 15*b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(15\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} + \frac{5\,b^6 - \frac{27\,b^6}{\cos(fx+e)^2} + \frac{135\,b^6}{\cos(fx+e)^4}}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}\right)b}{45\,f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="maxima")
[Out] 2/45*(15*(b/cos(f*x + e))^(3/2) + (5*b^6 - 27*b^6/cos(f*x + e)^2 + 135*b^6/
cos(f*x + e)^4)/(b/cos(f*x + e))^(9/2))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(5\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^4 - 27\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^2 + 135\sqrt{b\cos(fx+e)}b^4\right.}{45\,b^2 f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="giac")
[Out] 2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 27*sqrt(b*cos(f*x + e))*b
^4*cos(f*x + e)^2 + 135*sqrt(b*cos(f*x + e))*b^4 + 15*b^5/(sqrt(b*cos(f*x +
 e))*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \int \sin(e+f\,x)^7\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}dx$$

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2), x)
```

3.398 $\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{4b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] -2/5*b^5/f/(b*sec(f*x+e))^(5/2)+2/3*b*(b*sec(f*x+e))^(3/2)/f+4*b^3/f/(b*sec
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{4b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(5*f*(b*Sec[e + f*x])^(5/2)) + (4*b^3)/(f*Sqrt[b*Sec[e + f*x]]) +
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b^5\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{x^{7/2}}-\frac{2}{b^2x^{3/2}}+\frac{\sqrt{x}}{b^4}\right)dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \frac{b(151+108\cos(2(e+fx))-3\cos(4(e+fx)))(b\sec(e+fx))^{3/2}}{60f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
[Out] (b*(151 + 108*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(3/2)
)/(60*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 436, normalized size of antiderivative = 6.92

$$-\frac{\sqrt{b\sec(fx+e)}\,b^2\left(6(\cos^3(fx+e))-15\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\sqrt{-\frac{}{(\mathrm{co}}}\right.}{}$$

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/15/f*(b*sec(f*x+e))^(1/2)*b^2*(6*cos(f*x+e)^3-15*ln((2*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+15
*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*cos(f*x+e)-15*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+15*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(
cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-60*cos(f*x+e)-10*sec(f*
x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\,b^2\cos\left(fx+e\right)^4-30\,b^2\cos\left(fx+e\right)^2-5\,b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -2/15*(3*b^2*cos(f*x + e)^4 - 30*b^2*cos(f*x + e)^2 - 5*b^2)*sqrt(b/cos(f*x
 + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \frac{2\left(5\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} - \frac{3\left(b^4-\frac{10\,b^4}{\cos(fx+e)^2}\right)}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}\right)b}{15\,f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="maxima")
[Out] 2/15*(5*(b/cos(f*x + e))^(3/2) - 3*(b^4 - 10*b^4/cos(f*x + e)^2)/(b/cos(f*x
 + e))^(5/2))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx =$$
$$-\frac{2\left(3\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2 - 30\sqrt{b\cos(fx+e)}b^2 - \frac{5\,b^3}{\sqrt{b\cos(fx+e)}\cos(fx+e)}\right)\mathrm{sgn}(\cos(fx+e))}{15\,f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="giac")
[Out] -2/15*(3*sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 30*sqrt(b*cos(f*x + e))*
b^2 - 5*b^3/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \int \sin(e+f\,x)^5\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}dx$$

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2), x)
```

3.399 $\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f+2*b^3/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 14}

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{3/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{b^2}\right)\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{b(5+3\cos(2(e+fx)))(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
[Out] (b*(5 + 3*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(35) = 70.

Time = 65.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 7.78

method	result
default	$\frac{\sqrt{b\sec(fx+e)}\,b^2\left(12\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cos^2(fx+e))+3\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\cos}{}$

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/6/f*(b*sec(f*x+e))^(1/2)*b^2/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(12*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2+3*ln(2*(2*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-3*ln((2*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
```

```
*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)+12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{2\left(3\,b^2\cos\left(fx+e\right)^2+b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="fricas")
[Out] 2/3*(3*b^2*cos(f*x + e)^2 + b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{2\left(\frac{3\,b^2}{\sqrt{\frac{b}{\cos(fx+e)}}}+\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\right)b}{3\,f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="maxima")
[Out] 2/3*(3*b^2/sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(3/2))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{2\left(3\sqrt{b\cos(fx+e)}b+\frac{b^2}{\sqrt{b\cos(fx+e)}\cos(fx+e)}\right)b\mathrm{sgn}(\cos(fx+e))}{3f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*cos(f*x + e))*b + b^2/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*b*
sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{b^2\sqrt{\frac{b}{\cos(e+f\,x)}}\left(\frac{13\cos(e+f\,x)}{3}+\cos(3\,e+3\,f\,x)\right)}{f\,\left(\cos(2\,e+2\,f\,x)+1\right)}$$

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2),x)
```

```
[Out] (b^2*(b/cos(e + f*x))^(1/2)*((13*cos(e + f*x))/3 + cos(3*e + 3*f*x)))/(f*(c
os(2*e + 2*f*x) + 1))
```

3.400 $\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx=\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 30}

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx=\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \sqrt{x}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b(b\sec(fx+e))^{\frac{3}{2}}}{3f}$	17
default	$\frac{2b(b\sec(fx+e))^{\frac{3}{2}}}{3f}$	17

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{2\,b^2\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f\cos(fx+e)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="fricas")
[Out] 2/3*b^2*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b \sec(e+fx))^{5/2} \sin(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int (b \sec(e+fx))^{5/2} \sin(e+fx)\,dx = \frac{2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\cos(fx+e)}{3f}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="maxima")
[Out] 2/3*(b/cos(f*x + e))^(5/2)*cos(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int (b \sec(e+fx))^{5/2} \sin(e+fx)\,dx = \frac{2\,b^3\mathrm{sgn}(\cos(fx+e))}{3\sqrt{b\cos(fx+e)}f\cos(fx+e)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="giac")
[Out] 2/3*b^3*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f*cos(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (b \sec(e+fx))^{5/2} \sin(e+fx)\,dx = \frac{4\,b^2\,\cos(e+f\,x)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{3\,f\,\left(\cos(2\,e+2\,f\,x)+1\right)}$$

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(5/2),x)
[Out] (4*b^2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(3*f*(cos(2*e + 2*f*x) + 1))
```

3.401 $\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(5/2)*arctanh((b*sec(f*x+e
))^(1/2)/b^(1/2))/f+2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2702, 327, 335, 304, 209, 212}

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (b^(5/2)*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{2b(b\sec(e+fx))^{3/2}}{3f} - \frac{b^3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b^3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{(b\sec(e+fx))^{5/2}\left(6\arctan\left(\sqrt{\sec(e+fx)}\right) + 3\log\left(1-\sqrt{\sec(e+fx)}\right) - 3\log\left(1+\sqrt{\sec(e+fx)}\right) \cdots\right)}{6f\sec^{\frac{5}{2}}(e+fx)}$$

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
[Out] ((b*Sec[e + f*x])^(5/2)*(6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[
e + f*x]]] - 3*Log[1 + Sqrt[Sec[e + f*x]]] + 4*Sec[e + f*x]^(3/2)))/(6*f*Se
c[e + f*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(62) = 124.

Time = 1.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.72

method	result
default	$\frac{\sqrt{b\sec(fx+e)}\,b^2\left(3\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-3\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+\cdots}{\cos(fx+e)+1}\right)\cdots\right)}{6f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}$

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(b*sec(f*x+e))^(1/2)*b^2/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(3*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln
(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)+4*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.21

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \left[\frac{6\sqrt{-b}b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)\cos(fx+e)+3\sqrt{-b}b^2\cos(fx+e)\log\left(\frac{b\cos(fx\ldots}{\ldots}\right.\ldots}{12f\cos(fx+e)}\right.$$

$$-\frac{6b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)\cos(fx+e)-3b^{\frac{5}{2}}\cos(fx+e)\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2+\cos(fx+e)\right)\ldots}{\cos(fx+e)^2-2\cos(\ldots}\right.\ldots}{12f\cos(fx+e)}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-b)*b^2*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e
) + 1)/b)*cos(f*x + e) + 3*sqrt(-b)*b^2*cos(f*x + e)*log((b*cos(f*x + e)^2
- 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*b^2*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)), -1/12*(6*b^(5/2)*arctan(1/2*sqrt(b/cos(f*x + e))
*(cos(f*x + e) - 1)/sqrt(b))*cos(f*x + e) - 3*b^(5/2)*cos(f*x + e)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*b^2*
sqrt(b/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(6\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)+3\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)+4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\right)b}{6\,f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 1/6*(6*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 3*b^(3/2)*log(-(sqrt(
b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 4*(b/cos(f*x
 + e))^(3/2))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^6\left(\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{2}{\sqrt{b\cos(fx+e)}b^3\cos(fx+e)}\right)\mathrm{sgn}(\cos(fx+e))}{3\,f}$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] 1/3*b^6*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(
sqrt(b*cos(f*x + e))/sqrt(b))/b^(7/2) + 2/(sqrt(b*cos(f*x + e))*b^3*cos(f*x
 + e)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x),x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x), x)
```

3.402 $\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{7b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

```
[Out] 7/4*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-7/4*b^(5/2)*arctanh((b*s
ec(f*x+e))^(1/2)/b^(1/2))/f+7/6*b*(b*sec(f*x+e))^(3/2)/f-1/2*cot(f*x+e)^2*(
b*sec(f*x+e))^(7/2)/b/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2702, 294, 327, 335, 304, 209, 212}

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{7b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (7*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (7*b^(5/2)*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (7*b*(b*Sec[e + f*x])^(3/2))/(6*f)
- (Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{7\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4f}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2f}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} - \frac{(7b^3)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} + \frac{(7b^3)\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f}$$

$$= \frac{7b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^3\left(-12\csc^2(e+fx)+42\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+21\left(\log\left(1-\sqrt{\sec(e+fx)}\right.\right.\right.}{24f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-12*Csc[e + f*x]^2 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]]
 + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[
e + f*x]] + 16*Sec[e + f*x]^2))/(24*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(89) = 178.

Time = 31.80 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.44

method	result
default	$-\frac{\sqrt{b\sec(fx+e)}\,b^2\left(21\left(\cos^3(fx+e)\right)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+3\left(\cos^3(fx+e)\right)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos}{(\cos(f}}}{\cos(fx+e)+1}\right.\right.}{}$

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/f*(b*sec(f*x+e))^(1/2)*b^2*(21*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2))+3*cos(f*x+e)^3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos
(f*x+e)+1))-24*cos(f*x+e)^3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1
))+28*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-21*cos(f*x+e)^2*arc
tan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*cos(f*x+e)^2*ln((2*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+24*cos(f*x+e)^2*ln(2*(2*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)+1)/(cos(f*x+e)+1))-16*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(89) = 178$.

Time = 0.34 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.96

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \left[\frac{42\left(b^2\cos(fx+e)^3 - b^2\cos(fx+e)\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right) + 21\left(b^2\cos\left(f\right.\right.}{}\right.$$

$$-\frac{42\left(b^2\cos(fx+e)^3 - b^2\cos(fx+e)\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right) - 21\left(b^2\cos(fx+e)^3 - b^2\cos\left(fx\right.\right.}{48\left(f\cos\left(fx+\right.\right.}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-b)*arctan(1/2*sqrt(
-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(b^2*cos(f*x + e)^3 - b
^2*cos(f*x + e))*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(7*b^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/48*(42*(b^2*cos(f*x + e)^3
 - b^2*cos(f*x + e))*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e)
- 1)/sqrt(b)) - 21*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(7*b
^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^3 - f*cos(
f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(\frac{12\,b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} + 42\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right) + 21\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right) + 16\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\right.}{24\,f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^2*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 42*b^(3/2)
*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 21*b^(3/2)*log(-(sqrt(b) - sqrt(b/c
os(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*(b/cos(f*x + e))^(3/2)
)*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^8\left(\frac{6\sqrt{b\cos(fx+e)}}{\left(b^2\cos(fx+e)^2-b^2\right)b^4} + \frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5} - \frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}} + \frac{8}{\sqrt{b\cos(fx+e)}b^5\cos(fx+e)}\right)\mathrm{sg}}{12\,f}$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*b^8*(6*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^4) + 21*arct
an(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arctan(sqrt(b*cos(f*x
 + e))/sqrt(b))/b^(11/2) + 8/(sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)))*sgn(c
os(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3, x)
```

3.403 $\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{77b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}$$

```
[Out] 77/32*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-77/32*b^(5/2)*arctanh(
(b*sec(f*x+e))^(1/2)/b^(1/2))/f+77/48*b*(b*sec(f*x+e))^(3/2)/f-11/16*cot(f*
x+e)^2*(b*sec(f*x+e))^(7/2)/b/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(11/2)/b^3/
f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2702, 294, 327, 335, 304, 209, 212}

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{77b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f} + \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf}$$

```
[In] Int[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (77*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (77*b^(5/2)*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) + (77*b*(b*Sec[e + f*x])^(3/2))/(
48*f) - (11*Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(16*b*f) - (Cot[e + f*x]
^4*(b*Sec[e + f*x])^(11/2))/(4*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{13/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{11\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{77\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{32bf}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{(77b)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{32f}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{(77b)\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{16f}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} - \frac{(77b^3)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f} + \frac{(77b^3)\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f}$$

$$= \frac{77b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^3\left(-180\csc^2(e+fx)-48\csc^4(e+fx)+462\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+231\left(\text{lo}\right.\right.}{192f\sqrt{b\sec(e}}$$

```
[In] Integrate[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-180*Csc[e + f*x]^2 - 48*Csc[e + f*x]^4 + 462*ArcTan[Sqrt[Sec[e + f*x
]]]*Sqrt[Sec[e + f*x]] + 231*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Se
c[e + f*x]]])*Sqrt[Sec[e + f*x]] + 128*Sec[e + f*x]^2))/(192*f*Sqrt[b*Sec[e
 + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(115) = 230.

Time = 246.76 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.89

method	result
default	$-\frac{\sqrt{b\sec(fx+e)}\,b^2\left(231\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)(\cot^2(fx+e))+57\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos}{\cos(fx+e)+1}\right.\right.}{}$

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/f*(b*sec(f*x+e))^(1/2)*b^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(231
*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cot(f*x+e)^2+57*ln((2*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cot(f*x+e)^2-288*ln(2*(2*cos(f*x+e)*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)+1)/(cos(f*x+e)+1))*cot(f*x+e)^2-231*arctan(1/2/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2))*cot(f*x+e)*csc(f*x+e)-57*ln((2*cos(f*x+e)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
+1)/(cos(f*x+e)+1))*cot(f*x+e)*csc(f*x+e)+288*ln(2*(2*cos(f*x+e)*(-cos(f*x+
```

```
e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)+1)/(cos(f*x+e)+1))*cot(f*x+e)*csc(f*x+e)-308*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*cot(f*x+e)^3*csc(f*x+e)+484*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*c
ot(f*x+e)*csc(f*x+e)^3-128*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e)*
csc(f*x+e)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(115) = 230$.

Time = 0.36 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.79

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \left[\frac{462\left(b^2\cos(fx+e)^5-2b^2\cos(fx+e)^3+b^2\cos(fx+e)\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx\right.}{2b}}{}\right.$$

$$-\frac{462\left(b^2\cos(fx+e)^5-2b^2\cos(fx+e)^3+b^2\cos(fx+e)\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)-231\left(b^2\cos\right.}{}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*
sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 2
31*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(-b)*
log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/c
os(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 8*(77*b^2*cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f
*x + e)))/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e)), -1/384*
(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)
*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 231*(b^2*cos
(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)*log((b*cos(f
*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
+ 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(77*b^2*
cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f*x + e)))/(f*
cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(462\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)+231\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)+128\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}+\frac{12\left(15\,b^4\left(\frac{b}{\cos(f\ldots}\right.\right.}{b^4-\frac{\ldots}{\cos\ldots}}\right.}{192\,f}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 1/192*(462*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 231*b^(3/2)*log(-
(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 128*(b
/cos(f*x + e))^(3/2) + 12*(15*b^4*(b/cos(f*x + e))^(3/2) - 19*b^2*(b/cos(f*
x + e))^(7/2))/(b^4 - 2*b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4))*b/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^{10}\left(\frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}-\frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{15}{2}}}+\frac{6\left(15\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-19\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^6}+\ldots\right.}{96\,f}$$

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] 1/96*b^10*(231*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^7) - 231*a
rctan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(15/2) + 6*(15*sqrt(b*cos(f*x + e))*b
^2*cos(f*x + e)^2 - 19*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2
)^2*b^6) + 64/(sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5, x)
```

3.404 $\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = -\frac{80b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21f} + \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)^5/f+40/21*b^3*sin(f*x+e)/f/(b*sec(f*x
+e))^(1/2)+20/21*b^3*sin(f*x+e)^3/f/(b*sec(f*x+e))^(1/2)-80/21*b^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2
))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2704, 2707, 3856, 2720}

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} - \frac{80b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21f} + \frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-80*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])
/(21*f) + (40*b^3*Sin[e + f*x])/(21*f*Sqrt[b*Sec[e + f*x]]) + (20*b^3*Sin[e
 + f*x]^3)/(21*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e
+ f*x]^5)/(3*f)
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{3}\left(10b^2\right)\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx \\
&= \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} \\
&\quad - \frac{1}{7}\left(20b^2\right)\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx \\
&= \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} \\
&\quad + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{21}\left(40b^2\right)\int \sqrt{b\sec(e+fx)}\,dx
\end{aligned}$$

$$= \frac{40b^3 \sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3 \sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f} - \frac{1}{21}\left(40b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx$$

$$= -\frac{80b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21f} + \frac{40b^3 \sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3 \sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = -\frac{b^2\sqrt{b\sec(e+fx)}\left(320\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 58\sin(2(e+fx)) + 3\sin(4(e+fx)) - 56\,\text{t}\right.}{84f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] -1/84*(b^2*Sqrt[b*Sec[e + f*x]]*(320*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)
/2, 2] - 58*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)] - 56*Tan[e + f*x]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1396.80 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.34

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\,b^2\left(40i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)+40i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F\right.}{21f}$

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/f*(b*sec(f*x+e))^(1/2)*b^2*(40*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+40
*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(
-cot(f*x+e)+csc(f*x+e)),I)-3*cos(f*x+e)^3*sin(f*x+e)+16*sin(f*x+e)*cos(f*x+
e)+7*tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = -\frac{2\left(-20i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+20i\sqrt{2}b^{\frac{5}{2}}\cos(fx+\right.}{}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -2/21*(-20*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*
x + e) + I*sin(f*x + e)) + 20*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPIn
verse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + (3*b^2*cos(f*x + e)^4 - 16*b^
2*cos(f*x + e)^2 - 7*b^2)*sqrt(b/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e
))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx=\int \sin(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2), x)
```

3.405 $\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = -\frac{8b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)^3/f+4/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e
))^(1/2)-8/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2704, 2707, 3856, 2720}

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} - \frac{8b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-8*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (4*b^3*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f
*x])^(3/2)*Sin[e + f*x]^3)/(3*f)
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \left(2b^2\right)\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx \\
&= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \frac{1}{3}\left(4b^2\right)\int \sqrt{b\sec(e+fx)}\,dx \\
&= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} \\
&\quad - \frac{1}{3}\left(4b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= -\frac{8b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} \\
&\quad + \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = -\frac{b^2\sqrt{b\sec(e+fx)}\left(8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-\sin(2(e+fx))-2\tan(e+fx)\right)}{3f}$$

[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]

[Out] -1/3*(b^2*Sqrt[b*Sec[e + f*x]]*(8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - Sin[2*(e + f*x)] - 2*Tan[e + f*x]))/f

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 201.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.55

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\,b^2\left(4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)+4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F}{3f}$

[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)

[Out] 2/3/f*(b*sec(f*x+e))^(1/2)*b^2*(4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+sin(f*x+e)*cos(f*x+e)+tan(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = -\frac{2\left(-2i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+2i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e\right.}{3f\cos}$$

[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="fricas")

[Out] -2/3*(-2*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 2*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInver

```
se(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - (b^2*cos(f*x + e)^2 + b^2)*sqrt(
b/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2), x)
```

3.406 $\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = -\frac{4b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f-4/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2704, 3856, 2720}

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f} - \frac{4b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-4*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} - \frac{1}{3}\left(2b^2\right)\int\sqrt{b\sec(e+fx)}\,dx \\
&= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} - \frac{1}{3}\left(2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= -\frac{4b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int(b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \frac{2b^2\sqrt{b\sec(e+fx)}\left(-2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\tan(e+fx)\right)}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2
] + Tan[e + f*x]))/(3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.91 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.03

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\,b^2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)+2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F}{3f}$

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(b*sec(f*x+e))^(1/2)*b^2*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+2*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-co
t(f*x+e)+csc(f*x+e)),I)+tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = -\frac{2\left(-i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{w}\right.}{3f\cos(fx+e)}$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -2/3*(-I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x +
e) + I*sin(f*x + e)) + I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-
4, 0, cos(f*x + e) - I*sin(f*x + e)) - b^2*sqrt(b/cos(f*x + e))*sin(f*x + e
))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2), x)
```

3.407 $\int (b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{2b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{2b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(
3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}+\frac{1}{3}b^2\int\sqrt{b\sec(e+fx)}\,dx\\ &= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}+\frac{1}{3}\left(b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int(b\sec(e+fx))^{5/2}\,dx=\frac{2b^2\sqrt{b\sec(e+fx)}\left(\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\tan(e+fx)\right)}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(5/2),x]
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] +
 Tan[e + f*x]))/(3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\sqrt{b\sec(fx+e)}\, b^{2}\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, F}{3f}$

[In] int((b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/3/f*(b*sec(f*x+e))^(1/2)*b^2*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-tan(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}b^{\frac{5}{2}}\cos(}{3f\cos(}$$

[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*b^2*sqrt(b/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))

Sympy [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(e+fx))^{\frac{5}{2}}\,dx$$

[In] integrate((b*sec(f*x+e))**(5/2),x)

[Out] Integral((b*sec(e + f*x))**(5/2), x)

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\,dx = \int \left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2),x)
[Out] int((b/cos(e + f*x))^(5/2), x)
```

3.408 $\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*csc(f*x+e)*(b*sec(f*x+e))^(3/2)/f-5/3*b^3*csc(f*x+e)/f/(b*sec(f*x+e))
^(1/2)+5/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2706, 2705, 3856, 2720}

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-5*b^3*Csc[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) + (2*b*Csc[e + f*x]
*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{3}\left(5b^2\right)\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx \\
&= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{6}\left(5b^2\right)\int \sqrt{b\sec(e+fx)}\,dx \\
&= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f} \\
&\quad + \frac{1}{6}\left(5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} \\
&\quad + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b\left(2-3\cot^2(e+fx)+5\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)(b\sec(e+fx))^{3/}}{3f}$$

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
[Out] (b*(2 - 3*Cot[e + f*x]^2 + 5*Cos[e + f*x]^(3/2)*Csc[e + f*x]*EllipticF[(e +
 f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.62

method	result
default	$-\frac{i b^2 \sqrt{b\sec(fx+e)}\left(5\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)+5\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F\right.}{3f}$

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*I/f*b^2*(b*sec(f*x+e))^(1/2)*(5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+5*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot
(f*x+e)+csc(f*x+e)),I)-5*I*cot(f*x+e)+2*I*csc(f*x+e)*sec(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)) -}{}$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] 1/6*(-5*I*sqrt(2)*b^(5/2)*cos(f*x + e)*sin(f*x + e)*weierstrassPInverse(-4,
 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*b^(5/2)*cos(f*x + e)*sin(f
```

```
*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(5*b^
2*cos(f*x + e)^2 - 2*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)*sin(f*x + e
))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin(e+f\,x)^2}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2, x)
```

3.409 $\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{2f} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] b*csc(f*x+e)*(b*sec(f*x+e))^(3/2)/f-1/3*b*csc(f*x+e)^3*(b*sec(f*x+e))^(3/2)
/f-5/2*b^3*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+5/2*b^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^
(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2705, 2706, 3856, 2720}

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{2f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}$$

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-5*b^3*Csc[e + f*x])/(2*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(2*f) + (b*Csc[e + f*x]*(
b*Sec[e + f*x])^(3/2))/f - (b*Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{3}{2}\int\csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx\\
&= \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}\\
&\quad+\frac{1}{2}\left(5b^2\right)\int\csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx\\
&= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}\\
&\quad-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{1}{4}\left(5b^2\right)\int\sqrt{b\sec(e+fx)}\,dx
\end{aligned}$$

$$= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f} + \frac{1}{4}\left(5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}dx$$

$$= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{2f} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int\csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b\left(4-\cot^2(e+fx)\left(11+2\csc^2(e+fx)\right)+15\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\right.\right.}{6f}$$

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b*(4 - Cot[e + f*x]^2*(11 + 2*Csc[e + f*x]^2) + 15*Cos[e + f*x]^(3/2)*Csc[
e + f*x]*EllipticF[(e + f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(6
*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 95.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.66

method	result
default	$\frac{ib^2\sqrt{b\sec(fx+e)}\left(15\cos(fx+e)(\sin^2(fx+e))\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(-\cot(fx+e)+\csc(fx+e)),i)+15\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{6f(\cos^2(f}$

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I/f*b^2*(b*sec(f*x+e))^(1/2)/(cos(f*x+e)^2-1)*(15*cos(f*x+e)*sin(f*x+e)
^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(
-cot(f*x+e)+csc(f*x+e)),I)+15*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^2+15*I*cos
(f*x+e)^2*cot(f*x+e)-21*I*cot(f*x+e)+4*I*csc(f*x+e)*sec(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{15\sqrt{2}\left(i\,b^2\cos\left(fx+e\right)^3-i\,b^2\cos\left(fx+e\right)\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(\right.}{}$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(15*sqrt(2)*(I*b^2*cos(f*x + e)^3 - I*b^2*cos(f*x + e))*sqrt(b)*sin(f
*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*sqrt
(2)*(-I*b^2*cos(f*x + e)^3 + I*b^2*cos(f*x + e))*sqrt(b)*sin(f*x + e)*weier
strassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(15*b^2*cos(f*x +
e)^4 - 21*b^2*cos(f*x + e)^2 + 4*b^2)*sqrt(b/cos(f*x + e)))/((f*cos(f*x + e
)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin(e+f\,x)^4}\,dx$$

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4, x)
```

3.410 $\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^7}{15f(b\sec(e+fx))^{15/2}} - \frac{6b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{6b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/15*b^7/f/(b*sec(f*x+e))^(15/2)-6/11*b^5/f/(b*sec(f*x+e))^(11/2)+6/7*b^3/f
/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2702, 276}

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^7}{15f(b\sec(e+fx))^{15/2}} - \frac{6b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{6b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*b^7)/(15*f*(b*Sec[e + f*x])^(15/2)) - (6*b^5)/(11*f*(b*Sec[e + f*x])^(11
/2)) + (6*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(
3/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^7\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{17/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{b^7\text{Subst}\left(\int \left(-\frac{1}{x^{17/2}}+\frac{3}{b^2x^{13/2}}-\frac{3}{b^4x^{9/2}}+\frac{1}{b^6x^{5/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}\\
&\quad+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-7410+4035\cos(2(e+fx))-798\cos(4(e+fx))+77\cos(6(e+fx)))}{18480f(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
[Out] (b*(-7410 + 4035*Cos[2*(e + f*x)] - 798*Cos[4*(e + f*x)] + 77*Cos[6*(e + f*
x)]))/(18480*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\frac{2\left(\cos^{7}\left(f x+e\right)\right)}{15}-\frac{6\left(\cos^{5}\left(f x+e\right)\right)}{11}+\frac{6\left(\cos^{3}\left(f x+e\right)\right)}{7}-\frac{2\cos\left(f x+e\right)}{3}}{f\sqrt{b\sec\left(f x+e\right)}}$	55

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/f/(b*sec(f*x+e))^(1/2)*(77*cos(f*x+e)^7-315*cos(f*x+e)^5+495*cos(f*x
+e)^3-385*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\cos\left(fx+e\right)^{8}-315\cos\left(fx+e\right)^{6}+495\cos\left(fx+e\right)^{4}-385\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos(fx+e)}}}{1155\,bf}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*cos(f*x + e)^8 - 315*cos(f*x + e)^6 + 495*cos(f*x + e)^4 - 385*c
os(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\,b^6 - \frac{315\,b^6}{\cos(fx+e)^2} + \frac{495\,b^6}{\cos(fx+e)^4} - \frac{385\,b^6}{\cos(fx+e)^6}\right)b}{1155\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{15}{2}}}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*b^6 - 315*b^6/cos(f*x + e)^2 + 495*b^6/cos(f*x + e)^4 - 385*b^6/
cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(15/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\sqrt{b\cos(fx+e)}b^7\cos(fx+e)^7 - 315\sqrt{b\cos(fx+e)}b^7\cos(fx+e)^5 + 495\sqrt{b\cos(fx+e)}b^7\cos(\ldots\right.}{1155\,b^8 f\,\mathrm{sgn}\left(\cos(fx+e)\right)}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^7 - 315*sqrt(b*cos(f*x + e
))*b^7*cos(f*x + e)^5 + 495*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^3 - 385*s
qrt(b*cos(f*x + e))*b^7*cos(f*x + e))/(b^8*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^7}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2), x)
```

3.411 $\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/11*b^5/f/(b*sec(f*x+e))^(11/2)+4/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b
*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b^5)/(11*f*(b*Sec[e + f*x])^(11/2)) + (4*b^3)/(7*f*(b*Sec[e + f*x])^(7/
2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^5 \text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^5 \text{Subst}\left(\int \left(\frac{1}{x^{13/2}} - \frac{2}{b^2 x^{9/2}} + \frac{1}{b^4 x^{5/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{4b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-415+180\cos(2(e+fx))-21\cos(4(e+fx)))}{924f(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
[Out] (b*(-415 + 180*Cos[2*(e + f*x)] - 21*Cos[4*(e + f*x)]))/(924*f*(b*Sec[e + f
*x])^(3/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{2\left(21\left(\cos^5(fx+e)\right)-66\left(\cos^3(fx+e)\right)+77\cos(fx+e)\right)}{231f\sqrt{b\sec(fx+e)}}$	45

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/231/f/(b*sec(f*x+e))^(1/2)*(21*cos(f*x+e)^5-66*cos(f*x+e)^3+77*cos(f*x+e
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(21\cos\left(fx+e\right)^6-66\cos\left(fx+e\right)^4+77\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{231\,bf}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/231*(21*cos(f*x + e)^6 - 66*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(21\,b^4-\frac{66\,b^4}{\cos(fx+e)^2}+\frac{77\,b^4}{\cos(fx+e)^4}\right)b}{231\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -2/231*(21*b^4 - 66*b^4/cos(f*x + e)^2 + 77*b^4/cos(f*x + e)^4)*b/(f*(b/cos
(f*x + e))^(11/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(21\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^5 - 66\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^3 + 77\sqrt{b\cos(fx+e)}b^5\cos(\right.}{231\,b^6f\mathrm{sgn}(\cos(fx+e))}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -2/231*(21*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 66*sqrt(b*cos(f*x + e)
)*b^5*cos(f*x + e)^3 + 77*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e))/(b^6*f*sgn
(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^5}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2), x)
```

3.412 $\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2702, 14}

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
[Out] (2*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{9/2}}+\frac{1}{b^2x^{5/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{2b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$
\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-11+3\cos(2(e+fx)))}{21f(b\sec(e+fx))^{3/2}}
$$

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
[Out] (b*(-11 + 3*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\frac{2\left(\cos^3(fx+e)\right)}{7}-\frac{2\cos(fx+e)}{3}}{f\sqrt{b\sec(fx+e)}}$	35

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/f/(b*sec(f*x+e))^(1/2)*(3*cos(f*x+e)^3-7*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^4-7\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{21\,bf}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\,b^2-\frac{7\,b^2}{\cos(fx+e)^2}\right)b}{21\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 2/21*(3*b^2 - 7*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3 - 7\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{21\,b^4 f\mathrm{sgn}\left(\cos(fx+e)\right)}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e)^3 - 7*sqrt(b*cos(f*x + e))*b^
3*cos(f*x + e))/(b^4*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^3}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2), x)
```

3.413 $\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{3f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{3f(b\sec(fx+e))^{\frac{3}{2}}}$	17
default	$-\frac{2b}{3f(b\sec(fx+e))^{\frac{3}{2}}}$	17

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos{(fx+e)}^2}{3\,bf}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2/(b*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\cos{(fx+e)}}{3\,f\sqrt{\frac{b}{\cos(fx+e)}}}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -2/3*cos(f*x + e)/(f*sqrt(b/cos(f*x + e)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\sqrt{b\cos{(fx+e)}}\cos{(fx+e)}}{3\,bf\mathrm{sgn}\left(\cos{(fx+e)}\right)}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(b*cos(f*x + e))*cos(f*x + e)/(b*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\cos\left(e+f\,x\right)^2\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{3\,b\,f}$$

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(1/2),x)
[Out] -(2*cos(e + f*x)^2*(b/cos(e + f*x))^(1/2))/(3*b*f)
```

3.414 $\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

```
[Out] -arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-arctanh((b*sec(f*x+e))^(1/2
)/b^(1/2))/f/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2702, 335, 218, 212, 209}

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

```
[In] Int[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(Sqrt[b]*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(Sqrt[b]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx,x,b\sec(e+fx)\right)}{bf}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx,x,\sqrt{b\sec(e+fx)}\right)}{bf}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,x,\sqrt{b\sec(e+fx)}\right)}{f}-\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx,x,\sqrt{b\sec(e+fx)}\right)}{f}\\
&= -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(2\arctan\left(\sqrt{\sec(e+fx)}\right)-\log\left(1-\sqrt{\sec(e+fx)}\right)+\log\left(1+\sqrt{\sec(e+fx)}\right)\right)\sqrt{\sec(e+fx)}}{2f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/2*((2*ArcTan[Sqrt[Sec[e + f*x]]] - Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 +
 Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(47) = 94.

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.41

method	result	size
default	$\frac{\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\sec(fx+e)}}$	142

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)+1)/(cos(f*x+e)+1)))/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)/(b*sec(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(47) = 94.

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.29

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)-\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{4bf}\right.$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1
)/b) - sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)))/(b*f), 1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(co
s(f*x + e) - 1)/sqrt(b)) + sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} - \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{2f}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(3/2) - log(-(sqrt(b) - sq
rt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] (arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) + arctan(sqrt(b*cos(f*x + e
))/sqrt(b))/sqrt(b))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)), x)
```

3.415 $\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}$$

```
[Out] -1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-1/4*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f/b^(1/2)-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2702, 294, 335, 218, 212, 209}

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}$$

```
[In] Int[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/4*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(Sqrt[b]*f) - ArcTanh[Sqrt[b*Sec[
e + f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]])/(2
*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\left(-2\arctan\left(\sqrt{\sec(e+fx)}\right) + \log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(1+\sqrt{\sec(e+fx)}\right) - \frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}\right)\sqrt{\sec(}}{8f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]
])/(8*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.02

method	result
default	$-\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+}{(\cos(fx+e)}}}{\cos(fx+e)+1}\right.\right.}{8f\sqrt{-}}$

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x+e)*arctan
(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*cos(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/
(cos(f*x+e)+1))*cos(f*x+e)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(73) = 146$.

Time = 0.34 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.88

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \left[\frac{2\left(\cos(fx+e)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2-\left(\cos(fx+e\right.}{16\left(bf\cos(fx+e)^2-bf\right)}\right.$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - (cos(f
*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)))/(b*f*cos(f*x + e)^2 - b*f), 1/16*(2*(cos(f*x + e)
^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b))
 + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sqrt(b)*log
((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f
*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b
*f*cos(f*x + e)^2 - b*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} - \frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{8f}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*(4*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 2*arctan(sqrt(b/
cos(f*x + e))/sqrt(b))/b^(3/2) + log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqr
t(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^2\left(\frac{2\sqrt{b\cos(fx+e)}\cos(fx+e)}{\left(b^2\cos(fx+e)^2-b^2\right)b} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}\right)}{4f\operatorname{sgn}(\cos(fx+e))}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*(2*sqrt(b*cos(f*x + e))*cos(f*x + e)/((b^2*cos(f*x + e)^2 - b^2)*b)
 + arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + arctan(sqrt(b*cos
(f*x + e))/sqrt(b))/b^(5/2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)), x)
```

3.416 $\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}$$

```
[Out] -1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(5/2)/b^3/f-5/32*arctan((b*sec(f*x+e))^(1/
2)/b^(1/2))/f/b^(1/2)-5/32*arctanh((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-
5/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2702, 294, 335, 218, 212, 209}

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf}$$

```
[In] Int[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*Cot[e + f*x]^2*Sqrt[b*Sec[e +
 f*x]])/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(5/2))/(4*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f} + \frac{5\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3f}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{32bf}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f} + \frac{5\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{16bf}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f} - \frac{5\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f} - \frac{5\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32f}$$

$$= -\frac{5\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(10\arctan\left(\sqrt{\sec(e+fx)}\right) - 5\log\left(1-\sqrt{\sec(e+fx)}\right) + 5\log\left(1+\sqrt{\sec(e+fx)}\right) + 4(-5+\csc^2(e \cdots\right.}{64f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/64*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*L
og[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)*Sqr
t[Sec[e + f*x]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(99) = 198.

Time = 0.22 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.80

method	result
default	$\frac{\left(20\left(\cos^3(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-5\left(\sin^2(fx+e)\right)\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-5\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)}}}{\mathrm{co}}\right.\right.}{}$

[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/64/f*(20*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-5*sin(f*x+e)^2*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-5*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2*cos(f*x+e)+5*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*sin(f*x+e)^2+5*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2-36*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)^4

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(99) = 198.

Time = 0.34 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.66

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \left[\frac{10\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)-5\left(\cos(fx+e)^4-2\right.}{128\,(b}\right.$$

[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/128*(10*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) - 5*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f), 1/128*(1

```
0*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x
 + e))*(cos(f*x + e) - 1)/sqrt(b)) + 5*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(
b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x
 + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))
/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(\frac{4\left(5\,b^2\sqrt{\frac{b}{\cos(fx+e)}}-9\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\right)}{b^4-\frac{2\,b^4}{\cos(fx+e)^2}+\frac{b^4}{\cos(fx+e)^4}}-\frac{10\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{5\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{64\,f}$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 1/64*b*(4*(5*b^2*sqrt(b/cos(f*x + e)) - 9*(b/cos(f*x + e))^(5/2))/(b^4 - 2*
b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4) - 10*arctan(sqrt(b/cos(f*x + e))/s
qrt(b))/b^(3/2) + 5*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b
/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^4\left(\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}} + \frac{2\left(5\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3 - 9\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{\left(b^2\cos(fx+e)^2 - b^2\right)^2 b^4}\right)}{32\,f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*b^4*(5*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^4) + 5*arctan
(sqrt(b*cos(f*x + e))/sqrt(b))/b^(9/2) + 2*(5*sqrt(b*cos(f*x + e))*b^3*cos(
f*x + e)^3 - 9*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e))/((b^2*cos(f*x + e)^2
- b^2)^2*b^4))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)), x)
```

3.417 $\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

```
[Out] -8/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-20/117*b*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(3/2)-2/13*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)+16/39*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/c
os(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 3856, 2719}

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} + \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (16*EllipticE[(e + f*x)/2, 2])/(39*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]
]) - (8*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(3/2)) - (20*b*Sin[e + f*x]^
```

```
3)/(117*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^5)/(13*f*(b*Sec[e + f
*x])^(3/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}+\frac{10}{13}\int\frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}+\frac{20}{39}\int\frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}+\frac{8}{39}\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}+\frac{8\int\sqrt{\cos(e+fx)}\,dx}{39\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{768E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} - 317\sin(2(e+fx)) + 76\sin(4(e+fx)) - 9\sin(6(e+fx))}{1872f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
[Out] ((768*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 317*Sin[2*(e + f*x)]
+ 76*Sin[4*(e + f*x)] - 9*Sin[6*(e + f*x)])/(1872*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.90 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.93

method	result
default	$-\frac{2\left(\cos^6(fx+e)\right)\sin(fx+e)}{13}-\frac{2\left(\cos^5(fx+e)\right)\sin(fx+e)}{13}+\frac{16i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)}{39}-\frac{16i\sqrt{\frac{}{\cos}}}{}$

```
[In] int(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/117/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(-9*cos(f*x+e)^6*sin(f*x+e)-9*c
os(f*x+e)^5*sin(f*x+e)+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)-24*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e
)+csc(f*x+e)),I)*cos(f*x+e)+28*cos(f*x+e)^4*sin(f*x+e)+48*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*
x+e)),I)-48*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+28*cos(f*x+e)^3*sin(f*x+e)+24*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x
+e)+csc(f*x+e)),I)*sec(f*x+e)-24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-31*si
n(f*x+e)*cos(f*x+e)^2-31*sin(f*x+e)*cos(f*x+e)+24*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(\left(9\cos\left(fx+e\right)^6-28\cos\left(fx+e\right)^4+31\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)-12i\sqrt{2}\sqrt{b}\text{weierstr}\right.}{}$$

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/117*((9*cos(f*x + e)^6 - 28*cos(f*x + e)^4 + 31*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))*sin(f*x + e) - 12*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 12*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(
f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^6\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**6/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

```
[In] int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2), x)
```

3.418 $\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

```
[Out] -4/15*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-2/9*b*sin(f*x+e)^3/f/(b*sec(f*x+e
))^(3/2)+8/15*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 3856, 2719}

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} - \frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} + \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]
) - (4*b*Sin[e + f*x])/(15*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)
/(9*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx \\
&= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{4}{15}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx \\
&= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{4\int\sqrt{\cos(e+fx)}\,dx}{15\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\
&= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} - 68\sin(2(e+fx)) + 10\sin(4(e+fx))}{360f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
[Out] ((192*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 68*Sin[2*(e + f*x)] +
 10*Sin[4*(e + f*x)])/(360*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.75

method	result
default	$\frac{8i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)}{15}-\frac{8i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)}{15}$

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(12*I*EllipticE(I*(-cot(f*x+e)+c
sc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)-12*I*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+5*cos(f*x+e)^4*sin(f*x+e)+
24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(-cot(f*x+e)+csc(f*x+e)),I)-24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+5*cos(f*x+e)^3*si
n(f*x+e)+12*I*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sec(f*x+e)-12*I*EllipticF(I*(-cot(f*x
+e)+csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*sec(f*x+e)-11*sin(f*x+e)*cos(f*x+e)^2-11*sin(f*x+e)*cos(f*x+e)+12*sin(f*
x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(\left(5\cos(fx+e)^4-11\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)+6i\sqrt{2}\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstr}\right.}{}$$

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((5*cos(f*x + e)^4 - 11*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sin(f*x +
 e) + 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^4{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(e+f\,x)}^4}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2), x)
```

3.419 $\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/5*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+4/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/
(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 3856, 2719}

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
 - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
```

```
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2}{5}\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2\int\sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{4E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int\frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\sqrt{b\sec(e+fx)}\left(-8\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|\,2\right)+\sin(e+fx)+\sin(3(e+fx))\right)}{10bf}$$

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
[Out] -1/10*(Sqrt[b*Sec[e + f*x]]*(-8*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2
] + Sin[e + f*x] + Sin[3*(e + f*x)]))/(b*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.22

method	result
default	$-\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e}{}$

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos
(f*x+e)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-
4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*
(-cot(f*x+e)+csc(f*x+e)),I)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-2*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*
x+e)+csc(f*x+e)),I)*sec(f*x+e)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e
)-2*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e)-i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\right.}{}$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sin(f*x + e) - I*sqrt(2)*sqrt(b)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
+ e))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin^2{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin{(e+f\,x)}^2}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2), x)
```

3.420 $\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx=\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx=\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[1/Sqrt[b*Sec[e + f*x]],x]
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.87

method	result
risch	$-\frac{i\sqrt{2}}{f\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(fx+e)}+b\right)}{b\sqrt{e^{i(fx+e)}\left(b\,e^{2i(fx+e)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(fx+e)}-i\right)}\sqrt{ie^{i(fx+e)}}\left(-2iE\left(\sqrt{-i\left(e^{i(fx+e)}+i\right)},\frac{\sqrt{2}}{2}\cdots\right)\right)}{\sqrt{b\,e^{3i(fx+e)}+b\,e^{i(fx+e)}}}\right)}{f\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)}$
default	$2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc($

```
[In] int(1/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -I/f*2^(1/2)/(b*exp(I*(f*x+e))/(exp(I*(f*x+e))^2+1))^(1/2)-I/f*(-2*(b*exp(I
*(f*x+e))^2+b)/b/(exp(I*(f*x+e))*(b*exp(I*(f*x+e))^2+b))^(1/2)+I*(-I*(exp(I
*(f*x+e))+I))^(1/2)*2^(1/2)*(I*(exp(I*(f*x+e))-I))^(1/2)*(I*exp(I*(f*x+e)))
^(1/2)/(b*exp(I*(f*x+e))^3+b*exp(I*(f*x+e)))^(1/2)*(-2*I*EllipticE((-I*(exp
(I*(f*x+e))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(f*x+e))+I))^(1/2
),1/2*2^(1/2))))*2^(1/2)/(b*exp(I*(f*x+e))/(exp(I*(f*x+e))^2+1))^(1/2)*(b*e
xp(I*(f*x+e))*(exp(I*(f*x+e))^2+1))^(1/2)/(exp(I*(f*x+e))^2+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) - i\sqrt{2}\sqrt{b}\text{weierst}}{bf}$$

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*
x + e) + I*sin(f*x + e))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b \sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b \sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b \sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(b/cos(e + f*x))^(1/2),x)
[Out] int(1/(b/cos(e + f*x))^(1/2), x)
```

3.421 $\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2719}

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/
(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
```

```
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx \\ &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{\int \sqrt{\cos(e+fx)}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{-\cot(e+fx) - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
[Out] (-Cot[e + f*x] - EllipticE[(e + f*x)/2, 2]/Sqrt[Cos[e + f*x]])/(f*Sqrt[b*Se
c[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

method	result
default	$\frac{i\left(-\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)+\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i\right.}{}$

[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] I/f/(b*sec(f*x+e))^(1/2)*(-(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)+(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-co
t(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)+(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)+I*csc
(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\sin(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+}{}$$

[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/2*(-I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + I*sqrt(2)*sqrt(b)*sin(f*x + e
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e))) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2)/(b*f*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^2{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin{(e+f\,x)}^2\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)), x)
```

3.422 $\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -1/2*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e)
)^(3/2)-1/2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1
/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2719}

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/2*(b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^3)/(3*f*
(b*Sec[e + f*x])^(3/2)) - EllipticE[(e + f*x)/2, 2]/(2*f*Sqrt[Cos[e + f*x]]
*Sqrt[b*Sec[e + f*x]])
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{1}{2}\int\frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{1}{4}\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{\int\sqrt{\cos(e+fx)}\,dx}{4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int\frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(-3+\csc^2(e+fx)+2\csc^4(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)\tan(e+fx)}{6f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
[Out] -1/6*((-3 + Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e
+ f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.21

method	result
default	$\frac{3i\left(\sin^2(fx+e)\right)E(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-3i\left(\sin^2(fx+e)\right)F(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\cdots}}{}$

[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/f/(b*sec(f*x+e))^(1/2)/(cos(f*x+e)^2-1)*(3*I*sin(f*x+e)^2*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-3*I*sin(f*x+e)^2*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)*tan(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)*tan(f*x+e)+3*sin(f*x+e)+2*cot(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.66

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx =$$
$$-\frac{3\sqrt{2}\left(i\cos\left(fx+e\right)^2-i\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right.\cdots}{}$$

[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] -1/12*(3*sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(-I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(3*cos(f*x + e)^4 - 5*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b*f*cos(f*x + e)^2 - b*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc^4{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{1}{\sin{(e+f\,x)}^4\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)), x)
```

3.423 $\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -7/20*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-7/30*b*csc(f*x+e)^3/f/(b*sec(f*x+
e))^(3/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)-7/20*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f
*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2705, 3856, 2719}

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Csc[e + f*x])/(20*f*(b*Sec[e + f*x])^(3/2)) - (7*b*Csc[e + f*x]^3)/(3
0*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^5)/(5*f*(b*Sec[e + f*x])^(3/2
```

```
)) - (7*EllipticE[(e + f*x)/2, 2])/(20*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]])
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{10}\int\frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{20}\int\frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7}{40}\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7\int\sqrt{\cos(e+fx)}\,dx}{40\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}\\
&\quad-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(-21+7\csc^2(e+fx)+2\csc^4(e+fx)+12\csc^6(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\right.\right.}{60f\sqrt{b\sec(e+fx)}}$$

[In] Integrate[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]

[Out] -1/60*((-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 12*Csc[e + f*x]^6 + 21*Sqrt[Cos[e + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.71

method	result
default	$-\frac{21i(\sin^4(fx+e))E(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-21i(\sin^4(fx+e))F(i(-\cot(fx+e)+\csc(fx+e)),}{}$

[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/60/f/(cos(f*x+e)-1)^2/(cos(f*x+e)+1)^2/(b*sec(f*x+e))^(1/2)*(21*I*sin(f*x+e)^4*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-21*I*sin(f*x+e)^4*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^3*tan(f*x+e)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^3*tan(f*x+e)+21*sin(f*x+e)^3+14*sin(f*x+e)*cos(f*x+e)+12*cot(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{21\sqrt{2}\left(i\cos{(fx+e)}^4 - 2i\cos{(fx+e)}^2 + i\right)\sqrt{b}\sin{(fx+e)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}}{}$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(21*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin(
f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I
*sin(f*x + e))) + 21*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*s
qrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))) + 2*(21*cos(f*x + e)^6 - 56*cos(f*x + e)^4 + 47*
cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x
+ e)^2 + b*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^6{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**6/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc{(fx+e)}^6}{\sqrt{b\sec{(fx+e)}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^6\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)),x)

[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)), x)

3.424 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^7}{17f(b\sec(e+fx))^{17/2}} - \frac{6b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{2b^3}{3f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] 2/17*b^7/f/(b*sec(f*x+e))^(17/2)-6/13*b^5/f/(b*sec(f*x+e))^(13/2)+2/3*b^3/f
/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2702, 276}

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^7}{17f(b\sec(e+fx))^{17/2}} - \frac{6b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{2b^3}{3f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^7)/(17*f*(b*Sec[e + f*x])^(17/2)) - (6*b^5)/(13*f*(b*Sec[e + f*x])^(13
/2)) + (2*b^3)/(3*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(
5/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^7\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{19/2}}\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{b^7\text{Subst}\left(\int \left(-\frac{1}{x^{19/2}}+\frac{3}{b^2x^{15/2}}-\frac{3}{b^4x^{11/2}}+\frac{1}{b^6x^{7/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}} \\
&\quad +\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-10766+8365\cos(2(e+fx))-1890\cos(4(e+fx))+195\cos(6(e+fx)))}{53040f(b\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
[Out] (b*(-10766 + 8365*Cos[2*(e + f*x)] - 1890*Cos[4*(e + f*x)] + 195*Cos[6*(e +
 f*x)]))/(53040*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2\left(\cos^8(fx+e)\right)}{17}-\frac{6\left(\cos^6(fx+e)\right)}{13}+\frac{2\left(\cos^4(fx+e)\right)}{3}-\frac{2\left(\cos^2(fx+e)\right)}{5}}{fb\sqrt{b\sec(fx+e)}}$	60

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3315/f/b/(b*sec(f*x+e))^(1/2)*(195*cos(f*x+e)^8-765*cos(f*x+e)^6+1105*cos
(f*x+e)^4-663*cos(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\cos\left(fx+e\right)^9 - 765\cos\left(fx+e\right)^7 + 1105\cos\left(fx+e\right)^5 - 663\cos\left(fx+e\right)^3\right)}{3315\,b^2 f}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/3315*(195*cos(f*x + e)^9 - 765*cos(f*x + e)^7 + 1105*cos(f*x + e)^5 - 663
*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\,b^6 - \frac{765\,b^6}{\cos(fx+e)^2} + \frac{1105\,b^6}{\cos(fx+e)^4} - \frac{663\,b^6}{\cos(fx+e)^6}\right)b}{3315\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{17}{2}}}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] 2/3315*(195*b^6 - 765*b^6/cos(f*x + e)^2 + 1105*b^6/cos(f*x + e)^4 - 663*b^
6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(17/2))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\sqrt{b\cos\left(fx+e\right)}b^8\cos\left(fx+e\right)^8 - 765\sqrt{b\cos\left(fx+e\right)}b^8\cos\left(fx+e\right)^6 + 1\right.}{3315\,b^{10}f\mathrm{sgn}\left(\mathrm{co}\right.}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^8 - 765*sqrt(b*cos(f*x +
e))*b^8*cos(f*x + e)^6 + 1105*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^4 - 663
*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^2)/(b^10*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^7}{{\left(\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2), x)
```

3.425 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/13*b^5/f/(b*sec(f*x+e))^(13/2)+4/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b
*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^5)/(13*f*(b*Sec[e + f*x])^(13/2)) + (4*b^3)/(9*f*(b*Sec[e + f*x])^(9/
2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^5\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{15/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{x^{15/2}} - \frac{2}{b^2x^{11/2}} + \frac{1}{b^4x^{7/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= -\frac{2b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{4b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))}{2340f(b\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)]))/(2340*f*(b*Sec[e +
f*x])^(5/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{2\left(45\left(\cos^6(fx+e)\right)-130\left(\cos^4(fx+e)\right)+117\left(\cos^2(fx+e)\right)\right)}{585fb\sqrt{b\sec(fx+e)}}$	50

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585/f/b/(b*sec(f*x+e))^(1/2)*(45*cos(f*x+e)^6-130*cos(f*x+e)^4+117*cos(f
*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,\cos\left(fx+e\right)^{7}-130\,\cos\left(fx+e\right)^{5}+117\,\cos\left(fx+e\right)^{3}\right)\sqrt{\frac{b}{\cos(fx+e)}}}{585\,b^{2}f}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/585*(45*cos(f*x + e)^7 - 130*cos(f*x + e)^5 + 117*cos(f*x + e)^3)*sqrt(b
/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,b^{4}-\frac{130\,b^{4}}{\cos(fx+e)^{2}}+\frac{117\,b^{4}}{\cos(fx+e)^{4}}\right)b}{585\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{13}{2}}}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -2/585*(45*b^4 - 130*b^4/cos(f*x + e)^2 + 117*b^4/cos(f*x + e)^4)*b/(f*(b/c
os(f*x + e))^(13/2))
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(45\sqrt{b\cos\left(fx+e\right)}b^6\cos\left(fx+e\right)^6-130\sqrt{b\cos\left(fx+e\right)}b^6\cos\left(fx+e\right)^4+117\sqrt{b\cos\left(fx+e\right)}b^6\cos\left(\right.\right.}{585\,b^8f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -2/585*(45*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^6 - 130*sqrt(b*cos(f*x + e
))*b^6*cos(f*x + e)^4 + 117*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^2)/(b^8*f
*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2), x)
```

3.426 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] 2/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2702, 14}

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
[Out] (2*b^3)/(9*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{11/2}}+\frac{1}{b^2x^{7/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-13+5\cos(2(e+fx)))}{45f(b\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
[Out] (b*(-13 + 5*Cos[2*(e + f*x)]))/(45*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{2\left(\cos^4(fx+e)\right)}{9}-\frac{2\left(\cos^2(fx+e)\right)}{5}}{fb\sqrt{b\sec(fx+e)}}$	40

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45/f/b/(b*sec(f*x+e))^(1/2)*(5*cos(f*x+e)^4-9*cos(f*x+e)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\,\cos\left(fx+e\right)^5 - 9\,\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45\,b^2 f}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/45*(5*cos(f*x + e)^5 - 9*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\,b^2 - \frac{9\,b^2}{\cos(fx+e)^2}\right)b}{45\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] 2/45*(5*b^2 - 9*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^4-9\sqrt{b\cos\left(fx+e\right)}b^4\cos\left(fx+e\right)^2\right)}{45\,b^6f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] 2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 9*sqrt(b*cos(f*x + e))*b^
4*cos(f*x + e)^2)/(b^6*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2), x)
```

3.427 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 30}

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{5f(b\sec(fx+e))^{\frac{5}{2}}}$	17
default	$-\frac{2b}{5f(b\sec(fx+e))^{\frac{5}{2}}}$	17

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^{3}}{5\,b^{2}f}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/5*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/(b^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\,\cos\left(fx+e\right)}{5\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] -2/5*cos(f*x + e)/(f*(b/cos(f*x + e))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{b\cos\left(fx+e\right)}\cos\left(fx+e\right)^{2}}{5\,b^{2}f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] -2/5*sqrt(b*cos(f*x + e))*cos(f*x + e)^2/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\cos\left(e+f\,x\right)^3\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{5\,b^2\,f}$$

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(3/2),x)
[Out] -(2*cos(e + f*x)^3*(b/cos(e + f*x))^(1/2))/(5*b^2*f)
```

3.428 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

```
[Out] arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f-arctanh((b*sec(f*x+e))^(1/2)
/b^(1/2))/b^(3/2)/f+2/b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2702, 331, 335, 304, 209, 212}

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(3/2)*f) - ArcTanh[Sqrt[b*Sec[e + f
*x]]/Sqrt[b]]/(b^(3/2)*f) + 2/(b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}\\ &= \frac{2}{bf\sqrt{b\sec(e+fx)}}+\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{b^3f}\end{aligned}$$

$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$

$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf}$$

$$= \frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} - \frac{\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} + \frac{2}{bf\sqrt{b\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4}, 1, \frac{3}{4}, \sec^2(e+fx)\right)}{bf\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, Sec[e + f*x]^2])/(b*f*Sqrt[b*Sec[e + f*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(64) = 128.

Time = 0.18 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.53

method	result
default	$\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos}{(\cos(f}}}{\cos(fx+e)+1}\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\sec(fx+e)}\,b}$

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-ln((2
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
```

```
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(cos(f*x+e)+1)/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.

Time = 0.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.03

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)-\sqrt{-b}\log}{4b^2f}\right.$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos
(f*x + e) + b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) - sqrt(-b)*log(-(b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b^2*f)
, 1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(
f*x + e) - b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) + sqrt(b)*log(-(b*cos(
f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
 + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b^2*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(e+fx)}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}+\frac{4}{b^2\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{2f}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(5/2) + log(-(sqrt(b) - sqr
t(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2) + 4/(b^2*sqrt(
b/cos(f*x + e))))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\frac{b\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{b}\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right) + 2\sqrt{b\cos(fx+e)}}{b^2 f \operatorname{sgn}(\cos(fx+e))}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] (b*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) - sqrt(b)*arctan(sqrt(b*c
os(f*x + e))/sqrt(b)) + 2*sqrt(b*cos(f*x + e)))/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,{\left(\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)), x)
```

3.429 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}$$

```
[Out] -1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/4*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/b^(3/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2702, 296, 335, 304, 209, 212}

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}$$

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/4*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(3/2)*f) + ArcTanh[Sqrt[b*Sec[
e + f*x]]/Sqrt[b]]/(4*b^(3/2)*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3/2))/
(2*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4b^3f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f} - \frac{\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4bf} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4bf}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{-4\csc^2(e+fx) - 2\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)} + \left(-\log\left(1-\sqrt{\sec(\right.\right.}{8bf\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
[Out] (-4*Csc[e + f*x]^2 - 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (-Lo
g[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]]
)/(8*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(73) = 146$.

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.02

method	result
default	$\frac{\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e)-\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+4\sqrt{-}}{8f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{b\sec(fx}}$

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/8/f*(ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-cos(f*x+e
)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/
(b*sec(f*x+e))^(1/2)/b/(cos(f*x+e)^2-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(73) = 146$.

Time = 0.37 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.91

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[-\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2\,b}\right)+\left(\cos\left(fx+e\right)^2\right.}{16}\right.$$

[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) + (cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
+ e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos
(f*x + e))*cos(f*x + e))/(b^2*f*cos(f*x + e)^2 - b^2*f), 1/16*(2*(cos(f*x +
e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(
b)) + (cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(
b^2*f*cos(f*x + e)^2 - b^2*f)]

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^3\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)

[Out] Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b\left(\frac{4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^4-\frac{b^4}{\cos(fx+e)^2}} - \frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} - \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}\right)}{8f}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] 1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^4 - b^4/cos(f*x + e)^2) - 2*arctan(sqrt(
b/cos(f*x + e))/sqrt(b))/b^(5/2) - log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(s
qrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{b\cos(fx+e)}}{b^2\cos(fx+e)^2-b^2} - \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}}{4f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] 1/4*(2*sqrt(b*cos(f*x + e))/(b^2*cos(f*x + e)^2 - b^2) - arctan(sqrt(b*cos(
f*x + e))/sqrt(-b))/(sqrt(-b)*b) + arctan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(
3/2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)), x)
```

3.430 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f} - \frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}$$

```
[Out] -3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+3/32*arctanh((b*sec(f*
x+e))^(1/2)/b^(1/2))/b^(3/2)/f-3/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3/f
-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(3/2)/b^3/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2702, 294, 296, 335, 304, 209, 212}

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}-\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}$$

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) + (3*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) - (3*Cot[e + f*x]^2*(b*Sec[e + f*x
])^(3/2))/(16*b^3*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(3/2))/(4*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\text{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{32b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{16b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32bf} - \frac{3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32bf}$$

$$= -\frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2} f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2} f} - \frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{4\csc^2(e+fx)-16\csc^4(e+fx)-6\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+3\left(\right.}{64bf\sqrt{b\sec(e+}}$$

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 - 6*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[S
ec[e + f*x]] + 3*(-Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]
])*Sqrt[Sec[e + f*x]])/(64*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(99) = 198.

Time = 0.20 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.78

method	result
default	$-\frac{\left(-3\left(\sin^2(fx+e)\right)\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e}{\cos(fx+e)+1}\right.\right.}{}$

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(-3*sin(f*x+e)^2*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2))+3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2*co
s(f*x+e)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2+3*arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*sin(f*x+e)^2-3*ln((2*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2+12*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2))/(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/b*csc(f*x+e)^
4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(99) = 198$.

Time = 0.35 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.69

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[-\frac{6\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)}{}\right.$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos
(f*x + e)))/(b^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f), 1/128*
(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*
x + e))*(cos(f*x + e) - 1)/sqrt(b)) + 3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2
+ 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*
x + e) + 1)) - 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(b
^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{b\left(\frac{4\left(b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}+3\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}\right)}{b^6-\frac{2\,b^6}{\cos(fx+e)^2}+\frac{b^6}{\cos(fx+e)^4}}+\frac{6\,\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}\right)}{64\,f}$$

[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] -1/64*b*(4*(b^2*(b/cos(f*x + e))^(3/2) + 3*(b/cos(f*x + e))^(7/2))/(b^6 - 2*b^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) + 6*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(5/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2))/f

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{b^2\left(\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{2\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2+3\,\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^2}\right)}{32\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] -1/32*b^2*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(7/2) + 2*(sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 + 3*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2)^2*b^2))/(f*sgn(cos(f*x + e)))

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)), x)
```

3.431 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77b^2f} - \frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}} + \frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}} - \frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

```
[Out] -12/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)-2/11*b*sin(f*x+e)^3/f/(b*sec(f*x
+e))^(5/2)+8/77*sin(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+8/77*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x
+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2707, 3854, 3856, 2720}

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77b^2f} - \frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}} + \frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}} - \frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*b
^2*f) - (12*b*Sin[e + f*x])/(77*f*(b*Sec[e + f*x])^(5/2)) + (8*Sin[e + f*x]
```

```
)/(77*b*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(11*f*(b*Sec[e + f*x
])^(5/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{6}{11}\int\frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{12}{77}\int\frac{1}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{4\int\sqrt{b\sec(e+fx)}\,dx}{77b^2}\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}\\
&\quad+\frac{\left(4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{77b^2}
\end{aligned}$$

$$= \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77b^2f} - \frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}} + \frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}} - \frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(128\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 5\sin(2(e+fx)) - 24\right.}{1232f(b\sec(e+fx))^{3/2}}$$

[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]

[Out] (Sec[e + f*x]^2*(128*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 5*Sin[2*(e + f*x)] - 24*Sin[4*(e + f*x)] + 7*Sin[6*(e + f*x)]))/(1232*f*(b*Sec[e + f*x])^(3/2))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.40

method	result
default	$-\frac{2\left(-7(\cos^4(fx+e))\sin(fx+e)+4i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)+4i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\right.}{77f\sqrt{b\sec(fx+e)}\,b}$

[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/77/f/(b*sec(f*x+e))^(1/2)/b*(-7*cos(f*x+e)^4*sin(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)+13*sin(f*x+e)*cos(f*x+e)^2-4*sin(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(\left(7\cos(fx+e)^5 - 13\cos(fx+e)^3 + 4\cos(fx+e)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e) - \right.}{}$$

[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/77*((7*cos(f*x + e)^5 - 13*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e) - 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^2*f)

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)

[Out] Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2), x)
```

3.432 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21b^2f} - \frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)+4/21*sin(f*x+e)/b/f/(b*sec(f*x+e))
^(1/2)+4/21*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 3854, 3856, 2720}

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21b^2f} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(21*b
^2*f) - (2*b*Sin[e + f*x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (4*Sin[e + f*x])/
(21*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}+\frac{2}{7}\int\frac{1}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}+\frac{2\int\sqrt{b\sec(e+fx)}\,dx}{21b^2}\\
&= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{21b^2}\\
&= \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21b^2f}\\
&\quad-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(16\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+2\sin(2(e+fx))-3\sin(\right.}{84f(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(16*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 2*Sin[2*
(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f*(b*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.63

method	result
default	$-\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)+2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)\right.}{21f\sqrt{b\sec(fx+e)}\,b}$

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/f/(b*sec(f*x+e))^(1/2)/b*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+2*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+cs
c(f*x+e)),I)*sec(f*x+e)+3*sin(f*x+e)*cos(f*x+e)^2-2*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(3\cos(fx+e)^3-2\cos(fx+e)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(fx+\right.}{21\,b^2f}$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*((3*cos(f*x + e)^3 - 2*cos(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e
) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x +
 e)) - I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e)))/(b^2*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin^2{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^2}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin{(e+f\,x)}^2}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2), x)
```

3.433 $\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f} + \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/3*sin(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+2/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*
sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f} + \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^
2*f) + (2*Sin[e + f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\int\sqrt{b\sec(e+fx)}\,dx}{3b^2}\\ &= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2}\\ &= \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int\frac{1}{(b\sec(e+fx))^{3/2}}\,dx=\frac{\sec^2(e+fx)\left(2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sin(2(e+fx))\right)}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[(b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + Sin[2*(e
+ f*x)]))/(3*f*(b*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.00

method	result
default	$-\frac{2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i\right.}{3f\sqrt{b\sec(fx+e)}\,b}$

```
[In] int(1/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f/(b*sec(f*x+e))^(1/2)/b*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+I*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x
+e)),I)*sec(f*x+e)-sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(fx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + I*sqrt(2)*sqrt(b
)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^2*f)
```

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(b/cos(e + f*x))^(3/2),x)
[Out] int(1/(b/cos(e + f*x))^(3/2), x)
```

3.434 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{b^2f}$$

```
[Out] -csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x
+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2703, 3856, 2720}

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{b^2f} - \frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*Sqrt[b*Sec[e + f*x]])) - (Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(b^2*f)
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\,dx}{2b^2} \\
&= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2b^2} \\
&= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{b^2 f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{-\sqrt{\cos(e+fx)}\csc(e+fx) - \text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{f\cos^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-(Sqrt[Cos[e + f*x]]*Csc[e + f*x]) - EllipticF[(e + f*x)/2, 2])/(f*Cos[e +
 f*x]^(3/2)*(b*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.10

method	result
default	$\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\sec(}{f\sqrt{b\sec(fx+e)}\,b}$

[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f/(b*sec(f*x+e))^(1/2)/b*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-csc(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\sin\left(fx+e\right)\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))-i\sqrt{}}{}$$

[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/2*(I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)

[Out] Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)), x)
```

3.435 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6b^2f}$$

```
[Out] 1/6*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e))
^(1/2)-1/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2703, 2705, 3856, 2720}

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6b^2f} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} + \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Csc[e + f*x]/(6*b*f*Sqrt[b*Sec[e + f*x]]) - Csc[e + f*x]^3/(3*b*f*Sqrt[b*Se
c[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e +
 f*x]])/(6*b^2*f)
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{6b^2}\\
&= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\int \sqrt{b\sec(e+fx)}\,dx}{12b^2}\\
&= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{12b^2}\\
&= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\\
&\quad -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6b^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)-2\csc^3(e+fx)-\frac{\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}}{6bf\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x] - 2*Csc[e + f*x]^3 - EllipticF[(e + f*x)/2, 2]/Sqrt[Cos[e + f
*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

method	result
default	$\frac{-i(\sin^2(fx+e))F(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(-\cot(fx+e)+\csc}{6f\sqrt{b\sec(fx+e)}\,b(\cos^2(fx+e)-1)}$

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f/(b*sec(f*x+e))^(1/2)/b/(cos(f*x+e)^2-1)*(-I*sin(f*x+e)^2*EllipticF(I*
(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e
)+1))^(1/2)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)*tan(f*x+e)+cos(f*x+e)*cot(
f*x+e)+csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.45

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i}{}$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassPInver
se(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(-I*cos(f*x + e)^2 + I)*
sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
e)) + 2*(cos(f*x + e)^3 + cos(f*x + e))*sqrt(b/cos(f*x + e)))/((b^2*f*cos(f
*x + e)^2 - b^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^4{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin{(e+f\,x)}^4\,{\left(\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)), x)
```

3.436 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{12b^2f}$$

```
[Out] 1/12*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+1/30*csc(f*x+e)^3/b/f/(b*sec(f*x+e
))^(1/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(1/2)-1/12*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+
e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2703, 2705, 3856, 2720}

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{12b^2f} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Csc[e + f*x]/(12*b*f*Sqrt[b*Sec[e + f*x]]) + Csc[e + f*x]^3/(30*b*f*Sqrt[b*
Sec[e + f*x]]) - Csc[e + f*x]^5/(5*b*f*Sqrt[b*Sec[e + f*x]]) - (Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(12*b^2*f)
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}-\frac{\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx}{10b^2}\\
&= \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}-\frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{12b^2}\\
&= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}-\frac{\int \sqrt{b\sec(e+fx)}\,dx}{24b^2}\\
&= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{24b^2}\\
&= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{12b^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{5\csc(e+fx)+2\csc^3(e+fx)-12\csc^5(e+fx)-\frac{5\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}}{60bf\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (5*Csc[e + f*x] + 2*Csc[e + f*x]^3 - 12*Csc[e + f*x]^5 - (5*EllipticF[(e +
f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(60*b*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.59

method	result
default	$\frac{5i(\sin^4(fx+e))F(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(-\cot(fx+e)+\mathrm{cs}}{60f(\cos(fx+e)-1)^2(\cos(fx+e)+1)^2\sqrt{b}}$

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/f/(cos(f*x+e)-1)^2/(cos(f*x+e)+1)^2/(b*sec(f*x+e))^(1/2)/b*(5*I*sin(f*
x+e)^4*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(1/(cos(f*x+e)+1))^(1/2)+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^3*tan
(f*x+e)+5*cos(f*x+e)^3*cot(f*x+e)-12*cos(f*x+e)*cot(f*x+e)-5*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.49

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\sqrt{2}\left(-i\cos(fx+e)^4+2i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\mathrm{s}}{}$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/120*(5*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin(
f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt
```

```
(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(5*cos(f*x + e)^5 -
 12*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e)))/((b^2*f*cos(f*x
+ e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**6/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^6\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)), x)
```

3.437 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^7}{19f(b\sec(e+fx))^{19/2}} - \frac{2b^5}{5f(b\sec(e+fx))^{15/2}} + \frac{6b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] 2/19*b^7/f/(b*sec(f*x+e))^(19/2)-2/5*b^5/f/(b*sec(f*x+e))^(15/2)+6/11*b^3/f
/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^7}{19f(b\sec(e+fx))^{19/2}} - \frac{2b^5}{5f(b\sec(e+fx))^{15/2}} + \frac{6b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^7)/(19*f*(b*Sec[e + f*x])^(19/2)) - (2*b^5)/(5*f*(b*Sec[e + f*x])^(15/
2)) + (6*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^
(7/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^7\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{21/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{b^7\text{Subst}\left(\int \left(-\frac{1}{x^{21/2}}+\frac{3}{b^2x^{17/2}}-\frac{3}{b^4x^{13/2}}+\frac{1}{b^6x^{9/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}\\
&= \frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}\\
&\quad+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-15226+14287\cos(2(e+fx))-3542\cos(4(e+fx))+385\cos(6(e}{117040b^3f}$$

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-15226 + 14287*Cos[2*(e + f*x)] - 3542*Cos[4*(e + f*x)] +
385*Cos[6*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(117040*b^3*f)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2\left(\cos^{9}(fx+e)\right)}{19}-\frac{2\left(\cos^{7}(fx+e)\right)}{5}+\frac{6\left(\cos^{5}(fx+e)\right)}{11}-\frac{2\left(\cos^{3}(fx+e)\right)}{7}}{f\,b^{2}\sqrt{b\sec(fx+e)}}$	60

[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/7315/f/b^2/(b*sec(f*x+e))^(1/2)*(385*cos(f*x+e)^9-1463*cos(f*x+e)^7+1995*cos(f*x+e)^5-1045*cos(f*x+e)^3)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\cos\left(fx+e\right)^{10}-1463\cos\left(fx+e\right)^{8}+1995\cos\left(fx+e\right)^{6}-1045\cos\left(fx+\right.\right.}{7315\,b^3 f}$$

[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 2/7315*(385*cos(f*x + e)^10 - 1463*cos(f*x + e)^8 + 1995*cos(f*x + e)^6 - 1045*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\,b^6 - \frac{1463\,b^6}{\cos(fx+e)^2} + \frac{1995\,b^6}{\cos(fx+e)^4} - \frac{1045\,b^6}{\cos(fx+e)^6}\right)b}{7315\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{19}{2}}}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7315*(385*b^6 - 1463*b^6/cos(f*x + e)^2 + 1995*b^6/cos(f*x + e)^4 - 1045*
b^6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(19/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\sqrt{b\cos(fx+e)}b^9\cos(fx+e)^9 - 1463\sqrt{b\cos(fx+e)}b^9\cos(fx+e)^7 + 1\right.}{7315\,b^{12}f\,\mathrm{sgn}\,(\cos}$$

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/7315*(385*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^9 - 1463*sqrt(b*cos(f*x +
 e))*b^9*cos(f*x + e)^7 + 1995*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^5 - 10
45*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^3)/(b^12*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^7}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2), x)
```

3.438 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}} + \frac{4b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] -2/15*b^5/f/(b*sec(f*x+e))^(15/2)+4/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f/
(b*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 276}

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}} + \frac{4b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b^5)/(15*f*(b*Sec[e + f*x])^(15/2)) + (4*b^3)/(11*f*(b*Sec[e + f*x])^(1
1/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b^5 \text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{17/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^5 \text{Subst}\left(\int \left(\frac{1}{x^{17/2}} - \frac{2}{b^2 x^{13/2}} + \frac{1}{b^4 x^{9/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}} + \frac{4b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-711+532\cos(2(e+fx))-77\cos(4(e+fx)))\sqrt{b\sec(e+fx)}}{4620b^3 f}$$

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-711 + 532*Cos[2*(e + f*x)] - 77*Cos[4*(e + f*x)])*Sqrt[b*
Sec[e + f*x]])/(4620*b^3*f)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{2(77(\cos^7(fx+e))-210(\cos^5(fx+e))+165(\cos^3(fx+e)))}{1155 f b^2 \sqrt{b\sec(fx+e)}}$	50

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155/f/b^2/(b*sec(f*x+e))^(1/2)*(77*cos(f*x+e)^7-210*cos(f*x+e)^5+165*co
s(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\cos\left(fx+e\right)^8-210\cos\left(fx+e\right)^6+165\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos(fx+e)}}}{1155\,b^3f}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/1155*(77*cos(f*x + e)^8 - 210*cos(f*x + e)^6 + 165*cos(f*x + e)^4)*sqrt(
b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\,b^4-\frac{210\,b^4}{\cos(fx+e)^2}+\frac{165\,b^4}{\cos(fx+e)^4}\right)b}{1155\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{15}{2}}}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] -2/1155*(77*b^4 - 210*b^4/cos(f*x + e)^2 + 165*b^4/cos(f*x + e)^4)*b/(f*(b/
cos(f*x + e))^(15/2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\,\sqrt{b\cos\left(fx+e\right)}b^7\cos\left(fx+e\right)^7-210\,\sqrt{b\cos\left(fx+e\right)}b^7\cos\left(fx+e\right)^5+165\,\sqrt{b\cos\left(fx+e\right)}b^7\cos\left(f\right.\right.}{1155\,b^{10}f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -2/1155*(77*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^7 - 210*sqrt(b*cos(f*x +
e))*b^7*cos(f*x + e)^5 + 165*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^3)/(b^10
*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2), x)
```

3.439 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] 2/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2702, 14}

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
[Out] (2*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^3\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{b^3\text{Subst}\left(\int \left(-\frac{1}{x^{13/2}}+\frac{1}{b^2x^{9/2}}\right)\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-15+7\cos(2(e+fx)))\sqrt{b\sec(e+fx)}}{77b^3f}$$

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
[Out] (Cos[e + f*x]^4*(-15 + 7*Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(77*b^3*f)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{2\left(\cos^5(fx+e)\right)}{11}-\frac{2\left(\cos^3(fx+e)\right)}{7}}{f\,b^2\sqrt{b\sec(fx+e)}}$	40

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/77/f/b^2/(b*sec(f*x+e))^(1/2)*(7*cos(f*x+e)^5-11*cos(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\,\cos\left(fx+e\right)^6-11\,\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos(fx+e)}}}{77\,b^3f}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] 2/77*(7*cos(f*x + e)^6 - 11*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\,b^2-\frac{11\,b^2}{\cos(fx+e)^2}\right)b}{77\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 2/77*(7*b^2 - 11*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(11/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\sqrt{b\cos\left(fx+e\right)}b^5\cos\left(fx+e\right)^5-11\sqrt{b\cos\left(fx+e\right)}b^5\cos\left(fx+e\right)^3\right)}{77\,b^8 f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] 2/77*(7*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 11*sqrt(b*cos(f*x + e))*b
^5*cos(f*x + e)^3)/(b^8*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2), x)
```

3.440 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] -2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 30}

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{7f(b\sec(fx+e))^{\frac{7}{2}}}$	17
default	$-\frac{2b}{7f(b\sec(fx+e))^{\frac{7}{2}}}$	17

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^4}{7\,b^3 f}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/7*sqrt(b/cos(f*x + e))*cos(f*x + e)^4/(b^3*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(5/2),x)
[Out] Integral(sin(e + f*x)/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\cos(fx+e)}{7\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] -2/7*cos(f*x + e)/(f*(b/cos(f*x + e))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{b\cos(fx+e)}\cos(fx+e)^3}{7\,b^3 f\mathrm{sgn}(\cos(fx+e))}$$

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] -2/7*sqrt(b*cos(f*x + e))*cos(f*x + e)^3/(b^3*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\cos\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{7\,b^3\,f}$$

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(5/2),x)
[Out] -(2*cos(e + f*x)^4*(b/cos(e + f*x))^(1/2))/(7*b^3*f)
```

3.441 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f-arctanh((b*sec(f*x+e))^(1/2
)/b^(1/2))/b^(5/2)/f+2/3/b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2702, 331, 335, 218, 212, 209}

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(5/2)*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(b^(5/2)*f) + 2/(3*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}dx, x, \sqrt{b\sec(e+fx)}\right)}{b^2 f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}dx, x, \sqrt{b\sec(e+fx)}\right)}{b^2 f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}dx = \frac{\left(-6\arctan\left(\sqrt{\sec(e+fx)}\right) + 3\log\left(1-\sqrt{\sec(e+fx)}\right) - 3\log\left(1+\sqrt{\sec(e+\cdots\right.\right.}{6b^2 f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
[Out] ((-6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[e + f*x]]] - 3*Log[1 +
 Sqrt[Sec[e + f*x]]] + 4/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]])/(6*b^2*f*S
qrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(65) = 130.

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.93

method	result
default	$-3\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\sqrt{-\cdots}$

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f/(b*sec(f*x+e))^(1/2)/b^2*(-3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arc
tan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln((2*cos(f*x+e)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*cos(f*x+e)-3*ar
ctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*sec(f*x+e)-3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sec(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(65) = 130$.

Time = 0.41 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.94

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[\frac{8\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2 + 6\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right) - 3\sqrt{-b}\log}{12\,b^3 f}\right.$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 6*sqrt(-b)*arctan(2*sqrt(-b)
*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 3*sqrt(-b)*log(-
(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f
*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b
^3*f), 1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 6*sqrt(b)*arctan(2*sqr
t(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 3*sqrt(b)*lo
g(-(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos
(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/
(b^3*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{b\left(\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}} - \frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}} - \frac{4}{b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}\right)}{6\,f}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) -
sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2) - 4/(b^2*(b
/cos(f*x + e))^(3/2)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{3\,b\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 3\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right) + 2\sqrt{b\cos(fx+e)}\cos(fx+e)}{3\,b^3 f\,\mathrm{sgn}(\cos(fx+e))}$$

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) + 3*sqrt(b)*arctan(
sqrt(b*cos(f*x + e))/sqrt(b)) + 2*sqrt(b*cos(f*x + e))*cos(f*x + e))/(b^3*f
*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)), x)
```

3.442 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}$$

```
[Out] 3/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/4*arctanh((b*sec(f*x+e
))^(1/2)/b^(1/2))/b^(5/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2702, 296, 335, 218, 212, 209}

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}$$

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) + (3*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]
])/(2*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2} \, dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4b^3f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f} - \frac{3\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f} + \frac{3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4b^2f} + \frac{3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4b^2f}$$

$$= \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(6\arctan\left(\sqrt{\sec(e+fx)}\right) - 3\log\left(1-\sqrt{\sec(e+fx)}\right) + 3\log\left(1+\sqrt{\sec(e+fx)}\right)\right.}{8b^2f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f
*x]])/(8*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(73) = 146.

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.12

method	result
default	$\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-3\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)}}}{\cos(fx+e)+1}\right.}{8f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b}}$

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*cos(f*x+e)*arcta
n(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln((2*cos(f*x+e)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+
1)/(cos(f*x+e)+1))*cos(f*x+e)+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2))+3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b^2/(cos(f*x+e)^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.98

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[-\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)-8\sqrt{\frac{b}{\cos(fx+e)}}\,\mathrm{c}\cdots}{\cdots}\right.$$

$$-\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right)-8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2-3\left(\cos\left(fx+e\right)^2-\cdots\right.}{16\left(b^3f\cos\left(fx+e\right)^2-b^3f\right)}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 3*(co
s(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f), -1/16*(6*(cos(
f*x + e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/
sqrt(b)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 3*(cos(f*x + e)^2 - 1)*s
qrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sq
rt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^3{(e+fx)}}{(b\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
[Out] Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^4-\frac{b^4}{\cos(fx+e)^2}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}-\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}\right)}{8f}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 1/8*b*(4*sqrt(b/cos(f*x + e))/(b^4 - b^4/cos(f*x + e)^2) + 6*arctan(sqrt(b/
cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(s
qrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2\sqrt{b\cos(fx+e)}b\cos(fx+e)}{b^2\cos(fx+e)^2-b^2}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{4\,b^2f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] 1/4*(2*sqrt(b*cos(f*x + e))*b*cos(f*x + e)/(b^2*cos(f*x + e)^2 - b^2) - 3*a
rctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) - 3*arctan(sqrt(b*cos(f*x + e
))/sqrt(b))/sqrt(b))/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^3\,\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)), x)
```

3.443 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}$$

```
[Out] 3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/32*arctanh((b*sec(f*x
+e))^(1/2)/b^(1/2))/b^(5/2)/f-1/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/f-
1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(1/2)/b^3/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2702, 294, 296, 335, 218, 212, 209}

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}$$

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) + (3*ArcTanh[Sqrt[b
*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*
x]])/(16*b^3*f) - (Cot[e + f*x]^4*Sqrt[b*Sec[e + f*x]])/(4*b^3*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{32b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} - \frac{3\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{16b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f} + \frac{3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32b^2 f} + \frac{3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{32b^2 f}$$

$$= \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2} f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2} f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3 f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3 f}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(6\arctan\left(\sqrt{\sec(e+fx)}\right)-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(1+\sqrt{\sec(e+ \right.\right.}{64b^2f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (2*(5 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/Sec[e + f
*x]^(3/2))*Sqrt[Sec[e + f*x]])/(64*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(99) = 198.

Time = 0.19 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\left(12\left(\cos^3(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-3\left(\sin^2(fx+e)\right)\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e}{(\cos(fx+e)}}}{}\right.\right.}{}$

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*sin(f*x+e)^
2*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln((2*cos(f
*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2*cos(f*x+e)+3*arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*sin(f*x+e)^2+3*ln((2*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)+1)/(cos(f*x+e)+1))*sin(f*x+e)^2+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2))/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b^2
*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(99) = 198$.

Time = 0.39 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.72

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[-\frac{6\left(\cos(fx+e)^4 - 2\cos(fx+e)^2 + 1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)}{\quad}\right.$$

$$-\frac{6\left(\cos(fx+e)^4 - 2\cos(fx+e)^2 + 1\right)\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-1)}{2\sqrt{b}}\right) - 3\left(\cos(fx+e)^4 - 2\cos(fx+\right.}{128\left(b^3 f\cos(fx\right.}$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/c
os(f*x + e)))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f), -1/1
28*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos
(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 3*(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*s
qrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos
(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/cos(f*x + e)
))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{b\left(\frac{4\left(3\,b^2\sqrt{\frac{b}{\cos(fx+e)}}+\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\right)}{b^6-\frac{2\,b^6}{\cos(fx+e)^2}+\frac{b^6}{\cos(fx+e)^4}}-\frac{6\,\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}\right)}{64\,f}$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/64*b*(4*(3*b^2*sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(5/2))/(b^6 - 2*b
^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) - 6*arctan(sqrt(b/cos(f*x + e))/sqr
t(b))/b^(7/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/c
os(f*x + e))))/b^(7/2))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(3\,\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3+\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^2}}{32\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + 3*arctan(sq
rt(b*cos(f*x + e))/sqrt(b))/b^(5/2) + 2*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x
 + e)^3 + sqrt(b*cos(f*x + e))*b^3*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2
)^2*b^2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)), x)
```

3.444 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

```
[Out] -4/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+8/195*sin(f*x+e)/b/f/(b*sec(f*x+e
))^(3/2)-2/13*b*sin(f*x+e)^3/f/(b*sec(f*x+e))^(7/2)+8/65*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/c
os(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 3854, 3856, 2719}

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(65*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (4*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(7/2)) + (8*Sin[e + f*x])
```

```
/(195*b*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)/(13*f*(b*Sec[e + f
*x])^(7/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{6}{13}\int\frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx \\
&= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{4}{39}\int\frac{1}{(b\sec(e+fx))^{5/2}}\,dx \\
&= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} \\
&\quad -\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{4\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{65b^2} \\
&= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} \\
&\quad -\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{4\int\sqrt{\cos(e+fx)}\,dx}{65b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right) + \cos^{\frac{3}{2}}(e+fx)(-6\sin(e+fx) - 55\sin(3(e+fx)) + 15\sin}{1560f\cos^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{5/2}}$$

```
[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (192*EllipticE[(e + f*x)/2, 2] + Cos[e + f*x]^(3/2)*(-6*Sin[e + f*x] - 55*S
in[3*(e + f*x)] + 15*Sin[5*(e + f*x)]))/(1560*f*Cos[e + f*x]^(5/2)*(b*Sec[e
 + f*x])^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.86

method	result
default	$\frac{2\left(\cos^6(fx+e)\right)\sin(fx+e)}{13}+\frac{2\left(\cos^5(fx+e)\right)\sin(fx+e)}{13}+\frac{8i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)}{65}-\frac{8i\sqrt{\frac{}{\cos(f}}}{}$

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/195/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(15*cos(f*x+e)^6*sin(f*x+e)
+15*cos(f*x+e)^5*sin(f*x+e)+12*I*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-12*I*El
lipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-25*cos(f*x+e)^4*sin(f*x+e)+24*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+c
sc(f*x+e)),I)-24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-25*cos(f*x+e)^3*sin(f*x+e)+12*I*
EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*sec(f*x+e)-12*I*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sec(f*x+e)+
4*sin(f*x+e)*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)+12*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(\left(15\cos\left(fx+e\right)^6-25\cos\left(fx+e\right)^4+4\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)\right.}{}$$

[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 2/195*((15*cos(f*x + e)^6 - 25*cos(f*x + e)^4 + 4*cos(f*x + e)^2)*sqrt(b/co
s(f*x + e))*sin(f*x + e) + 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
 + e))))/(b^3*f)

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin^4\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)

[Out] Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2), x)
```

3.445 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}} + \frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/9*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+4/45*sin(f*x+e)/b/f/(b*sec(f*x+e))
^(3/2)+4/15*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1
/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2707, 3854, 3856, 2719}

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(15*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (2*b*Sin[e + f*x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (4*Sin[e + f*x])/
(45*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{2}{9}\int\frac{1}{(b\sec(e+fx))^{5/2}}\,dx\\
&= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{15b^2}\\
&= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int\sqrt{\cos(e+fx)}\,dx}{15b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{96E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}} - 4\sin(2(e+fx)) - 10\sin(4(e+fx))}{360b^2f\sqrt{b\sec(e+fx)}}$$

[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]

[Out] ((96*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 4*Sin[2*(e + f*x)] - 10*Sin[4*(e + f*x)])/(360*b^2*f*Sqrt[b*Sec[e + f*x]])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.63

method	result
default	$-\frac{2\left(6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)-6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\right.}{}$

[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/45/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+5*cos(f*x+e)^4*sin(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)+5*cos(f*x+e)^3*sin(f*x+e)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-2*sin(f*x+e)*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-6*sin(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(\left(5\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)-3i\sqrt{2}\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstra}}{}$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45*((5*cos(f*x + e)^4 - 2*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sin(f*x +
 e) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b^3*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2), x)
```

3.446 $\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/5*sin(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)+6/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/
2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

```
[In] Int[(b*Sec[e + f*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*Sin[e + f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{5b^2}\\ &= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \sqrt{\cos(e+fx)}\,dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b\sec(e+fx)}\left(12\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)+\sin(3(e+fx))\right)}{10b^3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^(-5/2),x]
[Out] (Sqrt[b*Sec[e + f*x]]*(12*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + Si
n[e + f*x] + Sin[3*(e + f*x)]))/(10*b^3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 420, normalized size of antiderivative = 5.83

method	result
default	$\frac{\frac{6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)\cos(fx+e)}{5}-\frac{6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)),i)\cos}{5}}{}$

```
[In] int(1/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(3*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*
cos(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*cos(f*x+e)+6*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),
I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-cot(f*x+e)+csc(f*x+e)),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-3*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot
(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*
x+e)+3*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e) + 3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstr}$$

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sin(f*x + e) + 3*I*sqrt(2)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f
*x + e))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b^3*f)
```

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(b/cos(e + f*x))^(5/2),x)
[Out] int(1/(b/cos(e + f*x))^(5/2), x)
```

3.447 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b
*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2703, 3856, 2719}

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*(b*Sec[e + f*x])^(3/2))) - (3*EllipticE[(e + f*x)/2, 2]
)/(b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
```

```
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3\int\frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}\\
&= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3\int\sqrt{\cos(e+fx)}\,dx}{2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}-\frac{3E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int\frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\frac{-\cot(e+fx)-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{b^2f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
[Out] (-Cot[e + f*x] - (3*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(b^2*f*S
qrt[b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.94

method	result
default	$-\frac{3i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(-\cot(fx+e)+\csc(fx+e)),i)-3i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(-\cot(fx+e)+\csc(fx+e)$

[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/f/(b*sec(f*x+e))^(1/2)/b^2*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sec(f*x+e)-2*cot(f*x+e)+3*csc(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.60

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{-3i\,\sqrt{2}\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx\right.$$

[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] 1/2*(-3*I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2)/(b^3*f*sin(f*x + e))

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)

[Out] Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(5/2), x)

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^2}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)), x)
```

3.448 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}} - \frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} + \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] 1/2*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e))
^(3/2)+1/2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2703, 2705, 3856, 2719}

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}} + \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] Csc[e + f*x]/(2*b*f*(b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^3/(3*b*f*(b*Sec[
e + f*x])^(3/2)) + EllipticE[(e + f*x)/2, 2]/(2*b^2*f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Sec[e + f*x]])
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}-\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}\\
&= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{4b^2}\\
&= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\int \sqrt{\cos(e+fx)}\,dx}{4b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(-3+5\csc^2(e+fx)-2\csc^4(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\tfrac{1}{2}(e+fx\right.\right.}{6b^3f}$$

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-3 + 5*Csc[e + f*x]^2 - 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e + f
*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(6*b^3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.02

method	result
default	$-\frac{3i(\sin^2(fx+e))E(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-3i(\sin^2(fx+e))F(i(-\cot(fx+e)+\csc(fx+e)),i)}{}$

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f/(b*sec(f*x+e))^(1/2)/b^2/(cos(f*x+e)^2-1)*(3*I*sin(f*x+e)^2*Elliptic
E(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f
*x+e)+1))^(1/2)-3*I*sin(f*x+e)^2*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f
*x+e)),I)*sin(f*x+e)*tan(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)*tan(
f*x+e)+3*sin(f*x+e)-2*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.59

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(-i\cos(fx+e)^2+i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)}{}$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*(-I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sq
rt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(3*cos(f*x + e
)^4 - cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b^3*f*cos(f*x + e)^2 - b^3*f)
*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^4{(e+fx)}}{(b\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
[Out] Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^4\,\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)), x)
```

3.449 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] 3/20*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)+1/10*csc(f*x+e)^3/b/f/(b*sec(f*x+e
))^(3/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(3/2)+3/20*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/co
s(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2703, 2705, 3856, 2719}

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} + \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*Csc[e + f*x])/(20*b*f*(b*Sec[e + f*x])^(3/2)) + Csc[e + f*x]^3/(10*b*f*(
b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^5/(5*b*f*(b*Sec[e + f*x])^(3/2)) + (3
```

```
*EllipticE[(e + f*x)/2, 2])/(20*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]])
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}-\frac{3\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{10b^2}\\
&= \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}-\frac{3\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{20b^2}\\
&= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{40b^2}\\
&= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}\\
&\quad -\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{3\int \sqrt{\cos(e+fx)}\,dx}{40b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(-3+\csc^2(e+fx)+6\csc^4(e+fx)-4\csc^6(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+f\right.}{20b^3f}$$

[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]

[Out] ((-3 + Csc[e + f*x]^2 + 6*Csc[e + f*x]^4 - 4*Csc[e + f*x]^6 + 3*Sqrt[Cos[e + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(20*b^3*f)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.55

method	result
default	$-\frac{3i(\sin^4(fx+e))F(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-3i(\sin^4(fx+e))E(i(-\cot(fx+e)+\csc(fx+e)),i)\sqrt{}}{}$

[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/20/f/(cos(f*x+e)-1)^2/(cos(f*x+e)+1)^2/(b*sec(f*x+e))^(1/2)/b^2*(3*I*sin(f*x+e)^4*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-3*I*sin(f*x+e)^4*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^3*tan(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-cot(f*x+e)+csc(f*x+e)),I)*sin(f*x+e)^3*tan(f*x+e)-3*sin(f*x+e)^3-2*sin(f*x+e)*cos(f*x+e)+4*cot(f*x+e))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.54

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{2}\left(-i\cos(fx+e)^4+2i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(}{}$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(3*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f
*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*
sin(f*x + e))) + 3*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt
(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
+ e) - I*sin(f*x + e))) - 2*(3*cos(f*x + e)^6 - 8*cos(f*x + e)^4 + cos(f*x
+ e)^2)*sqrt(b/cos(f*x + e)))/((b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)
^2 + b^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)), x)
```

3.450 $\int \sqrt{b \sec(e+fx)}(a \sin(e+fx))^{9/2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 449

$$\int \sqrt{b \sec(e+fx)}(a \sin(e+fx))^{9/2} \, dx = -\frac{21a^{9/2} \arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a \sin(e+fx)}}{\sqrt{a}\sqrt{b \cos(e+fx)}}\right)\sqrt{b \cos(e+fx)}\sqrt{b \sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}$$
$$+\frac{21a^{9/2} \arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a \sin(e+fx)}}{\sqrt{a}\sqrt{b \cos(e+fx)}}\right)\sqrt{b \cos(e+fx)}\sqrt{b \sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}$$
$$+\frac{21a^{9/2}\sqrt{b \cos(e+fx)} \log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a \sin(e+fx)}}{\sqrt{b \cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b \sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$
$$-\frac{21a^{9/2}\sqrt{b \cos(e+fx)} \log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a \sin(e+fx)}}{\sqrt{b \cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b \sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$
$$-\frac{7a^3 b(a \sin(e+fx))^{3/2}}{16f\sqrt{b \sec(e+fx)}}-\frac{ab(a \sin(e+fx))^{7/2}}{4f\sqrt{b \sec(e+fx)}}$$

```
[Out] -7/16*a^3*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e))^(1/2)-1/4*a*b*(a*sin(f*x+
e))^(7/2)/f/(b*sec(f*x+e))^(1/2)-21/64*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*
sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec
(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+21/64*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*
sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec
(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+21/128*a^(9/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*
(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e)
)^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)-21/128*a^(9/2)*ln(a^(1/2)+2^
(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))
*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2663, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = & -\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\sqrt{b}f} \\ & +\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{32\sqrt{2}\sqrt{b}f} \\ & +\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{64\sqrt{2}\sqrt{b}f} \\ & -\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{64\sqrt{2}\sqrt{b}f} \\ & -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\end{aligned}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-21*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqr
t[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*
Sqrt[b]*f) + (21*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/
(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/
(32*Sqrt[2]*Sqrt[b]*f) + (21*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]*f) - (21*a^(9/2)*Sqrt[b*Cos
[e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[
e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]
*f) - (7*a^3*b*(a*Sin[e + f*x])^(3/2))/(16*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(
a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}+\frac{1}{8}\left(7a^2\right)\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx\\
&= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{32}\left(21a^4\right)\int\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx\\
&= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{32}\left(21a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx\\
&= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\left(21a^5b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{16f}\\
&= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a-bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{32f}\\
&\quad+\frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a+bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{32f}
\end{aligned}$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$+ \frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64bf}$$

$$+ \frac{\left(21a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64bf}$$

$$+ \frac{\left(21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64\sqrt{2}\sqrt{b}f}$$

$$+ \frac{\left(21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64\sqrt{2}\sqrt{b}f}$$

$$= \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$

$$- \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$

$$- \frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$+ \frac{\left(21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\sqrt{b}f}$$

$$- \frac{\left(21a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\sqrt{b}f}$$

$$= -\frac{21a^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}$$

$$+ \frac{21a^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}$$

$$+ \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$

$$- \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}\sqrt{b}f}$$

$$- \frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.38

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \frac{a^4\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\left(4(-9+2\cos(2(e+fx)))\sin^2(e+fx)+21\sqrt{2}\right.}{}$$

[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]

[Out] (a^4*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(4*(-9 + 2*Cos[2*(e + f*x)])*Sin[e + f*x]^2 + 21*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - 21*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/(64*f)

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{2}\left(16\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{2}\left(\cos^3(fx+e)\right)\sin(fx+e)+16\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{2}\left(\cos^2(fx+e)\right)\sin(fx+e)-44\sqrt{-\frac{\sin(fx+e)}{(\cos(fx}}\right.}{}$

[In] int((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/128/f*2^(1/2)*(16*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)+16*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*cos(f*x+e)^2*sin(f*x+e)-44*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)-44*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+21*ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-21*ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))+42*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))+42*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2)*a^4*cos(f*x+e)/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.55

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/256*(16*(4*a^4*cos(f*x + e)^3 - 11*a^4*cos(f*x + e))*sqrt(a*sin(f*x + e))
*sqrt(b/cos(f*x + e))*sin(f*x + e) - 21*(-a^18*b^2/f^4)^(1/4)*f*log(9261/2*
a^14*b^2*cos(f*x + e)*sin(f*x + e) + 9261/2*((-a^18*b^2/f^4)^(1/4)*a^9*b*f*
cos(f*x + e)*sin(f*x + e) - (-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(
a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 9261/4*sqrt(-a^18*b^2/f^4)*(2*a^5*b*
f^2*cos(f*x + e)^2 - a^5*b*f^2)) + 21*(-a^18*b^2/f^4)^(1/4)*f*log(9261/2*a^
14*b^2*cos(f*x + e)*sin(f*x + e) - 9261/2*((-a^18*b^2/f^4)^(1/4)*a^9*b*f*co
s(f*x + e)*sin(f*x + e) - (-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(a*
sin(f*x + e))*sqrt(b/cos(f*x + e)) - 9261/4*sqrt(-a^18*b^2/f^4)*(2*a^5*b*f^
2*cos(f*x + e)^2 - a^5*b*f^2)) + 21*I*(-a^18*b^2/f^4)^(1/4)*f*log(9261/2*a^
14*b^2*cos(f*x + e)*sin(f*x + e) - 9261/2*(I*(-a^18*b^2/f^4)^(1/4)*a^9*b*f*
cos(f*x + e)*sin(f*x + e) + I*(-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqr
t(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 9261/4*sqrt(-a^18*b^2/f^4)*(2*a^5*
b*f^2*cos(f*x + e)^2 - a^5*b*f^2)) - 21*I*(-a^18*b^2/f^4)^(1/4)*f*log(9261/
2*a^14*b^2*cos(f*x + e)*sin(f*x + e) - 9261/2*(-I*(-a^18*b^2/f^4)^(1/4)*a^9
*b*f*cos(f*x + e)*sin(f*x + e) - I*(-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)^2
)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 9261/4*sqrt(-a^18*b^2/f^4)*(2
*a^5*b*f^2*cos(f*x + e)^2 - a^5*b*f^2)) - 21*(-a^18*b^2/f^4)^(1/4)*f*log(92
61*a^14*b^2 + 18522*((-a^18*b^2/f^4)^(1/4)*a^9*b*f*cos(f*x + e)^2 - (-a^18*
b^2/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/c
os(f*x + e))) + 21*(-a^18*b^2/f^4)^(1/4)*f*log(9261*a^14*b^2 - 18522*((-a^1
8*b^2/f^4)^(1/4)*a^9*b*f*cos(f*x + e)^2 - (-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x
 + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) + 21*I*(-a^1
8*b^2/f^4)^(1/4)*f*log(9261*a^14*b^2 - 18522*(I*(-a^18*b^2/f^4)^(1/4)*a^9*b
*f*cos(f*x + e)^2 + I*(-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*
sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) - 21*I*(-a^18*b^2/f^4)^(1/4)*f*l
og(9261*a^14*b^2 - 18522*(-I*(-a^18*b^2/f^4)^(1/4)*a^9*b*f*cos(f*x + e)^2 -
 I*(-a^18*b^2/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e)
)*sqrt(b/cos(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(9/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{9}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{9}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int (a\sin(e+fx))^{9/2}\sqrt{\frac{b}{\cos(e+fx)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2), x)
```

3.451 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 414

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = -\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f} - \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f} - \frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

```
[Out] -1/2*a*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e))^(1/2)-3/8*a^(5/2)*arctan(1-2
^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*
x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+3/8*a^(5/2)*arctan(1+2^(
1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+
e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+3/16*a^(5/2)*ln(a^(1/2)-2^
(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))
*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)-3/16*a^(5/2)*l
n(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)
*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2663, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = -\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{8\sqrt{2}\sqrt{b}f} - \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{8\sqrt{2}\sqrt{b}f} - \frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*Sq
rt[b]*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sq
rt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*
Sqrt[2]*Sqrt[b]*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]
*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]
*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (3*a^(5/2)*Sqrt[b*Cos[e + f*
x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x
]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (a
*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}+\frac{1}{4}\left(3a^2\right)\int\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx \\
&= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}+\frac{1}{4}\left(3a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx \\
&= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} \\
&\quad+\frac{\left(3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2f} \\
&= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} \\
&\quad-\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a-bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{4f} \\
&\quad+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a+bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{4f} \\
&= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} \\
&\quad+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8bf} \\
&\quad+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8bf} \\
&\quad+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8\sqrt{2}\sqrt{b}f} \\
&\quad+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8\sqrt{2}\sqrt{b}f}
\end{aligned}$$

$$= \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f}$$
$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f}$$
$$-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$
$$+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\sqrt{b}f}$$
$$-\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\sqrt{b}f}$$

$$= -\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f}$$
$$+\frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f}$$
$$+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f}$$
$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\sqrt{b}f}$$
$$-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.38

$$\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^2\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\left(4\sin^2(e+fx)-3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}\right.}{8f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/8*(a^2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(4*Sin[e +
 f*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*
x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e +
f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/f
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{2}\sin(fx+e)\cos(fx+e)+4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+3\ln\left(2\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\right.\right.}{}$

```
[In] int((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f*2^(1/2)*(4*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*
sin(f*x+e)*cos(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*sin(f*x+e)+3*ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cs
c(f*x+e)+2-2*cot(f*x+e))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*
(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(c
os(f*x+e)-1))-3*ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc
(f*x+e)+2-2*cot(f*x+e)))*(a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*a^2*cos(
f*x+e)/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.73

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*sqrt(a*sin(f*x + e))*a^2*sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*
x + e) + 3*(-a^10*b^2/f^4)^(1/4)*f*log(27/2*a^8*b^2*cos(f*x + e)*sin(f*x +
e) + 27/2*((-a^10*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)*sin(f*x + e) - (-a^10
*b^2/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e
)) - 27/4*sqrt(-a^10*b^2/f^4)*(2*a^3*b*f^2*cos(f*x + e)^2 - a^3*b*f^2)) - 3
*(-a^10*b^2/f^4)^(1/4)*f*log(27/2*a^8*b^2*cos(f*x + e)*sin(f*x + e) - 27/2*
((-a^10*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)*sin(f*x + e) - (-a^10*b^2/f^4)^
(3/4)*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 27/4*
sqrt(-a^10*b^2/f^4)*(2*a^3*b*f^2*cos(f*x + e)^2 - a^3*b*f^2)) - 3*I*(-a^10*
b^2/f^4)^(1/4)*f*log(27/2*a^8*b^2*cos(f*x + e)*sin(f*x + e) - 27/2*(I*(-a^1
0*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)*sin(f*x + e) + I*(-a^10*b^2/f^4)^(3/4
```

```
)*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 27/4*sqrt
(-a^10*b^2/f^4)*(2*a^3*b*f^2*cos(f*x + e)^2 - a^3*b*f^2)) + 3*I*(-a^10*b^2/
f^4)^(1/4)*f*log(27/2*a^8*b^2*cos(f*x + e)*sin(f*x + e) - 27/2*(-I*(-a^10*b
^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)*sin(f*x + e) - I*(-a^10*b^2/f^4)^(3/4)*f
^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 27/4*sqrt(-a
^10*b^2/f^4)*(2*a^3*b*f^2*cos(f*x + e)^2 - a^3*b*f^2)) + 3*(-a^10*b^2/f^4)^
(1/4)*f*log(27*a^8*b^2 + 54*((-a^10*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)^2 -
 (-a^10*b^2/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*
sqrt(b/cos(f*x + e))) - 3*(-a^10*b^2/f^4)^(1/4)*f*log(27*a^8*b^2 - 54*((-a^
10*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)^2 - (-a^10*b^2/f^4)^(3/4)*f^3*cos(f*
x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) - 3*I*(-a^1
0*b^2/f^4)^(1/4)*f*log(27*a^8*b^2 - 54*(I*(-a^10*b^2/f^4)^(1/4)*a^5*b*f*cos
(f*x + e)^2 + I*(-a^10*b^2/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a
*sin(f*x + e))*sqrt(b/cos(f*x + e))) + 3*I*(-a^10*b^2/f^4)^(1/4)*f*log(27*a
^8*b^2 - 54*(-I*(-a^10*b^2/f^4)^(1/4)*a^5*b*f*cos(f*x + e)^2 - I*(-a^10*b^2
/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(
f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int {\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2), x)
```

3.452 $\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 376

$$\begin{aligned}
&\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx \\
&= -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f} \\
&\quad+\frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f} \\
&\quad+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f} \\
&\quad-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f}
\end{aligned}$$

```
[Out] -1/2*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(
1/2))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+1
/2*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/
2))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+1/4
*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/
2)*tan(f*x+e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/
b^(1/2)-1/4*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^
(1/2)+a^(1/2)*tan(f*x+e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)
/f*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$$
$$= -\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}$$
$$+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}f}$$
$$+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{2\sqrt{2}\sqrt{b}f}$$
$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{2\sqrt{2}\sqrt{b}f}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[
b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*Sqrt[
b]*f)) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2
]*Sqrt[b]*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]
*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*
Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sq
rt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[
a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = \left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx$$

$$=\frac{\left(2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f}$$

$$=-\frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a-bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f}+\frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a+bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f}$$

$$=\frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2bf}+\frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2bf}+\frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2\sqrt{2}\sqrt{b}f}+\frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2\sqrt{2}\sqrt{b}f}$$

$$=\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f}-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f}+\frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}-\frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}$$

$$= -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f} + \frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f} + \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f} - \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}\sqrt{b}f}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.32

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \frac{\left(\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\right)\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e-}{\sqrt{2}f}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] ((ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))] - Ar
cTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])])*Cot[e +
 f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(Tan[e + f*x]^2)^(1/4))/(Sq
rt[2]*f)
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.96

method	result
default	$-\frac{\sqrt{2}\,\sqrt{b\sec(fx+e)}\left(\ln\left(2\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\cot(fx+e)+2\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\csc(fx+e)+2-2\cot(fx+e)\right)-2\,\mathrm{ar}\right.}{}$

```
[In] int((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*2^(1/2)*(b*sec(f*x+e))^(1/2)*(ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*
```

```
x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-2*arctan((2^(1/2)*(-sin(f*x+e)*
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))
-ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2
*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot
(f*x+e))-2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*cos(f*x+e)*(a*sin(f*x+e))^(1/2)/(
cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 1041, normalized size of antiderivative = 2.77

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-a^2*b^2/f^4)^(1/4)*log(1/2*a^2*b^2*cos(f*x + e)*sin(f*x + e) + 1/2*(f
^3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)^2 - a*b*f*(-a^2*b^2/f^4)^(1/4)*cos(f*x
 + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 1/4*(2*a*b*
f^2*cos(f*x + e)^2 - a*b*f^2)*sqrt(-a^2*b^2/f^4)) - 1/8*(-a^2*b^2/f^4)^(1/4
)*log(1/2*a^2*b^2*cos(f*x + e)*sin(f*x + e) - 1/2*(f^3*(-a^2*b^2/f^4)^(3/4)
*cos(f*x + e)^2 - a*b*f*(-a^2*b^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqr
t(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 1/4*(2*a*b*f^2*cos(f*x + e)^2 - a*
b*f^2)*sqrt(-a^2*b^2/f^4)) - 1/8*I*(-a^2*b^2/f^4)^(1/4)*log(1/2*a^2*b^2*cos
(f*x + e)*sin(f*x + e) + 1/2*(I*f^3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)^2 + I
*a*b*f*(-a^2*b^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))
*sqrt(b/cos(f*x + e)) + 1/4*(2*a*b*f^2*cos(f*x + e)^2 - a*b*f^2)*sqrt(-a^2*
b^2/f^4)) + 1/8*I*(-a^2*b^2/f^4)^(1/4)*log(1/2*a^2*b^2*cos(f*x + e)*sin(f*x
 + e) + 1/2*(-I*f^3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)^2 - I*a*b*f*(-a^2*b^2
/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x
+ e)) + 1/4*(2*a*b*f^2*cos(f*x + e)^2 - a*b*f^2)*sqrt(-a^2*b^2/f^4)) + 1/8*
(-a^2*b^2/f^4)^(1/4)*log(a^2*b^2 + 2*(f^3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)
*sin(f*x + e) - a*b*f*(-a^2*b^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x +
 e))*sqrt(b/cos(f*x + e))) - 1/8*(-a^2*b^2/f^4)^(1/4)*log(a^2*b^2 - 2*(f^3*
(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) - a*b*f*(-a^2*b^2/f^4)^(1/4)
*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) + 1/8*I*(-a^2*b
^2/f^4)^(1/4)*log(a^2*b^2 - 2*(I*f^3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)*sin(
f*x + e) + I*a*b*f*(-a^2*b^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e)
)*sqrt(b/cos(f*x + e))) - 1/8*I*(-a^2*b^2/f^4)^(1/4)*log(a^2*b^2 - 2*(-I*f^
3*(-a^2*b^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) - I*a*b*f*(-a^2*b^2/f^4)^(
1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)))
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{a\sin\left(e+fx\right)}\sqrt{b\sec\left(e+fx\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))*sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{a\,\sin\left(e+f\,x\right)}\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2), x)
```

3.453 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b}{af\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[Out] -2*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2658}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(a*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2b}{af\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{b\sec(e+fx)}\sin(2(e+fx))}{f(a\sin(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
[Out] -((Sqrt[b*Sec[e + f*x]]*Sin[2*(e + f*x)])/(f*(a*Sin[e + f*x])^(3/2)))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\cos(fx+e)\sqrt{b\sec(fx+e)}}{fa\sqrt{a\sin(fx+e)}}$	35

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/f*cos(f*x+e)*(b*sec(f*x+e))^(1/2)/a/(a*sin(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)}{a^2 f\sin\left(fx+e\right)}$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a^2*f*sin(f*x +
e))
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(e+fx)}}}{(a\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}}}{(a\sin{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}}}{(a\sin{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\,\cos\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{a\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}}$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
[Out] -(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(a*f*(a*sin(e + f*x))^(1/2))
```

3.454 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} - \frac{8b}{5a^3f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/5*b/a/f/(a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(1/2)-8/5*b/a^3/f/(b*sec(f*x
+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2664, 2658}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(5*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*f
*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
```

```
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}+\frac{4\int\frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{5a^2}\\ &= -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}}-\frac{8b}{5a^3f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int\frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx=\frac{2(-3+2\cos(2(e+fx)))\cot(e+fx)\sqrt{b\sec(e+fx)}}{5a^2f(a\sin(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
[Out] (2*(-3 + 2*Cos[2*(e + f*x)])*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]])/(5*a^2*f*(a
*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{b\sec(fx+e)}\,(4(\cos^2(fx+e))-5)\cot(fx+e)\csc(fx+e)}{5f\sqrt{a\sin(fx+e)}\,a^3}$	53

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5/f*(b*sec(f*x+e))^(1/2)*(4*cos(f*x+e)^2-5)/(a*sin(f*x+e))^(1/2)/a^3*cot(
f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{2\left(4\cos\left(fx+e\right)^{3}-5\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos(fx+e)}}}{5\left(a^{4}f\cos\left(fx+e\right)^{2}-a^{4}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas")
[Out] -2/5*(4*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*
x + e))/((a^4*f*cos(f*x + e)^2 - a^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{4\sqrt{\frac{b}{\cos(e+f\,x)}}\,(3\,\cos(e+f\,x)-4\,\cos(3\,e+3\,f\,x)+\cos(5\,e+5\,f\,x))}{5\,a^3\,f\,\sqrt{a\,\sin(e+f\,x)}\,(\cos(4\,e+4\,f\,x)-4\,\cos(2\,e+2\,f\,x)+3)}$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(7/2),x)
```

```
[Out] -(4*(b/cos(e + f*x))^(1/2)*(3*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(5*e +
 5*f*x)))/(5*a^3*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e + 2
*f*x) + 3))
```

3.455 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}} - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} - \frac{64b}{45a^5f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/9*b/a/f/(a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(1/2)-16/45*b/a^3/f/(a*sin(f
*x+e))^(5/2)/(b*sec(f*x+e))^(1/2)-64/45*b/a^5/f/(b*sec(f*x+e))^(1/2)/(a*sin
(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2664, 2658}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{64b}{45a^5f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{16b}{45a^3f(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}} - \frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

```
[Out] (-2*b)/(9*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2)) - (16*b)/(45*a^3
*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (64*b)/(45*a^5*f*Sqrt[b*S
ec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}} + \frac{8\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx}{9a^2} \\
&= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}} \\
&\quad - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} + \frac{32\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{45a^4} \\
&= -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}} - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} \\
&\quad - \frac{64b}{45a^5f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \frac{2b(-21+20\cos(2(e+fx))-4\cos(4(e+fx)))\csc^5(e+fx)\sqrt{a\sin(e+fx)}}{45a^6f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
[Out] (2*b*(-21 + 20*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sqrt[a
*Sin[e + f*x]])/(45*a^6*f*Sqrt[b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2\sqrt{b\sec(fx+e)}\,(32(\cos^4(fx+e))-72(\cos^2(fx+e))+45)\cot(fx+e)(\csc^3(fx+e))}{45f\sqrt{a\sin(fx+e)}\,a^5}$	65

[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)

[Out] -2/45/f*(b*sec(f*x+e))^(1/2)*(32*cos(f*x+e)^4-72*cos(f*x+e)^2+45)/(a*sin(f*x+e))^(1/2)/a^5*cot(f*x+e)*csc(f*x+e)^3

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{2\left(32\cos(fx+e)^5-72\cos(fx+e)^3+45\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}}{45\left(a^6f\cos(fx+e)^4-2a^6f\cos(fx+e)^2+a^6f\right)\sin(fx+e)}$$

[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="fricas")

[Out] -2/45*(32*cos(f*x + e)^5 - 72*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))/((a^6*f*cos(f*x + e)^4 - 2*a^6*f*cos(f*x + e)^2 + a^6*f)*sin(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(11/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{\mathrm{e}^{-e\,5\mathrm{i}-f\,x\,5\mathrm{i}}\,\sqrt{\frac{b}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{352\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{45\,a^5\,f}-\frac{256\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)}{45\,a^5\,f}+\frac{64\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)}{45\,a^5\,f}\right)}{16\,\sin\left(e+f\,x\right)^4\,\sqrt{a\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(11/2),x)
[Out] -(exp(- e*5i - f*x*5i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(1/2)*((352*cos(e + f*x)*exp(e*5i + f*x*5i))/(45*a^5*f) - (256*exp(e*5i + f
*x*5i)*cos(3*e + 3*f*x))/(45*a^5*f) + (64*exp(e*5i + f*x*5i)*cos(5*e + 5*f*
x))/(45*a^5*f)))/(16*sin(e + f*x)^4*(a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2))
```

3.456 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}} + \frac{5a^4\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/3*a*b*(a*sin(f*x+e))^(5/2)/f/(b*sec(f*x+e))^(1/2)-5/6*a^3*b*(a*sin(f*x+e
))^(1/2)/f/(b*sec(f*x+e))^(1/2)-5/12*a^4*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+
1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2
*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2663, 2665, 2653, 2720}

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \frac{5a^4\sqrt{\sin(2e+2fx)}\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{12f\sqrt{a\sin(e+fx)}} - \frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*b*Sqrt[a*Sin[e + f*x]])/(6*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(a*Sin[e
+ f*x])^(5/2))/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*a^4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{6}\left(5a^2\right)\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx\\
&= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5a^4\right)\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx\\
&= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{12}\left(5a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx
\end{aligned}$$

$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}} + \frac{\left(5a^4\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{12\sqrt{a\sin(e+fx)}}$$

$$= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}} + \frac{5a^4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.84 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \frac{a^3b\sqrt{a\sin(e+fx)}\left(2(-6+\cos(2(e+fx)))+5\csc^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\text{sec}\right.\right.}{12f\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*b*Sqrt[a*Sin[e + f*x]]*(2*(-6 + Cos[2*(e + f*x)]) + 5*Csc[e + f*x]^2*H
ypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(
12*f*Sqrt[b*Sec[e + f*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.71 (sec) , antiderivative size = 1739, normalized size of antiderivative = 13.59

method	result	size
default	Expression too large to display	1739

```
[In] int((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/f*2^(1/2)*(-6*I*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e
)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)
+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)-6*I*(-cot(f*x+e)+csc(f*x+e)+1)^
(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*Ellipti
cPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*2^(1/2)*cos(f
*x+e)^3*sin(f*x+e)+6*I*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x
```

+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+
e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*I*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(co
t(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)-6*(-cot(f*x+e
)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e
))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))
*cos(f*x+e)-6*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1
/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/
2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)+32*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(c
ot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)-6*(-cot(f*x+e)+csc(f*x+
e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*E
llipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*(-cot(f
*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f
*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1
/2))+32*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(c
ot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*
2^(1/2))-28*2^(1/2)*cos(f*x+e)*sin(f*x+e)-3*ln(-2*2^(1/2)*(-sin(f*x+e)*cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+3*ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*cos(f*x+e)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+6*arctan((2^(1/2)*(-sin(f*x+e)
*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)
)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-3*ln(-2*2^(1/2
)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-si
n(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-si
n(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*ln(2*2^(1/2)*(-sin(f*x+e)*cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)
*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1
/2)*a^3*csc(f*x+e)

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(7/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \sec(e+fx)}(a \sin(e+fx))^{7/2}\,dx = \int (a\,\sin\left(e+f\,x\right))^{7/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2), x)
```

3.457 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{a^2\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -a*b*(a*sin(f*x+e))^(1/2)/f/(b*sec(f*x+e))^(1/2)-1/2*a^2*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2663, 2665, 2653, 2720}

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \frac{a^2\sqrt{\sin(2e+2fx)}\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}} - \frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e -
 Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}a^2\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx\\
&= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{1}{2}\left(a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx\\
&= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}+\frac{\left(a^2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{2\sqrt{a\sin(e+fx)}}\\
&= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}+\frac{a^2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}{abf\left(-\tan^2(e+fx)\right)^{5/4}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
[Out] (Hypergeometric2F1[-1/2, -1/4, 1/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2)*
(a*Sin[e + f*x])^(5/2))/(a*b*f*(-Tan[e + f*x]^2)^(5/4))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(104) = 208.

Time = 1.88 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.32

method	result
default	$-\frac{\sqrt{2}\,\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,a\left(-\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\right.\right.}{}$

```
[In] int((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/f*2^(1/2)*(a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*a*(-(-cot(f*x+e)+c
sc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^
(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cot(f*x+e)-(-
cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-
csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*c
sc(f*x+e)+2^(1/2)*cos(f*x+e))
```

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int (a\,\sin(e+f\,x))^{3/2}\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2), x)
```

3.458 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),
2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2665, 2653, 2720}

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/
(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx\\ &= \frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{\sqrt{a\sin(e+fx)}}\\ &= \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{f\sqrt{a\sin(e+fx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$
$$=\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{3/4}}{f\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*Sqrt[b*Sec[e
 + f*x]]*(-Tan[e + f*x]^2)^(3/4))/(f*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.13

method	result
default	$\frac{\sqrt{2}\,(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(f\right.}{f\sqrt{a\sin(fx+e)}}$

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*2^(1/2)*(cos(f*x+e)+1)*(b*sec(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^
(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*Ellipti
cF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))/(a*sin(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = -\frac{\sqrt{i\,ab}F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)+\sqrt{-i\,ab}F(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)|}{af}$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(I*a*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + sqrt(
-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1))/(a*f)
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

3.459 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/3*b/a/f/(a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/2)-2/3*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2664, 2665, 2653, 2720}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}} - \frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(3*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e
 - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}+\frac{2\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3a^2}\\
&= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}\\
&\quad+\frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{3a^2}\\
&= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}\\
&\quad+\frac{\left(2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{3a^2\sqrt{a\sin(e+fx)}}
\end{aligned}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{2\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)(-$$
$$3a^2f\sqrt{a\sin(e+fx)}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
[Out] (2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/2, 3/4, 3/2,
 Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*a^2*f*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{2}\,\sqrt{b\sec(fx+e)}\left(-2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+c\right.\right.}{}$

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/f*2^(1/2)*(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^2*(-2*(-cot(f*x+
e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+
e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e
)-2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f
*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1
/2))+2^(1/2)*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2\left(\sqrt{i\,ab}\left(\cos\left(fx+e\right)^2-1\right)F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)+\sqrt{-i\,ab}\left(\cos\left(fx+e\right)^2-1\right)F(a\right.}{3\left(a^3f\cos\left(fx+e\right)^2-a^3\right.}$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos(f*x + e) + I*s
in(f*x + e)), -1) + sqrt(-I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos
(f*x + e) - I*sin(f*x + e)), -1) - sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)
)*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 - a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

3.460 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{7a^4f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/7*b/a/f/(a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)-4/7*b/a^3/f/(a*sin(f*x
+e))^(3/2)/(b*sec(f*x+e))^(1/2)-4/7*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*P
i+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+
2*e)^(1/2)/a^4/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2664, 2665, 2653, 2720}

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \frac{4\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{7a^4f\sqrt{a\sin(e+fx)}} - \frac{4b}{7a^3f(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}} - \frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*b)/(7*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) - (4*b)/(7*a^3*f
*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(7*a^4*f*Sqrt[a*Sin[e + f
*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}+\frac{6\int\frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2}\\
&= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}\\
&\quad -\frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}+\frac{4\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{7a^4}\\
&= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}-\frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}\\
&\quad +\frac{\left(4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{7a^4}
\end{aligned}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{\left(4\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{7a^4\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{7a^4f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int\frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2\cos(2(e+fx))(b\sec(e+fx))^{3/2}\left((-2+\cos(2(e+fx)))\csc^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\text{se}\right.\right.}{7a^3bf\left(-2+\sec^2(e+fx)\right)(a\sin(e+fx))^{3/2}}$$

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*Cos[2*(e + f*x)]*(b*Sec[e + f*x])^(3/2)*((-2 + Cos[2*(e + f*x)])*Csc[e
+ f*x]^2 + 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x
]^2)^(3/4)))/(7*a^3*b*f*(-2 + Sec[e + f*x]^2)*(a*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.76

method	result
default	$\frac{\sqrt{2}\sqrt{b\sec(fx+e)}\left(4\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\sqrt{\cot(fx+e)-\csc(fx+e)+1}\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+}\right.\right.}{}$

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/f*2^(1/2)*(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^4*(4*(-cot(f*x+e)
+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)
)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)+
4*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x
+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2
))+2*2^(1/2)*cot(f*x+e)^3-3*2^(1/2)*cot(f*x+e)*csc(f*x+e)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2\left(2\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{i\,ab}F(\arcsin(\cos(fx+e)+i\sin(fx+e))\,|\,-1)+2\left(\cos(fx\right.\right.}{7\,(a}$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
[Out] -2/7*(2*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(I*a*b)*elliptic_f(arcs
in(cos(f*x + e) + I*sin(f*x + e)), -1) + 2*(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)*sqrt(-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)
- (2*cos(f*x + e)^3 - 3*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x +
 e)))/(a^5*f*cos(f*x + e)^4 - 2*a^5*f*cos(f*x + e)^2 + a^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\left(a\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2),x)
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2), x)
```

3.461 $\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{20f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

```
[Out] -7/30*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/5*b*sin(f*x+e)^(7/2)/f/(b
*sec(f*x+e))^(3/2)-7/20*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellip
ticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin
(2*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2663, 2665, 2652, 2719}

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}} + \frac{7\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{20f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Sin[e + f*x]^(3/2))/(30*f*(b*Sec[e + f*x])^(3/2)) - (b*Sin[e + f*x]^(
7/2))/(5*f*(b*Sec[e + f*x])^(3/2)) + (7*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[S
in[e + f*x]])/(20*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{10}\int\frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{20}\int\frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7\int\sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{20\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{\left(7\sqrt{\sin(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{20\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{20f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\Big(23-26\cos(2(e+fx))+3\cos(4(e+fx))+42\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan}}{120f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/120*(b*(23 - 26*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 42*Hypergeometri
c2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(f*(b*Sec[e
+ f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(120) = 240.

Time = 0.98 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.46

method	result
default	$-\frac{\sqrt{2}\left(12\sqrt{2}\left(\cos^5(fx+e)\right)-38\sqrt{2}\left(\cos^3(fx+e)\right)-21\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(}\right.}{}$

```
[In] int(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/f*2^(1/2)/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)*(12*2^(1/2)*cos(f*x+
e)^5-38*2^(1/2)*cos(f*x+e)^3-21*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e
)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+
csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+42*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f
*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x
+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-21*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(c
ot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)+42*(-cot(f*x+e)+csc(f*x
+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*
EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)+47*2^(1/
2)*cos(f*x+e)-21*2^(1/2))
```

Fricas [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{9}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))*sqrt(
sin(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{9}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{9}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^{9/2}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2), x)
```

3.462 $\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{2f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

```
[Out] -1/3*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/2*(sin(e+1/4*Pi+f*x)^2)^(1
/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)
/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2663, 2665, 2652, 2719}

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/3*(b*Sin[e + f*x]^(3/2))/(f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*
f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
```

```
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{1}{2}\int\frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{\int\sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{\sin(e+fx)}\int\sqrt{\sin(2e+2fx)}\,dx}{2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}\\
&= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{2f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int\frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$=\frac{b\left(-1+\cos(2(e+fx))-3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{6f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-1 + Cos[2*(e + f*x)] - 3*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x
]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x
]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(96) = 192.

Time = 0.94 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.53

method	result
default	$\frac{\sqrt{2}\left(2\sqrt{2}\left(\cos^{3}(fx+e)\right)-6\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\mathrm{cs}\right.}{}$

```
[In] int(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*2^(1/2)/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)*(2*2^(1/2)*cos(f*x+e)^
3-6*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f
*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1
/2))+3*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(co
t(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2
^(1/2))-6*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*
(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/
2*2^(1/2))*sec(f*x+e)+3*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*
x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+
e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)-5*2^(1/2)*cos(f*x+e)+3*2^(1/2))
```

Fricas [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*s
ec(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^{5/2}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2), x)
```

3.463 $\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),
2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2665, 2652, 2719}

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (EllipticE[e - Pi/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*S
qrt[Sin[2*e + 2*f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\
&= \frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}} \\
&= \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
[Out] -((b*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1
/4))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(71) = 142.

Time = 0.84 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.27

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

[In] int(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/f*2^(1/2)/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)*(2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)-(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)+2^(1/2)*cos(f*x+e)-2^(1/2))

Fricas [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*sec(f*x + e)), x)

Sympy [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(e+fx\right)}}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

[In] integrate(sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(sin(e + f*x))/sqrt(b*sec(e + f*x)), x)

Maxima [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)

Giac [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(e+f\,x\right)}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2), x)
```

3.464 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

```
[Out] -2*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(1/2)+2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/
sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(
b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2664, 2665, 2652, 2719}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = -\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}} - \frac{2\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (-2*b)/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - 2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx \\
&= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\
&= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{\left(2\sqrt{\sin(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}} \\
&= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \frac{2b\left(-1+\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (2*b*(-1 + Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]
^2)^(1/4)))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(96) = 192.

Time = 0.76 (sec) , antiderivative size = 356, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\sqrt{2}\,(1-\cos(fx+e))\left(2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{2+2\cot(fx+e)-2\csc(fx+e)}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\text{c}\right.\right.}{f\left(\frac{-\cot(fx+e)+\csc(fx+e)}{(1-\cos(fx+e))^2\left(\csc^2(fx+e)\right)+1}\right)^{\frac{3}{2}}\left((1-\right.}$

```
[In] int(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)/(1/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(-cot(f*x+e)+csc(f*x+e)))
^(3/2)*(1-cos(f*x+e))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(2*(-cot(f*x+e)+csc
(f*x+e)+1)^(1/2)*(2+2*cot(f*x+e)-2*csc(f*x+e))^(1/2)*(cot(f*x+e)-csc(f*x+e)
)^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-(-cot(f*x+e
)+csc(f*x+e)+1)^(1/2)*(2+2*cot(f*x+e)-2*csc(f*x+e))^(1/2)*(cot(f*x+e)-csc(f
*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+(1-cos
(f*x+e))^2*csc(f*x+e)^2-1)/(-b*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)/((1-cos(f*
x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx =$$

$$-\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sqrt{\sin(fx+e)} + i\sqrt{ib}E(\arcsin(\cos(fx+e)+i\sin(fx+e))\,|-1)\sin(fx}{}$$

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e)) + I*sqrt(I*b)*el
liptic_e(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1)*sin(f*x + e) - I*sqrt(-
I*b)*elliptic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)*sin(f*x + e) - I
*sqrt(I*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1)*sin(f*x +
e) + I*sqrt(-I*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)*sin
(f*x + e))/(b*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx$$

```
[In] integrate(1/sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*sec(e + f*x))*sin(e + f*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sin(e+f\,x)^{3/2}\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)), x)
```

3.465 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{5f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

```
[Out] -2/5*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(5/2)-4/5*b/f/(b*sec(f*x+e))^(3/2)
/sin(f*x+e)^(1/2)+4/5*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipti
cE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2
*f*x+2*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2664, 2665, 2652, 2719}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = -\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{4b}{5f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}} - \frac{4\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{5f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(5/2)) - (4*b)/(5*f*(b*Sec[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (4*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
Sin[e + f*x]])/(5*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)}+\frac{2}{5}\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx\\
&= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)}\\
&\quad-\frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}-\frac{4}{5}\int\frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)}-\frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}\\
&\quad-\frac{4\int\sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{\left(4\sqrt{\sin(e+fx)}\right)\int\sqrt{\sin(2e+2fx)}\,dx}{5\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{5f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx$$
$$= \frac{2b\left(-2+\cos(2(e+fx))+2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sin^2(e+fx)\sqrt[4]{-\tan^2(e+fx)}\right)}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

```
[Out] (2*b*(-2 + Cos[2*(e + f*x)] + 2*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f
*x]^2]*Sin[e + f*x]^2*(-Tan[e + f*x]^2)^(1/4)))/(5*f*(b*Sec[e + f*x])^(3/2)
*Sin[e + f*x]^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(120) = 240.

Time = 0.89 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.83

method	result
default	$-\frac{\sqrt{2}\,(1-\cos(fx+e))\left(16\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{2+2\cot(fx+e)-2\csc(fx+e)}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)}\right.\right.}{}$

```
[In] int(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20/f*2^(1/2)/(1/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)*(-cot(f*x+e)+csc(f*x+e
)))^(7/2)*(1-cos(f*x+e))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^3*(16*(-cot(f*x+
e)+csc(f*x+e)+1)^(1/2)*(2+2*cot(f*x+e)-2*csc(f*x+e))^(1/2)*(cot(f*x+e)-csc(
```

```
f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(1-co
s(f*x+e))^2*csc(f*x+e)^2-8*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(2+2*cot(f*x+e)
-2*csc(f*x+e))^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+c
sc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e)^2-(1-cos(f*x+e)
)^6*csc(f*x+e)^6+9*(1-cos(f*x+e))^4*csc(f*x+e)^4-7*(1-cos(f*x+e))^2*csc(f*x
+e)^2-1)/(-b*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1))^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = -\frac{2\left(\left(i\cos\left(fx+e\right)^{2}-i\right)\sqrt{i\,b}E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\mid-1)\sin\left(fx+e\right)+\left(-i\cos\left(fx+e\right.\right.\right.}{}$$

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((I*cos(f*x + e)^2 - I)*sqrt(I*b)*elliptic_e(arcsin(cos(f*x + e) + I*s
in(f*x + e)), -1)*sin(f*x + e) + (-I*cos(f*x + e)^2 + I)*sqrt(-I*b)*ellipti
c_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)*sin(f*x + e) + (-I*cos(f*x +
 e)^2 + I)*sqrt(I*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1)*
sin(f*x + e) + (I*cos(f*x + e)^2 - I)*sqrt(-I*b)*elliptic_f(arcsin(cos(f*x
+ e) - I*sin(f*x + e)), -1)*sin(f*x + e) + (2*cos(f*x + e)^4 - 3*cos(f*x +
e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)))/((b*f*cos(f*x + e)^2 - b*f)*
sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)**(7/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^{7/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)), x)
```

3.466 $\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 363

$$\begin{aligned}\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad-\frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad-\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad+\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}\end{aligned}$$

```
[Out] 1/8*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)
/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/8*arctan(1+2^(1/2)*(
b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+
e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/16*ln(b^(1/2)+cot(f*x+e)*b^(1/2)-2^(1/2)*(
b*cos(f*x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2
)/(b*sec(f*x+e))^(1/2)+1/16*ln(b^(1/2)+cot(f*x+e)*b^(1/2)+2^(1/2)*(b*cos(f*
x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec
(f*x+e))^(1/2)-1/2*b*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2663, 2665, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} - \frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\sqrt{b}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\sqrt{b}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(4*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*A
rcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(4*
Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b]
 + Sqrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]
])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[S
qrt[b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e +
 f*x]]])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (b*Sqrt[
Sin[e + f*x]])/(2*f*(b*Sec[e + f*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
```

```
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{1}{4}\int\frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx\\
&= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{\int\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}-\frac{b\text{Subst}\left(\int\frac{x^2}{b^2+x^4}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{b\text{Subst}\left(\int\frac{b-x^2}{b^2+x^4}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{b+x^2}{b^2+x^4}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}-\frac{\sqrt{b}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{\sqrt{b}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx,x,\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}-\frac{\sqrt{b}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\sqrt{b}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.40

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(-4\sin^2(e+fx)+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\tan^2(e+fx)^{3/4}+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\right.}{8f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-4*Sin[e + f*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]
*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(3/4) + Sqrt[2]*ArcTanh[(Sqrt[2]
*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(3/4)
))/(8*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(281) = 562.

Time = 1.36 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\sqrt{2}\left(4\left(\sin^2(fx+e)\right)\cos(fx+e)\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)-\cos(fx+e)+1}{\cos(fx+e)-1}\right)\right.}{}$

```
[In] int(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/f*2^(1/2)*(4*sin(f*x+e)^2*cos(f*x+e)*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-2*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-2*cos(f*
```

```
x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*
x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))+cos(f*x+e)*ln(-2*2^(1/2)*(-sin(f*x+e)*co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-cos(f*x+e)*ln(2*2^(1/
2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-s
in(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))+2*a
rctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-c
os(f*x+e)+1)/(cos(f*x+e)-1))+2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-ln(-2*2^(1/2)*
(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))+ln(2*2
^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)
*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))
)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/sin(
f*x+e)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.47

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(b*f*(-1/(b^2*f^4))^(1/4)*log(2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)*
sin(f*x + e) - 2*cos(f*x + e)^2 + 2*(b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e
)^2 + f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e)
)*sqrt(sin(f*x + e)) + 1) - b*f*(-1/(b^2*f^4))^(1/4)*log(2*b*f^2*sqrt(-1/(b
^2*f^4))*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e)^2 - 2*(b*f^3*(-1/(b^2*f
^4))^(3/4)*cos(f*x + e)^2 + f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e
))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) + 1) + I*b*f*(-1/(b^2*f^4))^(1/4
)*log(-2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e
)^2 - 2*(I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - I*f*(-1/(b^2*f^4))^(
1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) + 1
) - I*b*f*(-1/(b^2*f^4))^(1/4)*log(-2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)
*sin(f*x + e) - 2*cos(f*x + e)^2 - 2*(-I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x
 + e)^2 + I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*
x + e))*sqrt(sin(f*x + e)) + 1) + b*f*(-1/(b^2*f^4))^(1/4)*log(2*(b*f^3*(-1
/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(
f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) - 1) - b*f*(-1/(b^2*f^4))
^(1/4)*log(-2*(b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - f*(-1/(b^2*f^4))
^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) -
```

```
 1) + I*b*f*(-1/(b^2*f^4))^(1/4)*log(-2*(I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f
*x + e)^2 + I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(
f*x + e))*sqrt(sin(f*x + e)) - 1) - I*b*f*(-1/(b^2*f^4))^(1/4)*log(-2*(-I*b
*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x
 + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) - 1) - 16*sqrt(
b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e)))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^{\frac{3}{2}}\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**(3/2)/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^{3/2}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2), x)
```

3.467 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx &= \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad - \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} \\ &\quad + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)
/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/2*arctan(1+2^(1/2)*(
b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+
e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/4*ln(b^(1/2)+cot(f*x+e)*b^(1/2)-2^(1/2)*(b
*cos(f*x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)
/(b*sec(f*x+e))^(1/2)+1/4*ln(b^(1/2)+cot(f*x+e)*b^(1/2)+2^(1/2)*(b*cos(f*x+
e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f
*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2665, 2655, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\sqrt{b}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\sqrt{b}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Arc
Tan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(Sqrt
[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b] + S
qrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]])/(
2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[Sqrt[
b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x
]]])/(2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

$$= \frac{b\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}x+x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{b\left(\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\right)\tan^2(e+fx)^{3/4}}{\sqrt{2}f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]

[Out] (b*(ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))] + ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])])*(Tan[e + f*x]^2)^(3/4))/(Sqrt[2]*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\sqrt{2}\,(\cos(fx+e)-1)\left(\ln\left(-2\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\cot(fx+e)-2\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\csc(fx+e)+2-2\cot(fx+e)\right)-2\,\mathrm{ar}\right.}{}$

[In] int(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/f*2^(1/2)*(cos(f*x+e)-1)*(ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))+2*arctan((-2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))/sin(f*x+e)^(3/2)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 843, normalized size of antiderivative = 2.57

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-1/(b^2*f^4))^(1/4)*log(2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)*sin(f*
x + e) - 2*cos(f*x + e)^2 + 2*(b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 +
f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt
(sin(f*x + e)) + 1) - 1/8*(-1/(b^2*f^4))^(1/4)*log(2*b*f^2*sqrt(-1/(b^2*f^4
))*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e)^2 - 2*(b*f^3*(-1/(b^2*f^4))^(
3/4)*cos(f*x + e)^2 + f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqr
t(b/cos(f*x + e))*sqrt(sin(f*x + e)) + 1) + 1/8*I*(-1/(b^2*f^4))^(1/4)*log(
-2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x + e)^2 -
2*(I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - I*f*(-1/(b^2*f^4))^(1/4)*c
os(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) + 1) - 1/
8*I*(-1/(b^2*f^4))^(1/4)*log(-2*b*f^2*sqrt(-1/(b^2*f^4))*cos(f*x + e)*sin(f
*x + e) - 2*cos(f*x + e)^2 - 2*(-I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^
2 + I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e)
)*sqrt(sin(f*x + e)) + 1) + 1/8*(-1/(b^2*f^4))^(1/4)*log(2*(b*f^3*(-1/(b^2*
f^4))^(3/4)*cos(f*x + e)^2 - f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x +
e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) - 1) - 1/8*(-1/(b^2*f^4))^(1/4)
*log(-2*(b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - f*(-1/(b^2*f^4))^(1/4)
*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) - 1) +
1/8*I*(-1/(b^2*f^4))^(1/4)*log(-2*(I*b*f^3*(-1/(b^2*f^4))^(3/4)*cos(f*x + e
)^2 + I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x +
e))*sqrt(sin(f*x + e)) - 1) - 1/8*I*(-1/(b^2*f^4))^(1/4)*log(-2*(-I*b*f^3*(
-1/(b^2*f^4))^(3/4)*cos(f*x + e)^2 - I*f*(-1/(b^2*f^4))^(1/4)*cos(f*x + e)*
sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)) - 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(e+fx\right)}\sqrt{\sin\left(e+fx\right)}}\,dx$$

```
[In] integrate(1/sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*sec(e + f*x))*sqrt(sin(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{\sin\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)), x)
```

3.468 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2658}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2\cos(fx+e)}{3f\sin(fx+e)^{\frac{3}{2}}\sqrt{b\sec(fx+e)}}$	30

```
[In] int(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/f*cos(f*x+e)/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sqrt{\sin(fx+e)}}{3\left(bf\cos(fx+e)^2-bf\right)}$$

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e))/(b*f*cos(f*x + e
)^2 - b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}\,(\sin(e+f\,x)+\sin(3\,e+3\,f\,x))}{3\,b\,f\,\sqrt{\sin(e+f\,x)}\,(\cos(2\,e+2\,f\,x)-1)}$$

```
[In] int(1/(sin(e + f*x)^(5/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] ((b/cos(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x)))/(3*b*f*sin(e + f
*x)^(1/2)*(cos(2*e + 2*f*x) - 1))
```

3.469 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{8b}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[Out] -2/7*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-8/21*b/f/(b*sec(f*x+e))^(3/2
)/sin(f*x+e)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2664, 2658}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = -\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(b*Sec
[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
```

```
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}+\frac{4}{7}\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx\\ &= -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}-\frac{8b}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx=\frac{2b(-5+2\cos(2(e+fx)))}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
[Out] (2*b*(-5 + 2*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(
7/2))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\frac{8\left(\cos^{3}(fx+e)\right)}{21}-\frac{2\cos(fx+e)}{3}}{f\sin(fx+e)^{\frac{7}{2}}\sqrt{b\sec(fx+e)}}$	43

```
[In] int(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/f/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2)*(4*cos(f*x+e)^3-7*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{2\left(4\cos(fx+e)^4-7\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin(fx+e)}}{21\left(bf\cos(fx+e)^4-2\,bf\cos(fx+e)^2+bf\right)}$$

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*
x + e))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{4\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(11\,\sin\left(e+f\,x\right)+4\,\sin\left(3\,e+3\,f\,x\right)-6\,\sin\left(5\,e+5\,f\,x\right)+\sin\left(7\,e+7\,f\,x\right)\right)}{21\,b\,f\,\sqrt{\sin\left(e+f\,x\right)}\,\left(15\,\cos\left(2\,e+2\,f\,x\right)-6\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(6\,e+6\,f\,x\right)-10\right)}$$

```
[In] int(1/(sin(e + f*x)^(9/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] (4*(b/cos(e + f*x))^(1/2)*(11*sin(e + f*x) + 4*sin(3*e + 3*f*x) - 6*sin(5*e
 + 5*f*x) + sin(7*e + 7*f*x)))/(21*b*f*sin(e + f*x)^(1/2)*(15*cos(2*e + 2*f
*x) - 6*cos(4*e + 4*f*x) + cos(6*e + 6*f*x) - 10))
```

3.470 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{64b}{231f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[Out] -2/11*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(11/2)-16/77*b/f/(b*sec(f*x+e))^(
3/2)/sin(f*x+e)^(7/2)-64/231*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2664, 2658}

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = -\frac{64b}{231f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{16b}{77f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (-2*b)/(11*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (16*b)/(77*f*(b*
Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (64*b)/(231*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(3/2))
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}+\frac{8}{11}\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx\\
&= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}-\frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}\\
&\quad+\frac{32}{77}\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx\\
&= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}-\frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}\\
&\quad-\frac{64b}{231f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int\frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx=\frac{2b(-45+28\cos(2(e+fx))-4\cos(4(e+fx)))}{231f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (2*b*(-45 + 28*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)]))/(231*f*(b*Sec[e + f*
x])^(3/2)*Sin[e + f*x]^(11/2))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{2\left(32\left(\cos^{5}\left(f x+e\right)\right)-88\left(\cos^{3}\left(f x+e\right)\right)+77\cos\left(f x+e\right)\right)}{231 f \sin\left(f x+e\right)^{\frac{11}{2}} \sqrt{b \sec\left(f x+e\right)}}$	53

```
[In] int(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231/f/sin(f*x+e)^(11/2)/(b*sec(f*x+e))^(1/2)*(32*cos(f*x+e)^5-88*cos(f*x
+e)^3+77*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{2\left(32\,\cos\left(fx+e\right)^{6}-88\,\cos\left(fx+e\right)^{4}+77\,\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{231\left(bf\cos\left(fx+e\right)^{6}-3\,bf\cos\left(fx+e\right)^{4}+3\,bf\cos\left(fx+e\right)^{2}-bf\right)}$$

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^6 - 88*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/co
s(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^6 - 3*b*f*cos(f*x + e)^4 +
 3*b*f*cos(f*x + e)^2 - b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)**(13/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{13}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(13/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{e^{-e\,6i-f\,x\,6i}\sqrt{\frac{b}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{e^{e\,6i+f\,x\,6i}\,992i}{231\,b\,f}+\frac{e^{e\,6i+f\,x\,6i}\,\cos(2\,e+2\,f\,x)\,608i}{231\,b\,f}-\frac{e^{e\,6i+f\,x\,6i}\,\cos(4\,e+4\,f\,x)\,320i}{231\,b\,f}+\frac{e^{e\,6i+f\,x\,6}}{}\right.}{32\sin(e+f\,x)^{11/2}}$$

```
[In] int(1/(sin(e + f*x)^(13/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] (exp(- e*6i - f*x*6i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(
1/2)*((exp(e*6i + f*x*6i)*992i)/(231*b*f) + (exp(e*6i + f*x*6i)*cos(2*e + 2
*f*x)*608i)/(231*b*f) - (exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*320i)/(231*b*f
) + (exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*64i)/(231*b*f))*1i)/(32*sin(e + f*
x)^(11/2))
```

3.471 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{64b}{385f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{256b}{1155f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

```
[Out] -2/15*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(15/2)-8/55*b/f/(b*sec(f*x+e))^(3
/2)/sin(f*x+e)^(11/2)-64/385*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-256/
1155*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used

$= \{2664, 2658\}$

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = -\frac{256b}{1155f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{64b}{385f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{8b}{55f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}} - \frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (-2*b)/(15*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2)) - (8*b)/(55*f*(b*S
ec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (64*b)/(385*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(7/2)) - (256*b)/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*
x]^(3/2))
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} + \frac{4}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx \\
&= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} \\
&\quad + \frac{32}{55}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx \\
&= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} \\
&\quad - \frac{64b}{385f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} + \frac{128}{385}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx
\end{aligned}$$

$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{64b}{385f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{256b}{1155f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{2b(-195+150\cos(2(e+fx))-36\cos(4(e+fx))+4\cos(6(e+fx)))}{1155f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)}$$

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (2*b*(-195 + 150*Cos[2*(e + f*x)] - 36*Cos[4*(e + f*x)] + 4*Cos[6*(e + f*x)
]))/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\frac{256\left(\cos^{7}(fx+e)\right)}{1155}-\frac{64\left(\cos^{5}(fx+e)\right)}{77}+\frac{8\left(\cos^{3}(fx+e)\right)}{7}-\frac{2\cos(fx+e)}{3}}{f\sin(fx+e)^{\frac{15}{2}}\sqrt{b\sec(fx+e)}}$	63

```
[In] int(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/f/sin(f*x+e)^(15/2)/(b*sec(f*x+e))^(1/2)*(128*cos(f*x+e)^7-480*cos(f
*x+e)^5+660*cos(f*x+e)^3-385*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{2\left(128\cos\left(fx+e\right)^{8}-480\cos\left(fx+e\right)^{6}+660\cos\left(fx+e\right)^{4}-385\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin\left(fx+e\right.}}{1155\left(bf\cos\left(fx+e\right)^{8}-4\,bf\cos\left(fx+e\right)^{6}+6\,bf\cos\left(fx+e\right)^{4}-4\,bf\cos\left(fx+e\right)^{2}+bf\right)}$$

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(128*cos(f*x + e)^8 - 480*cos(f*x + e)^6 + 660*cos(f*x + e)^4 - 385*
cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^8
 - 4*b*f*cos(f*x + e)^6 + 6*b*f*cos(f*x + e)^4 - 4*b*f*cos(f*x + e)^2 + b*f
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)**(17/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{17}{2}}}\,dx$$

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{\mathrm{e}^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\,\sqrt{\frac{b}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,1024\mathrm{i}}{77\,b\,f}+\frac{\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,384\mathrm{i}}{55\,b\,f}-\frac{\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,5248\mathrm{i}}{1155\,b\,f}+\frac{\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8}\cdots}{\cdots}\right.}{128\sin\left(e+f\,x\right)^{15/2}}$$

```
[In] int(1/(sin(e + f*x)^(17/2)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(
1/2)*((exp(e*8i + f*x*8i)*1024i)/(77*b*f) + (exp(e*8i + f*x*8i)*cos(2*e + 2
*f*x)*384i)/(55*b*f) - (exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*5248i)/(1155*b*
f) + (exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*256i)/(165*b*f) - (exp(e*8i + f*x
*8i)*cos(8*e + 8*f*x)*256i)/(1155*b*f))*1i)/(128*sin(e + f*x)^(15/2))
```

3.472 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 490

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{7a^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} + \frac{7a^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} + \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f} - \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f} - \frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

```
[Out] -7/192*a^3*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e))^(1/2)-1/48*a*(a*sin(f*x+
e))^(7/2)/b/f/(b*sec(f*x+e))^(1/2)+1/6*(a*sin(f*x+e))^(11/2)/a/b/f/(b*sec(f
*x+e))^(1/2)-7/256*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^
(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/
2)/f*2^(1/2)+7/256*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^
(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/
2)/f*2^(1/2)+7/512*a^(9/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/
(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e)
)^(1/2)/b^(5/2)/f*2^(1/2)-7/512*a^(9/2)*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f
```

```
*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*
(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2662, 2663, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = & -\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}b^{5/2}f} \\ & +\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{128\sqrt{2}b^{5/2}f} \\ & +\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{256\sqrt{2}b^{5/2}f} \\ & -\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{256\sqrt{2}b^{5/2}f} \\ & -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}}\end{aligned}$$

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-7*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(128*Sqrt[2]*
b^(5/2)*f) + (7*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(
Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(
128*Sqrt[2]*b^(5/2)*f) + (7*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqr
t[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f
*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)*f) - (7*a^(9/2)*Sqrt[b*Cos[
e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e
 + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)
*f) - (7*a^3*(a*Sin[e + f*x])^(3/2))/(192*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a
*Sin[e + f*x])^(7/2))/(48*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(11/
2)/(6*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
```

```
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx}{12b^2} \\
&= -\frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^2)\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx}{96b^2} \\
&= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} \\
&\quad + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^4)\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{128b^2} \\
&= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} \\
&\quad + \frac{\left(7a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{128b^2} \\
&= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} \\
&\quad + \frac{\left(7a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64bf}
\end{aligned}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} - \frac{\left(7a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{128b^2f} + \frac{\left(7a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a+bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{128b^2f}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{256b^3f} + \frac{\left(7a^5\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{256b^3f} + \frac{\left(7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{256\sqrt{2}b^{5/2}f} + \frac{\left(7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{256\sqrt{2}b^{5/2}f}$$

$$= \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f} - \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f} - \frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\left(7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}b^{5/2}f} - \frac{\left(7a^{9/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}b^{5/2}f}$$

$$= -\frac{7a^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f}$$
$$+\frac{7a^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f}$$
$$+\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f}$$
$$-\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}b^{5/2}f}$$
$$-\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.36

$$\int\frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{a^5\left(4(-3+14\cos(2(e+fx))-4\cos(4(e+fx)))\sin^2(e+fx)-21\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan}\right.}{768bf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/768*(a^5*(4*(-3 + 14*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Sin[e + f*x]
^2 - 21*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2
)^(1/4))]*(Tan[e + f*x]^2)^(1/4) + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*x
]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/(b*f*Sqrt[
b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 5.79 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{2}\left(128\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cos^5(fx+e))\sin(fx+e)+128\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cos^4(fx+e))\sin(fx+e)\sqrt{2}-240\sqrt{-\frac{\sin(fx}{(\cos}}\right.}{}$

```
[In] int((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536/f*2^(1/2)*(128*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*cos(f*x+e)^5*sin(f*x+e)+128*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)-240*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)-240*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*2^(1/2)*cos(f*x+e)^2*sin(f*x+e)+84*(-sin(f*x+e)*cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)+84*2^(1/2)*(-si
n(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+21*ln(-2*2^(1/2)*(-s
in(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x
+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-21*ln(2*2
^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)
*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))
+42*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x
+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))+42*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(a*sin(
f*x+e))^(1/2)*a^4/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)/(b*sec(f*x+e))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.45

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3072*(21*(-a^18/(b^6*f^4))^(1/4)*b^2*f*log(343/2*a^14*cos(f*x + e)*sin(f
*x + e) + 343/2*((-a^18/(b^6*f^4))^(1/4)*a^9*b*f*cos(f*x + e)*sin(f*x + e)
- (-a^18/(b^6*f^4))^(3/4)*b^4*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt
(b/cos(f*x + e)) - 343/4*(2*a^5*b^3*f^2*cos(f*x + e)^2 - a^5*b^3*f^2)*sqrt(
-a^18/(b^6*f^4))) - 21*(-a^18/(b^6*f^4))^(1/4)*b^2*f*log(343/2*a^14*cos(f*x
 + e)*sin(f*x + e) - 343/2*((-a^18/(b^6*f^4))^(1/4)*a^9*b*f*cos(f*x + e)*si
n(f*x + e) - (-a^18/(b^6*f^4))^(3/4)*b^4*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e))*sqrt(b/cos(f*x + e)) - 343/4*(2*a^5*b^3*f^2*cos(f*x + e)^2 - a^5*b^3
*f^2)*sqrt(-a^18/(b^6*f^4))) - 21*I*(-a^18/(b^6*f^4))^(1/4)*b^2*f*log(343/2
*a^14*cos(f*x + e)*sin(f*x + e) - 343/2*(I*(-a^18/(b^6*f^4))^(1/4)*a^9*b*f*
cos(f*x + e)*sin(f*x + e) + I*(-a^18/(b^6*f^4))^(3/4)*b^4*f^3*cos(f*x + e)^
2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 343/4*(2*a^5*b^3*f^2*cos(f*x
 + e)^2 - a^5*b^3*f^2)*sqrt(-a^18/(b^6*f^4))) + 21*I*(-a^18/(b^6*f^4))^(1/4
)*b^2*f*log(343/2*a^14*cos(f*x + e)*sin(f*x + e) - 343/2*(-I*(-a^18/(b^6*f^
4))^(1/4)*a^9*b*f*cos(f*x + e)*sin(f*x + e) - I*(-a^18/(b^6*f^4))^(3/4)*b^4
*f^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 343/4*(2*a
```

```
^5*b^3*f^2*cos(f*x + e)^2 - a^5*b^3*f^2)*sqrt(-a^18/(b^6*f^4))) + 21*(-a^18
/(b^6*f^4))^(1/4)*b^2*f*log(343*a^14 + 686*((-a^18/(b^6*f^4))^(1/4)*a^9*b*f
*cos(f*x + e)^2 - (-a^18/(b^6*f^4))^(3/4)*b^4*f^3*cos(f*x + e)*sin(f*x + e)
)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) - 21*(-a^18/(b^6*f^4))^(1/4)*b
^2*f*log(343*a^14 - 686*((-a^18/(b^6*f^4))^(1/4)*a^9*b*f*cos(f*x + e)^2 - (
-a^18/(b^6*f^4))^(3/4)*b^4*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x +
e))*sqrt(b/cos(f*x + e))) - 21*I*(-a^18/(b^6*f^4))^(1/4)*b^2*f*log(343*a^14
 - 686*(I*(-a^18/(b^6*f^4))^(1/4)*a^9*b*f*cos(f*x + e)^2 + I*(-a^18/(b^6*f^
4))^(3/4)*b^4*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/co
s(f*x + e))) + 21*I*(-a^18/(b^6*f^4))^(1/4)*b^2*f*log(343*a^14 - 686*(-I*(-
a^18/(b^6*f^4))^(1/4)*a^9*b*f*cos(f*x + e)^2 - I*(-a^18/(b^6*f^4))^(3/4)*b^
4*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)))
 - 16*(32*a^4*cos(f*x + e)^5 - 60*a^4*cos(f*x + e)^3 + 21*a^4*cos(f*x + e))
*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\,\sin\left(e+f\,x\right))^{9/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2), x)
```

3.473 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 453

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f} - \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f} - \frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

```
[Out] -1/16*a*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e))^(1/2)+1/4*(a*sin(f*x+e))^(7
/2)/a/b/f/(b*sec(f*x+e))^(1/2)-3/64*a^(5/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin
(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*
x+e))^(1/2)/b^(5/2)/f*2^(1/2)+3/64*a^(5/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(
f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x
+e))^(1/2)/b^(5/2)/f*2^(1/2)+3/128*a^(5/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*si
n(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/
2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)-3/128*a^(5/2)*ln(a^(1/2)+2^(1/2)*
b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*co
s(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2662, 2663, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{64\sqrt{2}b^{5/2}f} - \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{64\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*b
^(5/2)*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(S
qrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(3
2*Sqrt[2]*b^(5/2)*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[
2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x
]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f) - (3*a^(5/2)*Sqrt[b*Cos[e +
 f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e +
f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f)
- (a*(a*Sin[e + f*x])^(3/2))/(16*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x
])^(7/2)/(4*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}+\frac{\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx}{8b^2}\\
&=-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}+\frac{(3a^2)\int\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{32b^2}\\
&=-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\left(3a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{32b^2}\\
&=-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}\\
&\quad+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{16bf}\\
&=-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}\\
&\quad-\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a-bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{32b^2f}\\
&\quad+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a+bx^2}{a^2+b^2x^4}\,dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{32b^2f}
\end{aligned}
$$

$$=-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64b^3f}+\frac{\left(3a^3\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64b^3f}+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64\sqrt{2}b^{5/2}f}+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}dx,x,\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{64\sqrt{2}b^{5/2}f}$$

$$=\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f}-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f}-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}+\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}b^{5/2}f}-\frac{\left(3a^{5/2}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}b^{5/2}f}$$

$$=-\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f}+\frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f}-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}b^{5/2}f}-\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.36

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a^3\left(4-6\cos(2(e+fx))+2\cos(4(e+fx))+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\right.}{64bf\sqrt{b\sec(e+fx)}\sqrt{}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a^3*(4 - 6*Cos[2*(e + f*x)] + 2*Cos[4*(e + f*x)] + 3*Sqrt[2]*ArcTan[(-1 +
Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1
/4) - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e +
f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/(64*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[
e + f*x]])
```

Maple [A] (warning: unable to verify)

Time = 5.01 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.18

method	result
default	$\frac{\sqrt{2}\left(-16\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{2}\left(\cos^3(fx+e)\right)\sin(fx+e)-16\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{2}\left(\cos^2(fx+e)\right)\sin(fx+e)+12\sqrt{-\frac{\sin(fx}{(\cos}}\right.}{}$

```
[In] int((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/f*2^(1/2)*(-16*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2
)*cos(f*x+e)^3*sin(f*x+e)-16*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*2^(1/2)*cos(f*x+e)^2*sin(f*x+e)+12*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*2^(1/2)*sin(f*x+e)*cos(f*x+e)+12*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+3*ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))-3*ln(2*2^(1/2)*(-sin(f*x+e)*co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))+6*arctan((2^(1/2)*(-s
in(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(
f*x+e)-1))+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(a*sin(f*x+e))^(1/2)*a^2/(cos(f
*x+e)+1)/(b*sec(f*x+e))^(1/2)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 1188, normalized size of antiderivative = 2.62

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/256*(3*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27/2*a^8*cos(f*x + e)*sin(f*x +
e) + 27/2*(b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x + e)^2 - a^5*b*f*(-a^10/
(b^6*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos
(f*x + e)) - 27/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^3*f^2)*sqrt(-a^10/(
b^6*f^4))) - 3*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27/2*a^8*cos(f*x + e)*sin(
f*x + e) - 27/2*(b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x + e)^2 - a^5*b*f*(
-a^10/(b^6*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt
(b/cos(f*x + e)) - 27/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^3*f^2)*sqrt(-
a^10/(b^6*f^4))) + 3*I*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27/2*a^8*cos(f*x +
 e)*sin(f*x + e) - 27/2*(I*b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x + e)^2 +
 I*a^5*b*f*(-a^10/(b^6*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*
x + e))*sqrt(b/cos(f*x + e)) + 27/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^3
*f^2)*sqrt(-a^10/(b^6*f^4))) - 3*I*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27/2*a
^8*cos(f*x + e)*sin(f*x + e) - 27/2*(-I*b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos
(f*x + e)^2 - I*a^5*b*f*(-a^10/(b^6*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*
sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 27/4*(2*a^3*b^3*f^2*cos(f*x + e
)^2 - a^3*b^3*f^2)*sqrt(-a^10/(b^6*f^4))) + 3*b^2*f*(-a^10/(b^6*f^4))^(1/4)
*log(27*a^8 + 54*(b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e)
 - a^5*b*f*(-a^10/(b^6*f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqr
t(b/cos(f*x + e))) - 3*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27*a^8 - 54*(b^4*f
^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - a^5*b*f*(-a^10/(b^6*
f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) + 3*
I*b^2*f*(-a^10/(b^6*f^4))^(1/4)*log(27*a^8 - 54*(I*b^4*f^3*(-a^10/(b^6*f^4)
)^(3/4)*cos(f*x + e)*sin(f*x + e) + I*a^5*b*f*(-a^10/(b^6*f^4))^(1/4)*cos(f
*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) - 3*I*b^2*f*(-a^10/(b
^6*f^4))^(1/4)*log(27*a^8 - 54*(-I*b^4*f^3*(-a^10/(b^6*f^4))^(3/4)*cos(f*x
+ e)*sin(f*x + e) - I*a^5*b*f*(-a^10/(b^6*f^4))^(1/4)*cos(f*x + e)^2)*sqrt(
a*sin(f*x + e))*sqrt(b/cos(f*x + e))) - 16*(4*a^2*cos(f*x + e)^3 - 3*a^2*co
s(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{5/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2), x)
```

3.474 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 418

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

```
[Out] 1/2*(a*sin(f*x+e))^(3/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/8*arctan(1-2^(1/2)*b^
(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*a^(1/2)*(b*cos(f*x
+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)+1/8*arctan(1+2^(1/2)*b^(1
/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*a^(1/2)*(b*cos(f*x+e
))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)+1/16*ln(a^(1/2)-2^(1/2)*b^(
1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*a^(1/2)*
(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)-1/16*ln(a^(1/2)
+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+
e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2662, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{8\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{8\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/4*(Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sq
rt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*b^
(5/2)*f) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt
[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sq
rt[2]*b^(5/2)*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqr
t[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqr
t[b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Lo
g[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + S
qrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) + (a*Sin[e
 + f*x])^(3/2)/(2*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\int\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2bf}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} - \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{4b^2f} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{a+bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{4b^2f}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8b^3f} + \frac{\left(a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8b^3f} + \frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8\sqrt{2}b^{5/2}f} + \frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{8\sqrt{2}b^{5/2}f}$$

$$= \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f} - \frac{\left(\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f}$$

$$= -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a\left(4\sin^2(e+fx)+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}-\sqrt{2}\text{arctan}\right.}{8bf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(4*Sin[e + f*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*
(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*
(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4))
)/(8*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sqrt{2}\left(4\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{2}\sin(fx+e)\cos(fx+e)+4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+\ln\left(-2\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\right.\right.}{}$

```
[In] int((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f*2^(1/2)*(4*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)*s
in(f*x+e)*cos(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*sin(f*x+e)+ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(
f*x+e)+2-2*cot(f*x+e))-ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*csc(f*x+e)+2-2*cot(f*x+e))+2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))+2*arctan((2^(
1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-
1)/(cos(f*x+e)-1)))*(a*sin(f*x+e))^(1/2)/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(b^2*f*(-a^2/(b^6*f^4))^(1/4)*log(1/2*a^2*cos(f*x + e)*sin(f*x + e) +
1/2*(b^4*f^3*(-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)^2 - a*b*f*(-a^2/(b^6*f^4))
^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))
 - 1/4*(2*a*b^3*f^2*cos(f*x + e)^2 - a*b^3*f^2)*sqrt(-a^2/(b^6*f^4))) - b^2
*f*(-a^2/(b^6*f^4))^(1/4)*log(1/2*a^2*cos(f*x + e)*sin(f*x + e) - 1/2*(b^4*
f^3*(-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)^2 - a*b*f*(-a^2/(b^6*f^4))^(1/4)*co
s(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 1/4*(2
*a*b^3*f^2*cos(f*x + e)^2 - a*b^3*f^2)*sqrt(-a^2/(b^6*f^4))) - I*b^2*f*(-a^
2/(b^6*f^4))^(1/4)*log(1/2*a^2*cos(f*x + e)*sin(f*x + e) + 1/2*(I*b^4*f^3*(
-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)^2 + I*a*b*f*(-a^2/(b^6*f^4))^(1/4)*cos(f
*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1/4*(2*a*
```

```
b^3*f^2*cos(f*x + e)^2 - a*b^3*f^2)*sqrt(-a^2/(b^6*f^4))) + I*b^2*f*(-a^2/(
b^6*f^4))^(1/4)*log(1/2*a^2*cos(f*x + e)*sin(f*x + e) + 1/2*(-I*b^4*f^3*(-a
^2/(b^6*f^4))^(3/4)*cos(f*x + e)^2 - I*a*b*f*(-a^2/(b^6*f^4))^(1/4)*cos(f*x
 + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1/4*(2*a*b^
3*f^2*cos(f*x + e)^2 - a*b^3*f^2)*sqrt(-a^2/(b^6*f^4))) + b^2*f*(-a^2/(b^6*
f^4))^(1/4)*log(a^2 + 2*(b^4*f^3*(-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*
x + e) - a*b*f*(-a^2/(b^6*f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*
sqrt(b/cos(f*x + e))) - b^2*f*(-a^2/(b^6*f^4))^(1/4)*log(a^2 - 2*(b^4*f^3*(
-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - a*b*f*(-a^2/(b^6*f^4))^(1
/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))) + I*b^2*f*(-
a^2/(b^6*f^4))^(1/4)*log(a^2 - 2*(I*b^4*f^3*(-a^2/(b^6*f^4))^(3/4)*cos(f*x
+ e)*sin(f*x + e) + I*a*b*f*(-a^2/(b^6*f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*s
in(f*x + e))*sqrt(b/cos(f*x + e))) - I*b^2*f*(-a^2/(b^6*f^4))^(1/4)*log(a^2
 - 2*(-I*b^4*f^3*(-a^2/(b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - I*a*b*f
*(-a^2/(b^6*f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x
 + e))) + 16*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*x
 + e))/(b^2*f)
```

Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(e+fx)}}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(fx+e)}}}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2), x)
```

3.475 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 411

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$+\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1
/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2^(1/2)-1/
2*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2
))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2^(1/2)-1/4*
ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2
)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2
^(1/2)+1/4*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(
1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/
b^(5/2)/f*2^(1/2)-2/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2661, 2665, 2654, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$+\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\log\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)+\sqrt{a}\right)}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e +
f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b^(5/2)
*f) - (ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Co
s[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b
^(5/2)*f) - (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin
[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x
]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) + (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] + (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) - 2/(a*b*f*Sqrt[b
*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2661

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
```

```
f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\text{integral} = -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{abf}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{ab^2f} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a+bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{ab^2f}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2ab^3f} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2ab^3f} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}+2x}{-\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2\sqrt{2}a^{3/2}b^{5/2}f} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{b}}-2x}{-\frac{a}{b}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}-x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{2\sqrt{2}a^{3/2}b^{5/2}f}$$

$$= -\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$+\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$
$$-\frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$+\frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}$$

$$= \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$+\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a}+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$
$$-\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.35

$$\int\frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{4+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}}{2abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
[Out] -1/2*(4 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]
^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*x]
^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4))/(a*b*f*Sqrt[
b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)+4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\ln\left(-2\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cot(fx+e)-2\sqrt{2}\right.\right.}{}$

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*2^(1/2)*(4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*c
os(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+ln(-2*2
^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)
*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))
*sin(f*x+e)+2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*sin(f*x+e)+2*arctan((2^(1/2)*(-
sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos
(f*x+e)-1))*sin(f*x+e)-ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*csc(f*x+e)+2-2*cot(f*x+e))*sin(f*x+e))/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/
2)/(a*sin(f*x+e))^(1/2)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 1224, normalized size of antiderivative = 2.98

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/8*(a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(1/2*(a^4*b^4*f^3*(-1/(a^6*b^6*f
^4))^(3/4)*cos(f*x + e)^2 - a*b*f*(-1/(a^6*b^6*f^4))^(1/4)*cos(f*x + e)*sin
(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 1/2*cos(f*x + e)*sin
(f*x + e) + 1/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^3*f^2)*sqrt(-1/(a^6*b
^6*f^4)))*sin(f*x + e) - a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(-1/2*(a^4*b
^4*f^3*(-1/(a^6*b^6*f^4))^(3/4)*cos(f*x + e)^2 - a*b*f*(-1/(a^6*b^6*f^4))^(
1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) -
 1/2*cos(f*x + e)*sin(f*x + e) + 1/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^
3*f^2)*sqrt(-1/(a^6*b^6*f^4)))*sin(f*x + e) - I*a^2*b^2*f*(-1/(a^6*b^6*f^4)
)^(1/4)*log(1/2*(I*a^4*b^4*f^3*(-1/(a^6*b^6*f^4))^(3/4)*cos(f*x + e)^2 + I*
```

```
a*b*f*(-1/(a^6*b^6*f^4))^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x +
e))*sqrt(b/cos(f*x + e)) - 1/2*cos(f*x + e)*sin(f*x + e) - 1/4*(2*a^3*b^3*f
^2*cos(f*x + e)^2 - a^3*b^3*f^2)*sqrt(-1/(a^6*b^6*f^4)))*sin(f*x + e) + I*a
^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(1/2*(-I*a^4*b^4*f^3*(-1/(a^6*b^6*f^4)
)^(3/4)*cos(f*x + e)^2 - I*a*b*f*(-1/(a^6*b^6*f^4))^(1/4)*cos(f*x + e)*sin(
f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) - 1/2*cos(f*x + e)*sin(
f*x + e) - 1/4*(2*a^3*b^3*f^2*cos(f*x + e)^2 - a^3*b^3*f^2)*sqrt(-1/(a^6*b^
6*f^4)))*sin(f*x + e) - a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(2*(a^4*b^4*f
^3*(-1/(a^6*b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - a*b*f*(-1/(a^6*b^6*
f^4))^(1/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1)*
sin(f*x + e) + a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(-2*(a^4*b^4*f^3*(-1/(
a^6*b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - a*b*f*(-1/(a^6*b^6*f^4))^(1
/4)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1)*sin(f*x
+ e) - I*a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(-2*(I*a^4*b^4*f^3*(-1/(a^6*
b^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) + I*a*b*f*(-1/(a^6*b^6*f^4))^(1/4
)*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1)*sin(f*x +
e) + I*a^2*b^2*f*(-1/(a^6*b^6*f^4))^(1/4)*log(-2*(-I*a^4*b^4*f^3*(-1/(a^6*b
^6*f^4))^(3/4)*cos(f*x + e)*sin(f*x + e) - I*a*b*f*(-1/(a^6*b^6*f^4))^(1/4)
*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + 1)*sin(f*x + e
) - 16*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e))/(a^2*b^2*f*s
in(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}\,(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)), x)
```

3.476 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af(b\sec(e+fx))^{5/2}(a\sin(e+fx))^{5/2}}$$

```
[Out] -2/5*b/a/f/(b*sec(f*x+e))^(5/2)/(a*sin(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2658}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

```
[Out] (-2*b)/(5*a*f*(b*Sec[e + f*x])^(5/2)*(a*Sin[e + f*x])^(5/2))
```

Rule 2658

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2b}{5af(b\sec(e+fx))^{5/2}(a\sin(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2\cot^3(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}{5a^4b^2f}$$

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
[Out] (-2*Cot[e + f*x]^3*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])/(5*a^4*b^2*f)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\left(\cot^2(fx+e)\right)}{5f\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,a^3b}$	40

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5/f/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^3/b*cot(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(29) = 58.

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^3}{5\left(a^4b^2f\cos(fx+e)^2-a^4b^2f\right)\sin(fx+e)}$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas"
)
[Out] 2/5*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/((a^4*b^2*f*co
s(f*x + e)^2 - a^4*b^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}\,(a\sin(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}\,(a\sin(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.40

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)+\cos\left(5\,e+5\,f\,x\right)\right)}{5\,a^3\,b^2\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\cos\left(4\,e+4\,f\,x\right)-4\,\cos\left(2\,e+2\,f\,x\right)+3\right)}$$

```
[In] int(1/((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] ((b/cos(e + f*x))^(1/2)*(cos(3*e + 3*f*x) - 2*cos(e + f*x) + cos(5*e + 5*f*
x)))/(5*a^3*b^2*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e + 2*
f*x) + 3))
```

3.477 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{24b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/30*a*(a*sin(f*x+e))^(5/2)/b/f/(b*sec(f*x+e))^(1/2)+1/5*(a*sin(f*x+e))^(9
/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/12*a^3*(a*sin(f*x+e))^(1/2)/b/f/(b*sec(f*x
+e))^(1/2)-1/24*a^4*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF
(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f
/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2662, 2663, 2665, 2653, 2720}

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a^4\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{24b^2f\sqrt{a\sin(e+fx)}} - \frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/12*(a^3*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a*Sin[e +
 f*x])^(5/2))/(30*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(9/2)/(5*a*b
```

```
*f*Sqrt[b*Sec[e + f*x]]) + (a^4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e +
 f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(24*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx}{10b^2}$$

$$= -\frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^2\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx}{12b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{24b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{24b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{24b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{24b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$\int\frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}dx = -\frac{a^5\left(-4+17\cos(2(e+fx))-16\cos(4(e+fx))+3\cos(6(e+fx))-20\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2\right.\right.}{480bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/480*(a^5*(-4 + 17*Cos[2*(e + f*x)] - 16*Cos[4*(e + f*x)] + 3*Cos[6*(e +
f*x)] - 20*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^
2)^(3/4)))/(b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

method	result
default	$\frac{\sqrt{2}\,\sqrt{a\sin(fx+e)}\,a^3\left(12(\cos^4(fx+e))\sqrt{2}-22(\cos^2(fx+e))\sqrt{2}+5\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx}\right.}{}$

```
[In] int((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/f*2^(1/2)*(a*sin(f*x+e))^(1/2)*a^3/(b*sec(f*x+e))^(1/2)/b*(12*cos(f*x
+e)^4*2^(1/2)-22*cos(f*x+e)^2*2^(1/2)+5*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(c
ot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*csc(f*x+e)+5*(-cot(f*x+e)+csc(f*x+
e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*E
llipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)*csc(f*x+e
)+5*2^(1/2))
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e)/(b^2*sec(f*x + e)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\,\sin(e+f\,x))^{7/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2), x)
```

3.478 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] 1/3*(a*sin(f*x+e))^(5/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/6*a*(a*sin(f*x+e))^(1
/2)/b/f/(b*sec(f*x+e))^(1/2)-1/12*a^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4
*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*
x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2662, 2663, 2665, 2653, 2720}

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a^2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{12b^2f\sqrt{a\sin(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} - \frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(a*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])
^(5/2)/(3*a*b*f*Sqrt[b*Sec[e + f*x]]) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*S
qrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2663

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Dist[a^2*((m - 1)/(m - n)), Int[(a*Sin[e + f*x])^(m
 - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}+\frac{\int\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx}{6b^2}\\ &= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}+\frac{a^2\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{12b^2}\end{aligned}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{12b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{12b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int\frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}dx = \frac{a\sqrt{a\sin(e+fx)}\left(-2\cos(2(e+fx))+\csc^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\text{s}\right.\right.}{12bf\sqrt{b\sec(e+fx)}}$$

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sin[e + f*x]]*(-2*Cos[2*(e + f*x)] + Csc[e + f*x]^2*Hypergeometri
c2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(12*b*f*Sqrt[
b*Sec[e + f*x]])
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 1746, normalized size of antiderivative = 12.93

method	result	size
default	Expression too large to display	1746

```
[In] int((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/f*2^(1/2)*(-6*I*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+
e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e
```

```
)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)-6*I*(-cot(f*x+e)+csc(f*x+e)+1)
^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*Ellipt
icPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*I*(-cot(f*x+
e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+
e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2)
)-6*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f
*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(f*x+e)+8*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-c
sc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc
(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)+6*I*(-cot(f*x+e)+csc(f*x+e)+1)^(1/
2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi
((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)-6*(-cot
(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc
(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^
(1/2))*cos(f*x+e)+8*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)-6*(-cot(f*x+e)+csc(f*x+
e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*E
llipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*(-cot(f
*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f
*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-6*(-co
t(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-cs
c(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2
^(1/2))-4*2^(1/2)*cos(f*x+e)*sin(f*x+e)-3*ln(-2*2^(1/2)*(-sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+3*ln(2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*cos(f*x+e)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+6*arctan((2^(1/2)*(-sin(f*x+e)*c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*
(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-3*ln(-2*2^(1/2)*
(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)-2*2^(1/2)*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*ln(2*2^(1/2)*(-sin(f*x+e)*cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)+2*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)+2-2*cot(f*x+e))*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)+6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(a*sin(f*x+e))^(1/2)*a/(b*sec(f*x+e))^(1
/2)/b*sec(f*x+e)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e)/(b^2*sec(
f*x + e)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2), x)
```

3.479 $\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] (a*sin(f*x+e))^(1/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/2*(sin(e+1/4*Pi+f*x)^2)^(
1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^
(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2662, 2665, 2653, 2720}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}} + \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

```
[Out] Sqrt[a*Sin[e + f*x]]/(a*b*f*Sqrt[b*Sec[e + f*x]]) + (EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2662

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2b^2}\\
&= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{2b^2}\\
&= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{2b^2\sqrt{a\sin(e+fx)}}\\
&= \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2b^2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-1+\cos(2(e+fx))-\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)(-\tan^2(e+\right.}{2b^2f\sqrt{a\sin(e+fx)}}$$

[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]

[Out] -1/2*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Cos[2*(e + f*x)] - Hypergeome tric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(b^2*f*Sqr t[a*Sin[e + f*x]])

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.18

method	result
default	$\frac{\sqrt{2}\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+\sqrt{\right.}{2f\sqrt{b\sec}}$

[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/f*2^(1/2)/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/b*((-cot(f*x+e)+csc (f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1 /2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+(-cot(f*x+e)+cs c(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)+sin (f*x+e)*2^(1/2))

Fricas [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}\sqrt{a\sin(fx+e)}}\,dx$$

[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*b^2*sec(f*x + e)^2*si n(f*x + e)), x)

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin{(e+fx)}}\,(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*sin(e + f*x))*(b*sec(e + f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{(b\sec{(fx+e)})^{\frac{3}{2}}\sqrt{a\sin{(fx+e)}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{(b\sec{(fx+e)})^{\frac{3}{2}}\sqrt{a\sin{(fx+e)}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)), x)
```

3.480 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/3/a/b/f/(a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/2)+1/3*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2661, 2665, 2653, 2720}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{\sin(2e+2fx)}\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}} - \frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (EllipticF[e - P
i/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*b^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2661

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3a^2b^2}\\
&= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}\\
&\quad-\frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{3a^2b^2}\\
&= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}-\frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{3a^2b^2\sqrt{a\sin(e+fx)}}\\
&= -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}\\
&\quad-\frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(2+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{3/4}\right)}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/3*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(2 + Hypergeometric2F1[1/2, 3/4, 3/
2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(a^2*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.08

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+\right.}{3f\sqrt{b s}}$

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*2^(1/2)/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^2/b*((-cot(f*x+e
)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e
))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+(-cot(f*x+
e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+
e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e
)+2^(1/2)*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{i\,ab}\left(\cos(fx+e)^2-1\right)F(\arcsin(\cos(fx+e)+i\,\sin(fx+e))}{}$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/3*(sqrt(I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos(f*x + e) + I*si
n(f*x + e)), -1) + sqrt(-I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos(
f*x + e) - I*sin(f*x + e)), -1) + 2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e
))*cos(f*x + e))/(a^3*b^2*f*cos(f*x + e)^2 - a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{(a\,\sin\left(e+f\,x\right))^{5/2}\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)), x)
```

3.481 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}+\frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/7/a/b/f/(a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)+2/21/a^3/b/f/(a*sin(f*
x+e))^(3/2)/(b*sec(f*x+e))^(1/2)+2/21*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4
*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*
x+2*e)^(1/2)/a^4/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2661, 2664, 2665, 2653, 2720}

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = -\frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}} + \frac{2}{21a^3bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}} - \frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

```
[Out] -2/(7*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 2/(21*a^3*b*f*Sq
rt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (2*EllipticF[e - Pi/4 + f*x, 2
]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(21*a^4*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2661

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2b^2}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{21a^4b^2}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{21a^4b^2}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\left(2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{21a^4b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \frac{\cos(2(e+fx))\csc^4(e+fx)\sqrt{a\sin(e+fx)}\left((5+\cos(2(e+fx\right.}{21a^5bf\sqrt{b\mathrm{s}}}$$

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (Cos[2*(e + f*x)]*Csc[e + f*x]^4*Sqrt[a*Sin[e + f*x]]*((5 + Cos[2*(e + f*x)
])*Sec[e + f*x]^2 - 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^(7/4)))/(21*a^5*b*f*Sqrt[b*Sec[e + f*x]]*(-2 + Sec[e + f*x]^2
))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.69

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)-$

[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/21/f*2^(1/2)/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^4/b*(2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)+2^(1/2)*cot(f*x+e)^2*csc(f*x+e)+2*2^(1/2)*csc(f*x+e)^3)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \frac{2\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{i\,ab}F(\arcsin\left(\cos\left(fx-\right.\right.$$

[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")

[Out] 2/21*((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(I*a*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + (cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) - (cos(f*x + e)^3 + 2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)))/(a^5*b^2*f*cos(f*x + e)^4 - 2*a^5*b^2*f*cos(f*x + e)^2 + a^5*b^2*f)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{(b\sec\left(fx+e\right))^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{(b\sec\left(fx+e\right))^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{9/2}\,{\left(\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)), x)
```

3.482 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = & -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} \\ & +\frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}+\frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} \\ & -\frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}\end{aligned}$$

```
[Out] -2/11/a/b/f/(a*sin(f*x+e))^(11/2)/(b*sec(f*x+e))^(1/2)+2/77/a^3/b/f/(a*sin(
f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)+4/77/a^5/b/f/(a*sin(f*x+e))^(3/2)/(b*sec
(f*x+e))^(1/2)+4/77*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF
(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^6/b
^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2661, 2664, 2665, 2653, 2720\}$

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = -\frac{4\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}} + \frac{4}{77a^5bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}} + \frac{2}{77a^3bf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}} - \frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

```
[Out] -2/(11*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(11/2)) + 2/(77*a^3*b*f*
Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 4/(77*a^5*b*f*Sqrt[b*Sec[e +
 f*x]]*(a*Sin[e + f*x])^(3/2)) - (4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec
[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(77*a^6*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2661

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2664

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2665

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
```

```
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx}{11a^2b^2} \\
&= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} \\
&\quad + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{77a^4b^2} \\
&= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} \\
&\quad + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} \\
&\quad + \frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{4\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{77a^6b^2} \\
&= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} \\
&\quad + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} \\
&\quad + \frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} \\
&\quad - \frac{\left(4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx}{77a^6b^2} \\
&= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} \\
&\quad + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} \\
&\quad + \frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} \\
&\quad - \frac{\left(4\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{77a^6b^2\sqrt{a\sin(e+fx)}}
\end{aligned}
$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \frac{2\cot(2(e+fx))\csc(2(e+fx))\sqrt{a\sin(e+fx)}\Big((23+6\cos(2(}{}$$

[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]

[Out] (2*Cot[2*(e + f*x)]*Csc[2*(e + f*x)]*Sqrt[a*Sin[e + f*x]]*((23 + 6*Cos[2*(e + f*x)] - Cos[4*(e + f*x)])*Csc[e + f*x]^4 + 8*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(77*a^7*b*f*Sqrt[b*Sec[e + f*x]]*(-2 + Sec[e + f*x]^2))

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{2}\left(4\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x,method=_RETURNVERBOSE)

[Out] -1/77/f*2^(1/2)/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^6/b*(4*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+4*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*x+e)-2*2^(1/2)*cot(f*x+e)^4*csc(f*x+e)+5*2^(1/2)*cot(f*x+e)^2*csc(f*x+e)^3+4*2^(1/2)*csc(f*x+e)^5)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.32

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \frac{2\left(2\left(\cos\left(fx+e\right)^{6}-3\cos\left(fx+e\right)^{4}+3\cos\left(fx+e\right)^{2}-1\right)\sqrt{i a}\right.}{}$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="fricas
")
```

```
[Out] 2/77*(2*(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(I*a
*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + 2*(cos(f*x + e)
^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-I*a*b)*elliptic_f(arcsi
n(cos(f*x + e) - I*sin(f*x + e)), -1) - (2*cos(f*x + e)^5 - 5*cos(f*x + e)^
3 - 4*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)))/(a^7*b^2*f*c
os(f*x + e)^6 - 3*a^7*b^2*f*cos(f*x + e)^4 + 3*a^7*b^2*f*cos(f*x + e)^2 - a
^7*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^{\frac{13}{2}}}\,dx$$

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{(a\sin(e+fx))^{13/2}\left(\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

```
[In] int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)), x)
```

3.483 $\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+bx)}{bc(1+m)}$$

```
[Out] d*(cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 2657}

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{m+}{2}\right.}{bc(m+1)}$$

```
[In] Int[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(d^2(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int\frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$$

$$= \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))}{bc(1+m)}$$

Mathematica [A] (verified)

Time = 5.94 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int(d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx =$$

$$-\frac{2\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),\frac{1-m}{2},\frac{1}{4}(9-2m),\sec^2(a+bx)\right)(d\sec(a+bx))^{5/2}(c\sin(a+}{b(-5+2m)}$$

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-5 + 2*m))
```

Maple [F]

$$\int(d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(b*x+a))**(5/2)*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\left(\frac{d}{\cos(a+b\,x)}\right)^{5/2}\,dx$$

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2), x)
```

3.484 $\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}}{bc(1+m)}$$

```
[Out] d*(cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)*(d*sec(b*x+a))^(1/2)/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2667, 2657}

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{m+1}{2},\right.}{bc(m+1)}$$

```
[In] Int[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\right)\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$$

$$= \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] (verified)

Time = 5.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx =$$
$$-\frac{2\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m),\frac{1-m}{2},\frac{1}{4}(7-2m),\sec^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}}{b(-3+2m)}$$

```
[In] Integrate[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-3 + 2*m))
```

Maple [F]

$$\int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
[Out] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d*sec(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (c\,\sin\left(a+b\,x\right))^m\,\left(\frac{d}{\cos\left(a+b\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2), x)
```

3.485 $\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

```
[Out] (cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2666, 2657}

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[In] Int[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\text{integral} = \frac{\left((d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}$$

$$= \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{1-}}{bcd(1+m)}$$

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx =$$

$$-\frac{\csc^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),\frac{1-m}{2},\frac{1}{4}(5-2m),\sec^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+}{b(-1+2m)}$$

```
[In] Integrate[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
[Out] -((Csc[a + b*x]^2*Hypergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Se
c[a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m*Sin[2*(a + b*x)]*(-Ta
n[a + b*x]^2)^((1 - m)/2))/(b*(-1 + 2*m)))
```

Maple [F]

$$\int \sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m\,dx$$

```
[In] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
[Out] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m \, dx = \int \sqrt{d\sec(bx+a)}(c\sin(bx+a))^m \, dx$$

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m \, dx = \int (c\sin(a+bx))^m \sqrt{d\sec(a+bx)} \, dx$$

```
[In] integrate((d*sec(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*sec(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m \, dx = \int \sqrt{d\sec(bx+a)}(c\sin(bx+a))^m \, dx$$

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m \, dx = \int \sqrt{d\sec(bx+a)}(c\sin(bx+a))^m \, dx$$

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\,\sqrt{\frac{d}{\cos(a+b\,x)}}\,dx$$

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2), x)
```

3.486 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

```
[Out] (cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)*(d*sec(b*x+a))^(1/2)/b/c/d/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2666, 2657}

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
```

```
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\right)\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx}{d^2}$$

$$= \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

Mathematica [A] (verified)

Time = 18.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{4}(5+2m),\frac{3+m}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^{\frac{1}{4}+\frac{m}{2}}(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)\sqrt{d\sec(a+bx)}}$$

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (Hypergeometric2F1[(1 + m)/2, (5 + 2*m)/4, (3 + m)/2, -Tan[a + b*x]^2]*(Sec
[a + b*x]^2)^(1/4 + m/2)*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d
*Sec[a + b*x]])
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}dx$$

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d*sec(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(1/2),x)
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\,\sin\left(a+b\,x\right))^m}{\sqrt{\frac{d}{\cos(a+b\,x)}}}\,dx$$

```
[In] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2), x)
```

3.487 $\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

```
[Out] hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/
b/c/d/(1+m)/(cos(b*x+a)^2)^(1/4)/(d*sec(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2666, 2657}

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

```
[In] Int[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b
*x])^(1 + m))/(b*c*d*(1 + m)*(Cos[a + b*x]^2)^(1/4)*Sqrt[d*Sec[a + b*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx}{d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}} \\
&= \frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{2c\cos(2(a+bx))\,\text{Hypergeometric2F1}\left(\frac{1}{4}(-3-2m),\frac{1-m}{2},\frac{1}{4}(1-2m),\sec^2(a+bx\right.}{bd(3+2m)\sqrt{d\sec(a+bx)}\,(-2+\sec^2(a+}$$

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Cos[2*(a + b*x)]*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/
4, Sec[a + b*x]^2]*(c*Sin[a + b*x])^(-1 + m)*(-Tan[a + b*x]^2)^((1 - m)/2))
/(b*d*(3 + 2*m)*Sqrt[d*Sec[a + b*x]]*(-2 + Sec[a + b*x]^2))
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}dx$$

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d^2*sec(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**m/(d*sec(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{\left(\frac{d}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2), x)
```

3.488 $\int \sec^n(e+fx)\sin^m(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1-n)}$$

```
[Out] -hypergeom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+
n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2667, 2656}

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e
 + f*x]^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1
- n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = (\cos^n(e+fx)\sec^n(e+fx)) \int \cos^{-n}(e+fx)\sin^m(e+fx)\,dx$$

$$= -\frac{\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin^{-1+m}(e+fx)\sin^2(e+f}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.31

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+}{2}\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

```
[In] Integrate[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*Si
n[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1 + m
 - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]

$$\int (\sec^n(fx+e))(\sin^m(fx+e))\,dx$$

```
[In] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
[Out] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sec(fx+e)^n \sin(fx+e)^m\,dx$$

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sin^m(e+fx)\sec^n(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)**n*sin(f*x+e)**m,x)
[Out] Integral(sin(e + f*x)**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sec(fx+e)^n \sin(fx+e)^m\,dx$$

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx=\int \sec(fx+e)^n\sin(fx+e)^m\,dx$$

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx=\int \sin(e+f\,x)^m\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int(sin(e + f*x)^m*(1/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^m*(1/cos(e + f*x))^n, x)
```

3.489 $\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)(a\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1-n)}$$

```
[Out] -a*hypergeom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-
1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 2656}

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec
[e + f*x]^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = (\cos^n(e+fx)\sec^n(e+fx))\int \cos^{-n}(e+fx)(a\sin(e+fx))^m\,dx$$

$$= -\frac{a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)(a\sin(e+fx))^{-1+m}\sin^2(e}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

```
[In] Integrate[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*(a
*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)\right)^{m}dx$$

```
[In] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^{n}(e+fx)(a\sin(e+fx))^{m}\,dx=\int \left(a\sin\left(fx+e\right)\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^{n}(e+fx)(a\sin(e+fx))^{m}\,dx=\int \left(a\sin\left(e+fx\right)\right)^{m}\sec^{n}\left(e+fx\right)dx$$

```
[In] integrate(sec(f*x+e)**n*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^{n}(e+fx)(a\sin(e+fx))^{m}\,dx=\int \left(a\sin\left(fx+e\right)\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \sec(fx+e)^n\,dx$$

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \int (a\,\sin(e+f\,x))^m \left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.490 $\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1-n)}$$

```
[Out] -b*hypergeom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e)
)^(-1+n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2667, 2656}

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = -\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -((b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*
Sec[e + f*x])^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^m(e+fx)\,dx$$

$$= -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin^{-1+m}(e+fx)\sin^2(e}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.41 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int (b\sec(e+fx))^n\sin^m(e+fx)\,dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2}\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*Sin[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]

$$\int (b\sec (fx+e))^n (\sin^m (fx+e))\, dx$$

[In] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)

[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = \int (b\sec (fx+e))^n \sin (fx+e)^m \, dx$$

[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e))^n*sin(f*x + e)^m, x)

Sympy [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = \int (b\sec (e+fx))^n \sin^m (e+fx)\, dx$$

[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**m,x)

[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**m, x)

Maxima [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = \int (b\sec (fx+e))^n \sin (fx+e)^m \, dx$$

[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)

Giac [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^m\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \int \sin(e+f\,x)^m \left(\frac{b}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int(sin(e + f*x)^m*(b/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^m*(b/cos(e + f*x))^n, x)
```

3.491 $\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = -\frac{ab\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}(a\sin(e+fx))^{-1+m}\sin^2(e+f}{f(1-n)}$$

```
[Out] -a*b*hypergeom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+
e))^(-1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 2656}

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = -\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+f\right.}{f(1-n)}$$

```
[In] Int[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(
b*Sec[e + f*x])^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m
)/2))/(f*(1 - n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}(a\sin(e+fx))^m\,dx$$

$$= -\frac{ab\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}(a\sin(e+fx))^{-1+m}\,\text{si}}{f(1-n)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.14

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx$$

$$= \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*(a*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n,
 (3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4
*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]

$$\int (b\sec (fx+e))^{n} (a\sin (fx+e))^{m} dx$$

```
[In] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
[Out] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\sec (fx+e))^{n} (a\sin (fx+e))^{m}\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (a\sin (e+fx))^{m} (b\sec (e+fx))^{n}\,dx$$

```
[In] integrate((b*sec(f*x+e))**n*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\sec (fx+e))^{n} (a\sin (fx+e))^{m}\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\sec(fx+e))^n\,(a\sin(fx+e))^m\;dx$$

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (a\,\sin(e+f\,x))^m\,\left(\frac{b}{\cos(e+f\,x)}\right)^n\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n, x)
```

3.492 $\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = -\frac{b^5(b\sec(e+fx))^{-5+n}}{f(5-n)} + \frac{2b^3(b\sec(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^5*(b*sec(f*x+e))^(-5+n)/f/(5-n)+2*b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(b
*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 276}

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = -\frac{b^5(b\sec(e+fx))^{n-5}}{f(5-n)} + \frac{2b^3(b\sec(e+fx))^{n-3}}{f(3-n)} - \frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

```
[Out] -((b^5*(b*Sec[e + f*x])^(-5 + n))/(f*(5 - n))) + (2*b^3*(b*Sec[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^5\text{Subst}\left(\int x^{-6+n}\left(-1+\frac{x^2}{b^2}\right)^2\,dx,x,b\sec(e+fx)\right)}{f}\\
&= \frac{b^5\text{Subst}\left(\int\left(x^{-6+n}-\frac{2x^{-4+n}}{b^2}+\frac{x^{-2+n}}{b^4}\right)\,dx,x,b\sec(e+fx)\right)}{f}\\
&= -\frac{b^5(b\sec(e+fx))^{-5+n}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int(b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{b(89-28n+3n^2-4(7-8n+n^2)\cos(2(e+fx))+(3-4n+n^2)\cos(4(e+fx)))\,(b\sec(e+fx))^{-1+n}}{8f(-5+n)(-3+n)(-1+n)}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
[Out] (b*(89 - 28*n + 3*n^2 - 4*(7 - 8*n + n^2)*Cos[2*(e + f*x)] + (3 - 4*n + n^2
)*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(8*f*(-5 + n)*(-3 + n)*(-1 +
 n))
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{\left(\left(-\frac{3}{2}n^{2}+14n-\frac{25}{2}\right)\cos\left(3fx+3e\right)+\left(\frac{1}{2}n^{2}-2n+\frac{3}{2}\right)\cos\left(5fx+5e\right)+\cos\left(fx+e\right)\left(n^{2}-12n+75\right)\right)\left(\frac{b}{\cos(fx+e)}\right)^{n}}{8\left(n^{3}-9n^{2}+23n-15\right)f}$	89
default	$\frac{\cos\left(fx+e\right)\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f\left(-1+n\right)}-\frac{2\left(\cos^{3}\left(fx+e\right)\right)\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f\left(-3+n\right)}+\frac{\left(\cos^{5}\left(fx+e\right)\right)\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f\left(-5+n\right)}$	94

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
[Out] 1/8*((-3/2*n^2+14*n-25/2)*cos(3*f*x+3*e)+(1/2*n^2-2*n+3/2)*cos(5*f*x+5*e)+c
os(f*x+e)*(n^2-12*n+75))*(b/cos(f*x+e))^n/(n^3-9*n^2+23*n-15)/f
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = \frac{\left((n^2-4\,n+3)\cos\left(fx+e\right)^5-2\,(n^2-6\,n+5)\cos\left(fx+e\right)^3+(n^2-8\,n+15)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos(fx+e)}\right)^n}{fn^3-9\,fn^2+23\,fn-15\,f}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="fricas")
[Out] ((n^2 - 4*n + 3)*cos(f*x + e)^5 - 2*(n^2 - 6*n + 5)*cos(f*x + e)^3 + (n^2 -
 8*n + 15)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^3 - 9*f*n^2 + 23*f*n - 15*
f)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{\frac{b^n\cos(fx+e)^{-n}\cos(fx+e)^5}{n-5} - \frac{2\,b^n\cos(fx+e)^{-n}\cos(fx+e)^3}{n-3} + \frac{b^n\cos(fx+e)^{-n}\cos(fx+e)}{n-1}}{f}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="maxima")
[Out] (b^n*cos(f*x + e)^(-n)*cos(f*x + e)^5/(n - 5) - 2*b^n*cos(f*x + e)^(-n)*cos
(f*x + e)^3/(n - 3) + b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(n - 1))/f
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \int (b\sec(fx+e))^n\sin(fx+e)^5\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n\left(150\cos(e+f\,x) - 25\cos(3\,e+3\,f\,x) + 3\cos(5\,e+5\,f\,x) - 24\,n\cos(e+f\,x) + 28\,n\cos\right.}{16\,f\,\left(n^3-9\,n^2\right.}$$

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^n,x)
[Out] ((b/cos(e + f*x))^n*(150*cos(e + f*x) - 25*cos(3*e + 3*f*x) + 3*cos(5*e + 5
*f*x) - 24*n*cos(e + f*x) + 28*n*cos(3*e + 3*f*x) - 4*n*cos(5*e + 5*f*x) +
2*n^2*cos(e + f*x) - 3*n^2*cos(3*e + 3*f*x) + n^2*cos(5*e + 5*f*x)))/(16*f*
(23*n - 9*n^2 + n^3 - 15))
```

3.493 $\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \frac{b^3(b\sec(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 14}

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \frac{b^3(b\sec(e+fx))^{n-3}}{f(3-n)} - \frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] (b^3*(b*Sec[e + f*x])^(-3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))
/(f*(1 - n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b^3\text{Subst}\left(\int x^{-4+n}\left(-1+\frac{x^2}{b^2}\right)\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3\text{Subst}\left(\int\left(-x^{-4+n}+\frac{x^{-2+n}}{b^2}\right)\,dx,x,b\sec(e+fx)\right)}{f}\\ &= \frac{b^3(b\sec(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx = -\frac{b(5-n+(-1+n)\cos(2(e+fx)))(b\sec(e+fx))^{-1+n}}{2f(-3+n)(-1+n)}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
[Out] -1/2*(b*(5 - n + (-1 + n)*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(f*(
-3 + n)*(-1 + n))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{((1-n)\cos(3fx+3e)+\cos(fx+e)(n-9))\left(\frac{b}{\cos(fx+e)}\right)^n}{4(-3+n)f(-1+n)}$	54
default	$\frac{\cos(fx+e)e^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f(-1+n)}-\frac{(\cos^3(fx+e))e^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f(-3+n)}$	63

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*((1-n)*cos(3*f*x+3*e)+cos(f*x+e)*(n-9))*(b/cos(f*x+e))^n/(-3+n)/f/(-1+n
)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx = -\frac{\left((n-1)\cos\left(fx+e\right)^3-(n-3)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos(fx+e)}\right)^n}{fn^2-4\,fn+3\,f}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -((n - 1)*cos(f*x + e)^3 - (n - 3)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^2
- 4*f*n + 3*f)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx = -\frac{\frac{b^n\cos(fx+e)^{-n}\cos(fx+e)^3}{n-3}-\frac{b^n\cos(fx+e)^{-n}\cos(fx+e)}{n-1}}{f}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -(b^n*cos(f*x + e)^(-n)*cos(f*x + e)^3/(n - 3) - b^n*cos(f*x + e)^(-n)*cos(
f*x + e)/(n - 1))/f
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^3\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx$$
$$= -\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n\,(9\,\cos(e+f\,x)-\cos(3\,e+3\,f\,x)-n\,\cos(e+f\,x)+n\,\cos(3\,e+3\,f\,x))}{4\,f\,(n^2-4\,n+3)}$$

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^n,x)
```

```
[Out] -((b/cos(e + f*x))^n*(9*cos(e + f*x) - cos(3*e + 3*f*x) - n*cos(e + f*x) +
n*cos(3*e + 3*f*x)))/(4*f*(n^2 - 4*n + 3))
```

3.494 $\int (b\sec(e+fx))^n \sin(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = -\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2702, 30}

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = -\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] -((b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{b\text{Subst}\left(\int x^{-2+n}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \frac{b(b\sec(e+fx))^{-1+n}}{f(-1+n)}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
[Out] (b*(b*Sec[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{\cos(fx+e)\left(\frac{b}{\cos(fx+e)}\right)^n}{f(-1+n)}$
derivativedivides	$\frac{e^{n\ln(b\sec(fx+e))}}{f(-1+n)\sec(fx+e)}$
default	$\frac{e^{n\ln(b\sec(fx+e))}}{f(-1+n)\sec(fx+e)}$
norman	$\frac{\frac{e^{n\ln\left(\frac{b\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1-\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}}{f(-1+n)}-\frac{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)e^{n\ln\left(\frac{b\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1-\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}}{f(-1+n)}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$
risch	$\left(e^{2i(fx+e)}+1\right)^{-n}\cos(fx+e)\left(e^{i(fx+e)}\right)^n 2^n b^n e^{i\pi n\left(-\operatorname{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+\operatorname{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2\operatorname{csgn}(ib)+\operatorname{csgn}\left(\frac{ib\,e^{i(fx-}}{e^{2i(fx+e}}\right.\right.}$

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/f/(-1+n)*cos(f*x+e)*(b/cos(f*x+e))^n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\frac{\left(\frac{b}{\cos(fx+e)}\right)^n\cos(fx+e)}{fn-f}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="fricas")
[Out] (b/cos(f*x + e))^n*cos(f*x + e)/(f*n - f)
```

Sympy [F]

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\int (b\sec(e+fx))^n\sin(e+fx)\,dx$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e),x)
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\frac{b^n\cos(fx+e)^{-n}\cos(fx+e)}{f(n-1)}$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="maxima")
[Out] b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(f*(n - 1))
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \frac{\cos\left(e+f\,x\right)\,\left(\frac{b}{\cos(e+f\,x)}\right)^n}{f\,\left(n-1\right)}$$

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^n,x)
[Out] (cos(e + f*x)*(b/cos(e + f*x))^n)/(f*(n - 1))
```

3.495 $\int \csc(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{1+n}}{bf(1+n)}$$

```
[Out] -hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(1+n)/b/
f/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2702, 371}

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = -\frac{(b\sec(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\sec^2(e+fx)\right)}{bf(n+1)}$$

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*
x])^(1 + n))/(b*f*(1 + n)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^{1+n}}{bf(1+n)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \sec^2(e+fx)\right)\sec(e+fx)(b\sec(e+fx))^n}{f(1+n)}$$

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
[Out] -((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]*
(b*Sec[e + f*x])^n)/(f*(1 + n)))
```

Maple [F]

$$\int \csc(fx+e)\,(b\sec(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
[Out] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n \csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n \csc(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**n,x)
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n \csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n \csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x),x)
[Out] int((b/cos(e + f*x))^n/sin(e + f*x), x)
```

3.496 $\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{3+n}}{b^3f(3+n)}$$

```
[Out] hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(3+n)/b^3
/f/(3+n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2702, 371}

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \frac{(b\sec(e+fx))^{n+3}\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},\sec^2(e+fx)\right)}{b^3f(n+3)}$$

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x]
)^(3 + n))/(b^3*f*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= \frac{\text{Hypergeometric2F1}\left(2, \frac{3+n}{2}, \frac{5+n}{2}, \sec^2(e+fx)\right)(b\sec(e+fx))^{3+n}}{b^3 f(3+n)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(48) = 96.

Time = 2.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.19

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$$

$$= \frac{b(b\sec(e+fx))^{-1+n}\left(2\,\text{Hypergeometric2F1}(1,1-n,2-n,\cos(e+fx)) + 2\,\text{Hypergeometric2F1}(2,1-\right.}{}$$

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (b*(b*Sec[e + f*x])^(-1 + n)*(2*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[e +
f*x]] + 2*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[e + f*x]] + 2^n*Hypergeome
tric2F1[1 - n, -n, 2 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2]*(Sec[(e + f*
x)/2]^2)^(1 - n) + 2^n*Hypergeometric2F1[1 - n, 1 - n, 2 - n, (Cos[e + f*x]
*Sec[(e + f*x)/2]^2)/2]*Sec[e + f*x]^(1 - n)*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(-1 + n)))/(8*f*(-1 + n))
```

Maple [F]

$$\int \left(\csc^3(fx+e)\right)(b\sec(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
[Out] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n \csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n \csc^3(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**n,x)
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n \csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin(e+f\,x)^3}\,dx$$

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^3,x)
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^3, x)
```

3.497 $\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = -\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

```
[Out] -((b*Hypergeometric2F1[-5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int(b\cos(e+fx))^{-n}\sin^6(e+fx)\,dx$$
$$= -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int(b\sec(e+fx))^n\sin^6(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{7}{2},4-\frac{n}{2},\frac{9}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^7(e+fx)}{7f}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
[Out] (Hypergeometric2F1[7/2, 4 - n/2, 9/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*T
an[e + f*x]^7)/(7*f*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int(b\sec(fx+e))^n\left(\sin^6(fx+e)\right)dx$$

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*(b*sec
(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx=\text{Timed out}$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**6,x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^6\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^6(e+fx)\,dx=\int \sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^n, x)
```

3.498 $\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = -\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] -((b*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int(b\cos(e+fx))^{-n}\sin^4(e+fx)\,dx$$
$$= -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int(b\sec(e+fx))^n\sin^4(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{5}{2},3-\frac{n}{2},\frac{7}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^5(e+fx)}{5f}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
[Out] (Hypergeometric2F1[5/2, 3 - n/2, 7/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*T
an[e + f*x]^5)/(5*f*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int(b\sec(fx+e))^n\left(\sin^4(fx+e)\right)dx$$

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**4,x)
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec(fx+e))^n\sin(fx+e)^4\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\, dx = \int \sin(e+f\,x)^4 \left(\frac{b}{\cos(e+f\,x)}\right)^n\, dx$$

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^n, x)
```

3.499 $\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = -\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^2(e+fx)\,dx$$

$$= -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(\frac{3}{2},2-\frac{n}{2},\frac{5}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^3(e+fx)}{3f}$$

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
[Out] (Hypergeometric2F1[3/2, 2 - n/2, 5/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*T
an[e + f*x]^3)/(3*f*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int (b\sec(fx+e))^n\left(\sin^2(fx+e)\right)dx$$

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(e+fx))^n \sin^2(e+fx)\,dx$$

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**2,x)
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^2\,dx$$

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^n, x)
```

3.500 $\int (b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (b\sec(e+fx))^n\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1+
n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3857, 2722}

$$\int (b\sec(e+fx))^n\,dx = -\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
+ f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(\frac{\cos(e+fx)}{b}\right)^n (b\sec(e+fx))^n \int \left(\frac{\cos(e+fx)}{b}\right)^{-n} dx$$

$$= -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^n\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (b\sec(e+fx))^n\,dx$$

$$= \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^n\sqrt{-\tan^2(e+fx)}}{fn}$$

```
[In] Integrate[(b*Sec[e + f*x])^n,x]
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*(b*Sec
[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n)
```

Maple [F]

$$\int (b\sec(fx+e))^n\,dx$$

```
[In] int((b*sec(f*x+e))^n,x)
[Out] int((b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

[In] integrate((b*sec(f*x+e))^n,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e))^n, x)

Sympy [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n\,dx$$

[In] integrate((b*sec(f*x+e))**n,x)

[Out] Integral((b*sec(e + f*x))**n, x)

Maxima [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

[In] integrate((b*sec(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^n, x)

Giac [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

[In] integrate((b*sec(f*x+e))^n,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e))^n, x)

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\,dx=\int \left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

```
[In] int((b/cos(e + f*x))^n,x)
[Out] int((b/cos(e + f*x))^n, x)
```

3.501 $\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}$$

```
[Out] -b*csc(f*x+e)*hypergeom([3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f
*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2712, 2656}

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int(b\cos(e+fx))^{-n}\csc^2(e+fx)\,dx$$
$$= -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx}}{f(1-n)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int\csc^2(e+fx)(b\sec(e+fx))^n\,dx =$$
$$-\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{n}{2},\frac{1}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}}{f}$$

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -1/2*n, 1/2, -Tan[e + f*x]^2]*(b*Se
c[e + f*x])^n)/(f*(Sec[e + f*x]^2)^(n/2)))
```

Maple [F]

$$\int\left(\csc^2(fx+e)\right)(b\sec(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
[Out] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Sympy [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(e+fx))^n\csc^2(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**n,x)
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin\left(e+f\,x\right)^2}\,dx$$

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^2,x)
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^2, x)
```

3.502 $\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}$$

```
[Out] -b*csc(f*x+e)*hypergeom([5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f
*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int(b\cos(e+fx))^{-n}\csc^4(e+fx)\,dx$$
$$= -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx}}{f(1-n)}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int\csc^4(e+fx)(b\sec(e+fx))^n\,dx =$$
$$-\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-1-\frac{n}{2},-\frac{1}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}}{3f}$$

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
[Out] -1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -1 - n/2, -1/2, -Tan[e + f*x]^
2]*(b*Sec[e + f*x])^n)/(f*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]

$$\int\left(\csc^4(fx+e)\right)(b\sec(fx+e))^n\,dx$$

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
[Out] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Sympy [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n\csc^4(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**n,x)
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**4, x)
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^4\,dx$$

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin\left(e+f\,x\right)^4}\,dx$$

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^4, x)
```

3.503 $\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt{c\sin(a+bx)}}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -c*hypergeom([-1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1
+n)*(c*sin(b*x+a))^(1/2)/(1-n)/(sin(b*x+a)^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 2656}

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c\sqrt{c\sin(a+bx)}(b\sec(a+bx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[In] Int[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*Sqrt[c*Sin[a + b*x]])/((1 - n)*(Sin[a + b*x]^2)^(1/4)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int(b\cos(a+bx))^{-n}(c\sin(a+bx))^{3/2}\,dx$$

$$= -\frac{c\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt{c\sin(a+bx)}}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 31.91 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int(b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \frac{2\cos^2(a+bx)^{\frac{1}{2}(-1+n)}(b\sec(a+bx))^{-1+n}(c\sin(a+bx))^{5/2}\left(9\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{2}(-1+\right.\right.}{45c}$$

```
[In] Integrate[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
[Out] (2*(Cos[a + b*x]^2)^((-1 + n)/2)*(b*Sec[a + b*x])^(-1 + n)*(c*Sin[a + b*x])
^(5/2)*(9*Hypergeometric2F1[5/4, (-1 + n)/2, 9/4, Sin[a + b*x]^2] + 5*Hyper
geometric2F1[9/4, (1 + n)/2, 13/4, Sin[a + b*x]^2]*Sin[a + b*x]^2))/(45*c)
```

Maple [F]

$$\int (b \sec (bx+a))^n (c \sin (bx+a))^{\frac{3}{2}} dx$$

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int (b \sec(a+bx))^n (c \sin(a+bx))^{3/2} dx = \int (c \sin (bx+a))^{\frac{3}{2}} (b \sec (bx+a))^n dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b \sec(a+bx))^n (c \sin(a+bx))^{3/2} dx = \text{Timed out}$$

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b \sec(a+bx))^n (c \sin(a+bx))^{3/2} dx = \int (c \sin (bx+a))^{\frac{3}{2}} (b \sec (bx+a))^n dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

Giac [F]

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\sin(bx+a))^{\frac{3}{2}}\,(b\sec(bx+a))^n\,\,dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\,\sin(a+b\,x))^{3/2}\,\left(\frac{b}{\cos(a+b\,x)}\right)^n\,dx$$

```
[In] int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n,x)
[Out] int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n, x)
```

3.504 $\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -c*hypergeom([1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+
n)*(sin(b*x+a)^2)^(1/4)/(1-n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 2656}

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

```
[In] Int[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/((1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int (b\cos(a+bx))^{-n}\sqrt{c\sin(a+bx)}\,dx$$

$$= -\frac{c\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{(1-n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 10.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$$

$$= \frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{7}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\sin(2(a+}{3b}$$

```
[In] Integrate[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[3/4, (1 + n)/2, 7/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]]*Sin[2*(a + b*x)])/(3*b)
```

Maple [F]

$$\int (b\sec(bx+a))^n\sqrt{c\sin(bx+a)}dx$$

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\sin(bx+a)}(b\sec(bx+a))^n\,dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Sympy [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$$

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
[Out] Integral((b*sec(a + b*x))**n*sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\sin(bx+a)}(b\sec(bx+a))^n\,dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Giac [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\sin(bx+a)}(b\sec(bx+a))^n\,dx$$

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\,\sin\left(a+b\,x\right)}\,\left(\frac{b}{\cos\left(a+b\,x\right)}\right)^n\,dx$$

```
[In] int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n,x)
[Out] int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n, x)
```

3.505 $\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sin^2(a+bx)^{3/4}}{(1-n)(c\sin(a+bx))^{3/2}}$$

```
[Out] -c*hypergeom([3/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+
n)*(sin(b*x+a)^2)^(3/4)/(1-n)/(c*sin(b*x+a))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2667, 2656}

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

```
[In] Int[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(3/4))/((1 - n)*(c*Sin[a + b*x])^(3/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
```

```
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int\frac{(b\cos(a+bx))^{-n}}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= -\frac{c\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sin^2(a+bx)^{3/4}}{(1-n)(c\sin(a+bx))^{3/2}}$$

Mathematica [A] (verified)

Time = 10.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int\frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{5}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sin(2(a+bx))}{b\sqrt{c\sin(a+bx)}}$$

```
[In] Integrate[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[1/4, (1 + n)/2, 5/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*Sqrt[c*Sin[a + b*x]])
```

Maple [F]

$$\int\frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}dx$$

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$$

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
[Out] Integral((b*sec(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

```
[In] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
[Out] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

3.506 $\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{c(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -hypergeom([5/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+n)
*(sin(b*x+a)^2)^(1/4)/c/(1-n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2667, 2656}

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

```
[In] Int[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a +
b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/(c*(1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int\frac{(b\cos(a+bx))^{-n}}{(c\sin(a+bx))^{3/2}}\,dx$$

$$= -\frac{\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{c(1-n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] (verified)

Time = 10.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int\frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sin(2(a+bx))}{b(c\sin(a+bx))^{3/2}}$$

```
[In] Integrate[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
[Out] -(((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[-1/4, (1 + n)/2, 3/4, Si
n[a + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*(c*Sin[a + b*x])^(3/2
)))
```

Maple [F]

$$\int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}dx$$

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

Sympy [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
[Out] Integral((b*sec(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{(c\,\sin(a+b\,x))^{3/2}}\,dx$$

```
[In] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
[Out] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

3.507 $\int \sqrt{d \csc(e+fx)} \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \sqrt{d \csc(e+fx)} \sin^4(e+fx)\,dx = -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}$$

```
[Out] -2/7*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d*cos(f*x+e)/f/(d*csc(f*x+
e))^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*
x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int \sqrt{d \csc(e+fx)} \sin^4(e+fx)\,dx = -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d*Cos[e + f*x])/(2
1*f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 +
f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^4 \int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}\left(5d^2\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{5}{21}\int \sqrt{d\csc(e+fx)}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{1}{21}\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx =$$
$$-\frac{\sqrt{d\csc(e+fx)}\Big(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+fx}{84f}$$

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.52

method	result
default	$\frac{\sqrt{2}\left(5i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\sqrt{i(-\cot(fx+e)+\csc(fx+e))}F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx}\right.\right.}{}$

```
[In] int(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/f*2^(1/2)*(5*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-
csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(
f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+3*2^(1/2)*cos(f*x+e)^3*si
n(f*x+e)+5*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x
+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2),1/2*2^(1/2))-8*2^(1/2)*cos(f*x+e)*sin(f*x+e))*(d*csc(f*x
+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx$$
$$=\frac{2\left(3\cos(fx+e)^3-8\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin(fx+e)-5i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+e}{21f}$$

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e
)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e)))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc{(e+fx)}}\sin^4{(e+fx)}\,dx$$

```
[In] integrate(sin(f*x+e)**4*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc(fx+e)}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc(fx+e)}\sin(fx+e)^4\,dx$$

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sin\left(e+f\,x\right)^4\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2), x)
```

3.508 $\int \sqrt{d \csc(e+fx)} \sin^3(e+fx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx) \, dx = -\frac{2d^2 \cos(e+fx)}{5f(d \csc(e+fx))^{3/2}} + \frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d \csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2719}

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx) \, dx = \frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d \csc(e+fx)}} - \frac{2d^2 \cos(e+fx)}{5f(d \csc(e+fx))^{3/2}}$$

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d*EllipticE[(e - Pi
/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^3 \int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{1}{5}(3d)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{(3d)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = -\frac{2\sqrt{d\csc(e+fx)}\left(3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sqrt{\sin(e+fx)}+\cos(e+fx)\sin^2(e+fx)\right)}{5f}$$

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
+ f*x]] + Cos[e + f*x]*Sin[e + f*x]^2))/(5*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.31

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.}{}$

[In] int(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+(3*cos(f*x+e)+3)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))*(d*csc(f*x+e))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(f\right.}{}$$

[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

[In] integrate(sin(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx = \int \sqrt{d \csc(fx+e)} \sin(fx+e)^3\, dx$$

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx = \int \sqrt{d \csc(fx+e)} \sin(fx+e)^3\, dx$$

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx = \int \sin(e+f\,x)^3 \sqrt{\frac{d}{\sin(e+f\,x)}}\, dx$$

```
[In] int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2), x)
```

3.509 $\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

```
[Out] -2/3*d*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (-2*d*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^2\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx\\ &= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx\\ &= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\ &= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f}$$

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
[Out] -1/3*(Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[
e + f*x]] + Sin[2*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.24

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx\right.\right.}{}$

```
[In] int(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*2^(1/2)*(I*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f
*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(
-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)+I*(-I*(I+cot(f*x+e)-csc(f*x+
e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)-2^(1/2)
*cos(f*x+e)*sin(f*x+e))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx =$$
$$-\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))\ -}{3f}$$

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*weie
rstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*weier
strassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx$$

```
[In] integrate(sin(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2), x)
```

3.510 $\int \sqrt{d \csc(e+fx)} \sin(e+fx)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx)\, dx = \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3856, 2719}

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx)\, dx = \frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = -\frac{2dE\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
[Out] (-2*d*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 314, normalized size of antiderivative = 7.14

method	result
risch	$-\frac{\left(e^{2i(fx+e)}-1\right)\sqrt{2}\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}\,e^{-i(fx+e)}}{f}+\frac{\left(-\frac{2i\left(id\,e^{2i(fx+e)}-id\right)}{d\sqrt{e^{i(fx+e)}\left(id\,e^{2i(fx+e)}-id\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\sqrt{-2\,e^{i(fx+e)}+2}\sqrt{-e^{i(fx+e)}}\left(-2E\left(\cdots\right.\right.}{\sqrt{id\,e^{3i(fx+e)}-i\cdots}}\right.}{\cdots}$
default	$-\frac{\sqrt{2}\sqrt{d\csc(fx+e)}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot\cdots}\right.\right.}{\cdots}$

```
[In] int(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(exp(I*(f*x+e))^2-1)/f*2^(1/2)*(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))^(
1/2)/exp(I*(f*x+e))+1/f*(-2*I*(I*d*exp(I*(f*x+e))^2-I*d)/d/(exp(I*(f*x+e))*
(I*d*exp(I*(f*x+e))^2-I*d))^(1/2)-(exp(I*(f*x+e))+1)^(1/2)*(-2*exp(I*(f*x+e
))+2)^(1/2)*(-exp(I*(f*x+e)))^(1/2)/(I*d*exp(I*(f*x+e))^3-I*d*exp(I*(f*x+e)
))^(1/2)*(-2*EllipticE((exp(I*(f*x+e))+1)^(1/2),1/2*2^(1/2))+EllipticF((exp
(I*(f*x+e))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*d*exp(I*(f*x+e))/(exp(I*(f*x
+e))^2-1))^(1/2)*(I*d*exp(I*(f*x+e))*(exp(I*(f*x+e))^2-1))^(1/2)/exp(I*(f*x
+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx) \, dx$$
$$= \frac{\sqrt{2i\,d} \text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{-2i\,d} \text{weierstrassZ}}{f}$$

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) +
 I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F]

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx) \, dx = \int \sqrt{d \csc{(e+fx)}} \sin{(e+fx)} \, dx$$

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x), x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)\,dx$$

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)\,dx$$

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

```
[In] int(sin(e + f*x)*(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)*(d/sin(e + f*x))^(1/2), x)
```

3.511 $\int \sqrt{d \csc(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \sqrt{d \csc(e+fx)}\,dx = \frac{2\sqrt{d \csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2720}

$$\int \sqrt{d \csc(e+fx)}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d \csc(e+fx)}}{f}$$

```
[In] Int[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/f
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\ &= \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sqrt{d\csc(e+fx)}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{f}$$

```
[In] Integrate[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

method	result
default	$\frac{i(\cos(fx+e)+1)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)\right.}{f}$

```
[In] int((d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/f*(cos(f*x+e)+1)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-c
sc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f
*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*2^(1/2)*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \sqrt{d\csc(e+fx)}\,dx = \frac{-i\sqrt{2i\,d}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{-2i\,d}\text{weierstrassPInverse}(4,0,\cos(}{f}$$

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(e+fx)}\,dx$$

```
[In] integrate((d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\,dx$$

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\,dx$$

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2\sqrt{\sin(e+f\,x)}\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|\,2\right)\,\sqrt{\frac{d}{\sin(e+f\,x)}}\,\sqrt{\cos(e+f\,x)^2}}{f\,\cos(e+f\,x)}$$

```
[In] int((d/sin(e + f*x))^(1/2),x)
[Out] -(2*sin(e + f*x)^(1/2)*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2)
, 2)*(d/sin(e + f*x))^(1/2)*(cos(e + f*x)^2)^(1/2))/(f*cos(e + f*x))
```

3.512 $\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/
(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {16, 3853, 3856, 2719}

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d*EllipticE[(e - Pi/2 + f*x)/
2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d}\\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{(d\csc(e+fx))^{3/2}\left(2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\,2\right)\sin^{\frac{3}{2}}(e+fx)-\sin(2(e+fx))\right)}{df}$$

```
[In] Integrate[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/(d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 409, normalized size of antiderivative = 6.01

method	result
default	$-\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\,\left(-2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\right.}$

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*(-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*
(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*Ell
ipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+(-I*(I-
cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot
(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1
/2*2^(1/2))*cos(f*x+e)-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*
x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+(-I*(I-cot(f*x+e)+csc(f*x+e)))^
(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/
2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)+\sqrt{2i\,d}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx}{f}$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))
))/f
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(e+fx\right)}\csc\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\,dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\,dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin\left(e+f\,x\right)}\,dx$$

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x),x)
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x), x)
```

3.513 $\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2720}

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}$$

```
[In] Int[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d*f) + (2*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^2}\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}\\
&\quad + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2(d\csc(e+fx))^{3/2}\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3df}$$

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4,
 2]*Sin[e + f*x]^(3/2)))/(3*d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.07

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(\right.\right.}{}$

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*(I*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*
(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))
^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)+I*(-I*(
I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*Elliptic
F((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f
*x+e)))^(1/2)-2^(1/2)*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{-i\sqrt{2i\,d}\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\sin(fx+e)\,\text{weie}}{3\,f\sin(fx+e)}$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) + I
*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(
f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(f*sin(f*
x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(e+fx)}\csc^2(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin(e+f\,x)^2}\,dx$$

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2,x)
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

3.514 $\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d^2/f-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/f+6/5*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+
e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2719}

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e + f
*x])^(5/2))/(5*d^2*f) - (6*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*
Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^3}\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}+\frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d}\\
&= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}\\
&\quad -\frac{1}{5}(3d)\int\frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\
&= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}\\
&\quad -\frac{(3d)\int\sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

$$= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f} - \frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\left(3\cos(e+fx)+\cot(e+fx)\csc(e+fx)-3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sqrt{\sin(e+fx)}\right)}{5f}$$

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*Cos[e + f*x] + Cot[e + f*x]*Csc[e + f*x] - 3*El
lipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]]))/(5*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.30

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\left(6\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\left(\sqrt{i(-i+\cot(\right.\right.}{}$

```
[In] int(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*(6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*
(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*Ell
ipticE((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-3*(I*(-
I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(c
ot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)
,1/2*2^(1/2))*cos(f*x+e)+6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(I+cot(
f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(
-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))-3*(I*(-I+cot(f*x+e)-csc(f*x+e
)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))
^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))-3*2^(1/2
)-2^(1/2)*cot(f*x+e)*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))}{}$$

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -1/5*(3*(cos(f*x + e)^2 - 1)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(cos(f*x + e)^2 - 1)*sqrt
(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*s
in(f*x + e))) + 2*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(d/sin(f*x + e)))
/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(e+fx\right)}\csc^3\left(e+fx\right)\,dx$$

```
[In] integrate(csc(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\csc(fx+e)^3\,dx$$

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin(e+f\,x)^3}\,dx$$

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3,x)
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

3.515 $\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}$$

```
[Out] -2/7*d^4*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d^2*cos(f*x+e)/f/(d*csc(f*
x+e))^(1/2)-10/21*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/
2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*si
n(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10d\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*d^4*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d^2*Cos[e + f*x])/
(21*f*Sqrt[d*Csc[e + f*x]]) + (10*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/
2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^5 \int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx \\
&= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}\left(5d^3\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{1}{21}(5d)\int \sqrt{d\csc(e+fx)}\,dx \\
&= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{1}{21}\left(5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx \\
&= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{10d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{d\sqrt{d\csc(e+fx)}\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+fx))\right)}{84f}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] -1/84*(d*Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[
Sin[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{2}\left(5i\sin(fx+e)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-\cot(fx+e)+\csc(fx+e))}F\left(\sqrt{-i(i-\cot(fx+\right.\right.}{}$

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/f*2^(1/2)*(5*I*sin(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-
cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF(
(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-3*cos(f*x+e)^4*2^(1/2)+3*
2^(1/2)*cos(f*x+e)^3+8*cos(f*x+e)^2*2^(1/2)-8*2^(1/2)*cos(f*x+e))*d*csc(f*x
+e)*(d*csc(f*x+e))^(1/2)*(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{2\left(3\,d\cos(fx+e)^3-8\,d\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin(fx+e)-5i\sqrt{2i\,d}d\text{weierstrassPInverse}(4,0,\ldots}{21\ldots}$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*d*cos(f*x + e)^3 - 8*d*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*
x + e) - 5*I*sqrt(2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f
*x + e)) + 5*I*sqrt(-2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) - I*si
n(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx=\text{Timed out}$$

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^5\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^5\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int \sin(e+f\,x)^5\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2), x)
```

3.516 $\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d^2*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d^2*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^4 \int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{1}{5}\left(3d^2\right)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\
&= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{\left(3d^2\right)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{2(d\csc(e+fx))^{3/2}\left(3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{3}{2}}(e+fx)+\cos(e+fx)\sin^3(e+fx)\right)}{5f}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f
*x]^(3/2) + Cos[e + f*x]*Sin[e + f*x]^3))/(5*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.23

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(}\right.}{}$

[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)

[Out] 1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+(3*cos(f*x+e)+3)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))*(d*csc(f*x+e))^(1/2)*d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3\sqrt{2i\,d}d\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))+3\sqrt{-2i\,d}}{}$$

[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")

[Out] 1/5*(3*sqrt(2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(-2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(d*cos(f*x + e)^3 - d*cos(f*x + e))*sqrt(d/sin(f*x + e)))/f

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

```
[In] int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2), x)
```

3.517 $\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx=-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2d\sqrt{d\csc(e+fx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

```
[Out] -2/3*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx=\frac{2d\sqrt{\sin(e+fx)}\,\operatorname{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*d*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^3 \int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}d\int\sqrt{d\csc(e+fx)}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.75

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = -\frac{d\sqrt{d\csc(e+fx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] -1/3*(d*Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + Sin[2*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.12

method	result
default	$\frac{\sqrt{2}\left(i\sin(fx+e)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e}\right.\right.}{3f}$

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*2^(1/2)*(I*sin(f*x+e)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f
*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2
*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+cos(f*x+e)^2*2^(1/2)-2^(1/2)
*cos(f*x+e))*d*csc(f*x+e)*(d*csc(f*x+e))^(1/2)*(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx =$$
$$-\frac{2\,d\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}d\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))-}{3\,f}$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(2*d*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*d*
weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*d
*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^3\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^3\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int \sin(e+f\,x)^3\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2), x)
```

3.518 $\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx=\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipt
icE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3856, 2719}

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx=\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^2 \int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= \frac{d^2 \int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2d^2 E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = -\frac{2d^2 E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
[Out] (-2*d^2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[S
in[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 316, normalized size of antiderivative = 6.87

method	result
risch	$-\frac{\left(e^{2i(fx+e)}-1\right)\sqrt{2}\,d\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}\,e^{-i(fx+e)}}{f} + \frac{\left(-\frac{2i\left(id\,e^{2i(fx+e)}-id\right)}{d\sqrt{e^{i(fx+e)}\left(id\,e^{2i(fx+e)}-id\right)}} - \frac{\sqrt{e^{i(fx+e)}+1}\,\sqrt{-2\,e^{i(fx+e)}+2}\,\sqrt{-e^{i(fx+e)}}\,(-2 \ldots}{\sqrt{id\,e^{3i(fx+e} \ldots}\right.}{\ldots}$
default	$-\frac{\sqrt{2}\,d\sqrt{d\csc(fx+e)}\,\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i- \ldots\right.\right.}{\ldots}$

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(exp(I*(f*x+e))^2-1)/f*2^(1/2)*d*(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))
^(1/2)/exp(I*(f*x+e))+1/f*(-2*I*(I*d*exp(I*(f*x+e))^2-I*d)/d/(exp(I*(f*x+e)
)*(I*d*exp(I*(f*x+e))^2-I*d))^(1/2)-(exp(I*(f*x+e))+1)^(1/2)*(-2*exp(I*(f*x
+e))+2)^(1/2)*(-exp(I*(f*x+e)))^(1/2)/(I*d*exp(I*(f*x+e))^3-I*d*exp(I*(f*x+
e)))^(1/2)*(-2*EllipticE((exp(I*(f*x+e))+1)^(1/2),1/2*2^(1/2))+EllipticF((e
xp(I*(f*x+e))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*d*(I*d*exp(I*(f*x+e))/(exp(I*
(f*x+e))^2-1))^(1/2)*(I*d*exp(I*(f*x+e))*(exp(I*(f*x+e))^2-1))^(1/2)/exp(I*
(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{2i\,d}d\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{-2i\,d}d\text{we}}{f}$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (sqrt(2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e)
 + I*sin(f*x + e))) + sqrt(-2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

```
[In] int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2), x)
```

3.519 $\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3856, 2720}

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2d\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{f}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x
]])/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int\sqrt{d\csc(e+fx)}\,dx \\
&= \left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx \\
&= \frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = -\frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{f}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
[Out] (-2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

method	result
default	$\frac{id(\cos(fx+e)+1)\sqrt{d\csc(fx+e)}\,\sqrt{2}\,\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F}{f}$

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
[Out] I*d/f*(cos(f*x+e)+1)*(d*csc(f*x+e))^(1/2)*2^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)
))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{-i\sqrt{2i\,d}d\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}d\text{weierstrassPInverse}(}{f}$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
[Out] (-I*sqrt(2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))
+ I*sqrt(-2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))
)/f
```

Sympy [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\sin(e+fx)\,dx$$

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e),x)
[Out] Integral((d*csc(e + f*x))**(3/2)*sin(e + f*x), x)
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int \sin(e+f\,x)\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

```
[In] int(sin(e + f*x)*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)*(d/sin(e + f*x))^(3/2), x)
```

3.520 $\int (d\csc(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int (d\csc(e+fx))^{3/2}\,dx = -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*d*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2719}

$$\int (d\csc(e+fx))^{3/2}\,dx = -\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}} - \frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

```
[In] Int[(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - d^2\int\frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\
&= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{d^2\int\sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\
&= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int(d\csc(e+fx))^{3/2}\,dx = \frac{(d\csc(e+fx))^{3/2}\left(2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{3}{2}}(e+fx)-\sin(2(e+fx))\right)}{f}$$

```
[In] Integrate[(d*Csc[e + f*x])^(3/2),x]
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.82

method	result
default	$\frac{\sqrt{2}\, d \sqrt{d \csc(fx+e)} \left(2\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\, \sqrt{i(-\cot(fx+e)+\csc(fx+e))}\, E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}, \frac{\sqrt{2}}{2}\right) \sqrt{-i(i-}\right.}{}$

```
[In] int((d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*2^(1/2)*d*(d*csc(f*x+e))^(1/2)*(2*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*
(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))
^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)-(I*(-I-
cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot
(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1
/2*2^(1/2))*cos(f*x+e)+2*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+
e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^
(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)-(I*(-I-cot(f*x+e)+csc(f*x+e)))^
(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/
2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int (d\csc(e+fx))^{3/2}\, dx =$$

$$-\frac{2\, d\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e) + \sqrt{2i\, d}\, d\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx}{f}$$

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*d*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*d*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)
*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e))))/f
```

Sympy [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*csc(f*x+e))**(3/2),x)
[Out] Integral((d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\,dx$$

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\,dx = \int \left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}\,dx$$

```
[In] int((d/sin(e + f*x))^(3/2),x)
[Out] int((d/sin(e + f*x))^(3/2), x)
```

3.521 $\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f} + \frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/f-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {16, 3853, 3856, 2720}

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{2d\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}$$

```
[In] Int[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d}\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{1}{3}d\int\sqrt{d\csc(e+fx)}\,dx\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx\\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}\\
&\quad+\frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = -\frac{(d\csc(e+fx))^{5/2}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{5}{2}}(e+fx)+\sin(2(e+fx))\right)}{3df}$$

```
[In] Integrate[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
[Out] -1/3*((d*Csc[e + f*x])^(5/2)*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e +
 f*x]^(5/2) + Sin[2*(e + f*x)]))/(d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.15

method	result
default	$-\frac{\sqrt{2}\,d\sqrt{d\csc(fx+e)}\left(-i\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{i(-i+\right.\right.}{}$

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/f*2^(1/2)*d*(d*csc(f*x+e))^(1/2)*(-I*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1
/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)
*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-I*(
I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-
I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(
1/2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{-i\sqrt{2i\,d}d\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}d\sin(}{3\,f\sin(f}$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2*I*d)*d*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) +
 I*sin(f*x + e)) + I*sqrt(-2*I*d)*d*sin(f*x + e)*weierstrassPInverse(4, 0,
cos(f*x + e) - I*sin(f*x + e)) - 2*d*sqrt(d/sin(f*x + e))*cos(f*x + e))/(f*
sin(f*x + e))
```

Sympy [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\csc(e+fx)\,dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(3/2),x)
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)\,dx$$

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)}\,dx$$

```
[In] int((d/sin(e + f*x))^(3/2)/sin(e + f*x),x)
[Out] int((d/sin(e + f*x))^(3/2)/sin(e + f*x), x)
```

3.522 $\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx=-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d/f-6/5*d*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/f+6/5*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*
x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2719}

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx=-\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}$$

```
[In] Int[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e +
 f*x])^(5/2))/(5*d*f) - (6*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[
d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^2} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} + \frac{3}{5}\int (d\csc(e+fx))^{3/2}\,dx \\
&= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} \\
&\quad - \frac{1}{5}\left(3d^2\right)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\
&= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} \\
&\quad - \frac{\left(3d^2\right)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} \\
&\quad - \frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{(d\csc(e+fx))^{5/2}\left(-7\cos(e+fx)+3\cos(3(e+fx))+12E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{5}{2}}(e+fx)\right)}{10df}$$

```
[In] Integrate[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((d*Csc[e + f*x])^(5/2)*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*Elliptic
E[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.18

method	result
default	$\frac{\sqrt{2}\,d\sqrt{d\csc(fx+e)}\left(6\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cdots}\right.}{\cdots}$

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*2^(1/2)*d*(d*csc(f*x+e))^(1/2)*(6*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2
)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)-3*(I*
(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/
2),1/2*2^(1/2))*cos(f*x+e)+6*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(
f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/
2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)-3*(I*(-I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-3*2^(1
/2)-2^(1/2)*cot(f*x+e)*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx =$$

$$-\frac{3\left(d\cos\left(fx+e\right)^2-d\right)\sqrt{2i\,d}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))}{}$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(d*cos(f*x + e)^2 - d)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(d*cos(f*x + e)^2 - d)*
sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) -
 I*sin(f*x + e))) + 2*(3*d*cos(f*x + e)^3 - 4*d*cos(f*x + e))*sqrt(d/sin(f*
x + e)))/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\csc^2\left(e+fx\right)dx$$

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx=\int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx=\int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)^2}\,dx$$

```
[In] int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2,x)
[Out] int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

3.523 $\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21df}$$

```
[Out] -2/7*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/f/(d*csc(f*x+e)
)^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+
e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{21df}$$

```
[In] Int[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d^3 \int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}(5d)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{5\int\sqrt{d\csc(e+fx)}\,dx}{21d} \\
&= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{21d} \\
&= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21df}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+f}{84df}$$

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.41

method	result
default	$\frac{\sqrt{2}\left(5i\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(fx+e)+}}{}$

```
[In] int(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/f*2^(1/2)/(d*csc(f*x+e))^(1/2)*(5*I*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/
2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e
)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cot(f*x+e)+5*I*
(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*Ell
ipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2)*csc(f*x+e)+3*2^(1/2)*cos(f*x+e)^3-8*2^(1/2)*cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\left(3\cos(fx+e)^3-8\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin(fx+e)-5i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+}{21\,df}$$

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e
)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e)))/(d*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sin(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2), x)
```

3.524 $\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*d*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2719}

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= d^2 \int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx \\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3}{5}\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{-\frac{12E\left(\frac{1}{4}(-2e+\pi-2fx)|2\right)}{\sqrt{\sin(e+fx)}} - 2\sin(2(e+fx))}{10f\sqrt{d\csc(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 442, normalized size of antiderivative = 6.14

method	result
default	$\frac{\sqrt{2}\left(-6\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(f\right.\right.}{}$

[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/5/f*2^(1/2)*(-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-cs
c(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*
x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+3*(-I*(I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)
))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*
x+e)+2^(1/2)*cos(f*x+e)^3-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot
(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I
*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+3*(-I*(I-cot(f*x+e)+csc(f*x+
e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e))
)^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-4*2^(1/
2)*cos(f*x+e)+3*2^(1/2))/(d*csc(f*x+e))^(1/2)*csc(f*x+e)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx\right.}{}$$

[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")

[Out] 1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
f*x + e) - I*sin(f*x + e))))/(d*f)

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\int \frac{\sin^2{(e+fx)}}{\sqrt{d\csc{(e+fx)}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{d\csc{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\int \frac{\sin{(fx+e)}^2}{\sqrt{d\csc{(fx+e)}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\int \frac{\sin{(e+f\,x)}^2}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2), x)
```

3.525 $\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}$$

```
[Out] -2/3*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*
(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {16, 3854, 3856, 2720}

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3df} - \frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx\\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{\int\sqrt{d\csc(e+fx)}\,dx}{3d}\\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d}\\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int\frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{d\csc^2(e+fx)\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f(d\csc(e+fx))^{3/2}}$$

```
[In] Integrate[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
[Out] -1/3*(d*Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.15

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(fx+e)+\csc}\right.}{}$

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/f*2^(1/2)/(d*csc(f*x+e))^(1/2)*(I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*
(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*Ell
ipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cot(f*x+e)+I*(-I*(
I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-c
ot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)
,1/2*2^(1/2))*csc(f*x+e)-2^(1/2)*cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e) + i\sqrt{2i\,d}\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)) -}{3\,df}$$

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*weie
rstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*weier
strassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(1/2),x)

[Out] Integral(sin(e + f*x)/sqrt(d*csc(e + f*x)), x)

Maxima [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(fx+e)}{\sqrt{d\csc(fx+e)}}\,dx$$

[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)

Giac [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(fx+e)}{\sqrt{d\csc(fx+e)}}\,dx$$

[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

[In] int(sin(e + f*x)/(d/sin(e + f*x))^(1/2),x)

[Out] int(sin(e + f*x)/(d/sin(e + f*x))^(1/2), x)

3.526 $\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[1/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f
*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\ &= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\,2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[1/Sqrt[d*Csc[e + f*x]],x]
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 297, normalized size of antiderivative = 6.91

method	result
risch	$-\frac{i\sqrt{2}}{f\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}}+\frac{i\left(-\frac{2i\left(id\,e^{2i(fx+e)}-id\right)}{d\sqrt{e^{i(fx+e)}\left(id\,e^{2i(fx+e)}-id\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\,\sqrt{-2\,e^{i(fx+e)}+2}\,\sqrt{-e^{i(fx+e)}}\,\left(-2E\left(\sqrt{e^{i(fx+e)}+1},\frac{\sqrt{2}}{2}\right)+F\left(\sqrt{\right.\right.}{\sqrt{id\,e^{3i(fx+e)}-id\,e^{i(fx+e)}}}\right.}{f\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}\left(e^{2i(fx+e)}-1\right)}$
default	$-\frac{\sqrt{2}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc\right.\right.}{}$

```
[In] int(1/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -I/f*2^(1/2)/(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))^(1/2)+I/f*(-2*I*(I*d
*exp(I*(f*x+e))^2-I*d)/d/(exp(I*(f*x+e))*(I*d*exp(I*(f*x+e))^2-I*d))^(1/2)-
(exp(I*(f*x+e))+1)^(1/2)*(-2*exp(I*(f*x+e))+2)^(1/2)*(-exp(I*(f*x+e)))^(1/2
)/(I*d*exp(I*(f*x+e))^3-I*d*exp(I*(f*x+e)))^(1/2)*(-2*EllipticE((exp(I*(f*x
+e))+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(I*(f*x+e))+1)^(1/2),1/2*2^(1/2)))
)*2^(1/2)/(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))^(1/2)*(I*d*exp(I*(f*x+e
))*(exp(I*(f*x+e))^2-1))^(1/2)/(exp(I*(f*x+e))^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))) + \sqrt{-2i\,d}\text{weierstrassZe}}{df}$$

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) +
 I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(1/(d/sin(e + f*x))^(1/2),x)
[Out] int(1/(d/sin(e + f*x))^(1/2), x)
```

3.527 $\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{df}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/
f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3856, 2720}

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{df}$$

```
[In] Int[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d} \\ &= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d} \\ &= \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{df} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{df}$$

```
[In] Integrate[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.72

method	result
default	$\frac{i\sqrt{2}\,\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx-\right.}{f\sqrt{d\csc(fx+e)}}$

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/f*2^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+
e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2),1/2*2^(1/2))/(d*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{-i\sqrt{2i\,d}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\text{weierstrassPInverse}(4,0,\cos(f}{df}$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d
*f)
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)), x)
```

3.528 $\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d/f+2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/
(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2719}

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d*f) - (2*EllipticE[(e - Pi/2 + f*x
)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^2} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{-2\cot(e+fx) + \frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{f\sqrt{d\csc(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 415, normalized size of antiderivative = 5.93

method	result
default	$-\frac{\sqrt{2}\left(-2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc\right.}{}$

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*2^(1/2)*(-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc
(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+(-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^
(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e
)-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/
2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e
)))^(1/2),1/2*2^(1/2))+(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+
e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2))/(d*csc(f*x+e))^(1/2)*csc
(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e) + \sqrt{2i\,d}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx\right.}{df}$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))
))/(d*f)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)), x)
```

3.529 $\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^2/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2720}

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3df} - \frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}$$

```
[In] Int[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^2*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^3} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} \\
&\quad + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\csc^2(e+fx)\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3f\sqrt{d\csc(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Csc[e + f*x]^2*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.25

method	result
default	$\frac{\sqrt{2}\left(-i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx}\right.\right.}{}$

```
[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/f*2^(1/2)*(-I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-cs
c(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*
x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)-I*(-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)
+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1
/2))*sin(f*x+e)+2^(1/2)*cos(f*x+e))/(d*csc(f*x+e))^(1/2)/(cos(f*x+e)^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{-i\sqrt{2i\,d}\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\sin(fx+e)\,\text{weiers}}{3\,df\sin(fx+e)}$$

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) + I
*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(
f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(d*f*sin(
f*x + e))
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^3}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^3}{\sqrt{d\csc(fx+e)}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)), x)
```

3.530 $\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21d^2f}$$

```
[Out] -2/7*d*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/d/f/(d*csc(f*x+e)
)^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+
e)^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{10\sqrt{\sin(e+fx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{21d^2f} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} - \frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}$$

```
[In] Int[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*d*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= d^2 \int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx \\
&= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{5}{7}\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{5\int \sqrt{d\csc(e+fx)}\,dx}{21d^2} \\
&= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{21d^2} \\
&= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} \\
&\quad + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21d^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{d\csc(e+fx)}\Big(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+fx)}{84d^2f}$$

```
[In] Integrate[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d^2*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.73

method	result
default	$-\frac{\sqrt{2}\left(5i\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{i(-i+\cot(fx+e)-\csc\right.\right.}{}$

```
[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/f*2^(1/2)*(5*I*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)
-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot
(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+5*I*(I*(-I+cot(f*x+e)-cs
c(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f
*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))+3
*2^(1/2)*cos(f*x+e)^3*sin(f*x+e)-8*2^(1/2)*cos(f*x+e)*sin(f*x+e))/(d*csc(f*
x+e))^(1/2)/d/(cos(f*x+e)-1)/(cos(f*x+e)+1)*sin(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^3-8\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin\left(fx+e\right)-5i\sqrt{2i\,d}\text{weierstrass}}{}$$

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e
)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{\sin^2(e + fx)}{(d\csc(e + fx))^{3/2}}\,dx = \int \frac{\sin^2{(e + fx)}}{(d\csc{(e + fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e + fx)}{(d\csc(e + fx))^{3/2}}\,dx = \int \frac{\sin{(fx + e)}^2}{(d\csc{(fx + e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e + fx)}{(d\csc(e + fx))^{3/2}}\,dx = \int \frac{\sin{(fx + e)}^2}{(d\csc{(fx + e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^2}{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2), x)
```

3.531 $\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/
d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {16, 3854, 3856, 2719}

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}} - \frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[In] Int[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(5*d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx \\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d} \\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-\frac{12E\left(\frac{1}{4}(-2e+\pi-2fx)|2\right)}{\sqrt{\sin(e+fx)}} - 2\sin(2(e+fx))}{10df\sqrt{d\csc(e+fx)}}$$

```
[In] Integrate[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*d*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 445, normalized size of antiderivative = 6.01

method	result
default	$\frac{\sqrt{2}\left(-6\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(fx+e)+c}\right.}{}$

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*2^(1/2)*(-6*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(
f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*
(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)+3*(I*(-I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*
x+e)+2^(1/2)*cos(f*x+e)^3-6*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-cot(f
*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2
*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+3*(I*(-I-cot(f*x+e)+csc(f*x+
e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)+csc(f*x+e))
)^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-4*2^(1/
2)*cos(f*x+e)+3*2^(1/2))/(d*csc(f*x+e))^(1/2)/d*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstra}}{}$$

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
f*x + e) - I*sin(f*x + e))))/(d^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)}{{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(sin(e + f*x)/(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)/(d/sin(e + f*x))^(3/2), x)
```

3.532 $\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

```
[Out] -2/3*cos(f*x+e)/d/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f} - \frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

```
[In] Int[(d*Csc[e + f*x])^(-3/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2} \\
&= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2} \\
&= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2 f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{\csc^2(e+fx)\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f(d\csc(e+fx))^{3/2}}$$

```
[In] Integrate[(d*Csc[e + f*x])^(-3/2),x]
[Out] -1/3*(Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*
x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.40

method	result
default	$-\frac{\sqrt{2}\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(f\right.\right.}{}$

[In] int(1/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3/f*2^(1/2)*(I*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(
f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*
(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cos(f*x+e)+I*(-I*(I+cot(f*x+e)-csc(f*x
+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)-2^(1/2
)*cos(f*x+e)*sin(f*x+e))/(d*csc(f*x+e))^(1/2)/(cos(f*x+e)-1)/d/(cos(f*x+e)+
1)*sin(f*x+e)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))-i\sqrt{}}{3\,d^2f}$$

[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*weie
rstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*weier
strassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d^2*f)

Sympy [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))**(3/2),x)
[Out] Integral((d*csc(e + f*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(d/sin(e + f*x))^(3/2),x)
[Out] int(1/(d/sin(e + f*x))^(3/2), x)
```

3.533 $\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3856, 2719}

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d}\\ &= \frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.59

method	result
risch	$-\frac{i\sqrt{2}}{fd\sqrt{\frac{id\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}}}+\frac{i\left(-\frac{2i\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}{d\sqrt{\mathrm{e}^{i(fx+e)}\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}}-\frac{\sqrt{\mathrm{e}^{i(fx+e)}+1}\,\sqrt{-2\,\mathrm{e}^{i(fx+e)}+2}\,\sqrt{-\mathrm{e}^{i(fx+e)}}\left(-2E\left(\sqrt{\mathrm{e}^{i(fx+e)}+1},\frac{\sqrt{2}}{2}\right)+F\left(\right.\right.}{\sqrt{id\,\mathrm{e}^{3i(fx+e)}-id\,\mathrm{e}^{i(fx+e)}}}\right.}{fd\sqrt{\frac{id\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}}\left(\mathrm{e}^{2i(fx+e)}-1\right)}$
default	$-\frac{\sqrt{2}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc\right.\right.}{}$

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*2^(1/2)/d/(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))^(1/2)+I/f*(-2*I*(I
*d*exp(I*(f*x+e))^2-I*d)/d/(exp(I*(f*x+e))*(I*d*exp(I*(f*x+e))^2-I*d))^(1/2
)-(exp(I*(f*x+e))+1)^(1/2)*(-2*exp(I*(f*x+e))+2)^(1/2)*(-exp(I*(f*x+e)))^(1
/2)/(I*d*exp(I*(f*x+e))^3-I*d*exp(I*(f*x+e)))^(1/2)*(-2*EllipticE((exp(I*(f
*x+e))+1)^(1/2),1/2*2^(1/2))+EllipticF((exp(I*(f*x+e))+1)^(1/2),1/2*2^(1/2)
)))*2^(1/2)/d/(I*d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2-1))^(1/2)*(I*d*exp(I*(f
*x+e))*(exp(I*(f*x+e))^2-1))^(1/2)/(exp(I*(f*x+e))^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{2i\,d}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))}{}$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) +
 I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^2*f)
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(e+fx)}}{(d\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)), x)
```

3.534 $\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{d^2f}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^
2/f
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3856, 2720}

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

```
[In] Int[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d^2} \\
&= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d^2} \\
&= \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{d^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{d^2f}$$

```
[In] Integrate[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d^2*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

method	result
default	$-\frac{i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+}\right.}{f(\cos(fx+e)-1)\sqrt{d\csc(fx+e)}\,d}$

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1
/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+
e)))^(1/2),1/2*2^(1/2))*2^(1/2)/(cos(f*x+e)-1)/(d*csc(f*x+e))^(1/2)/d*sin(f
*x+e)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+i\sqrt{-2i\,d}\,\text{weierst}}{d^2f}$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d
^2*f)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^2{(e+fx)}}{(d\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^2}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\,\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)), x)
```

3.535 $\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d^2/f+2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/
d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d^2*f) - (2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^3} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-2\cot(e+fx) + \frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{df\sqrt{d\csc(e+fx)}}$$

```
[In] Integrate[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(d*f*Sqrt[d*Csc[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.77

method	result
default	$\frac{\sqrt{2}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e}\right.\right.}{}$

[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f*2^(1/2)*(2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2^(1/2))/(d*csc(f*x+e))^(1/2)/d*csc(f*x+e)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)+\sqrt{2i\,d}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e}{d^2f}$$

[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^2*f)

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^3{(e+fx)}}{(d\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**3/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^3}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^3}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin{(e+f\,x)}^3\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)), x)
```

3.536 $\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^3/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2720}

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}$$

```
[In] Int[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^3*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^4} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f} \\
&\quad+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\csc^3(e+fx)\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3f(d\csc(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Csc[e + f*x]^3*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{2}\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(f\right.\right.}{}$

```
[In] int(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*2^(1/2)*(I*sin(f*x+e)*cos(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2
)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*E
llipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+I*(-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2)*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)
+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1
/2))*sin(f*x+e)-2^(1/2)*cos(f*x+e))/d/(d*csc(f*x+e))^(1/2)/(cos(f*x+e)^2-1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2i\,d}\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{}}{}$$

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) + I
*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(
f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(d^2*f*si
n(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^4{(e+fx)}}{(d\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**4/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**4/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^4}{(d\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^4\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)), x)
```

3.537 $\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d^4/f-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/d^2/f+6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(
f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3853, 3856, 2719}

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[In] Int[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*d^2*f) - (2*Cos[e + f*x]*(d*Csc[e
 + f*x])^(5/2))/(5*d^4*f) - (6*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*d*f*Sqr
t[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^5} \\
&= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} + \frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d^3} \\
&= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d} \\
&= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} \\
&\quad - \frac{3\int \sqrt{\sin(e+fx)}\,dx}{5d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} \\
&\quad - \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\csc^4(e+fx)\left(-7\cos(e+fx)+3\cos(3(e+fx))+12E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\mathrm{s}\right.}{10f(d\csc(e+fx))^{3/2}}$$

```
[In] Integrate[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x]^4*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*EllipticE[(-2*e
+ Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*f*(d*Csc[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 453, normalized size of antiderivative = 4.31

method	result
default	$\frac{\sqrt{2}\left(6\sqrt{-i(i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{i(-\cot(fx+e)+\csc(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(i-\cot(fx+e)+\mathrm{c}\right.}{}$

```
[In] int(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*2^(1/2)/(d*csc(f*x+e))^(1/2)/d*(6*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2
)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*cot(f*x+e)-3*(-I
*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I
-cot(f*x+e)+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/
2),1/2*2^(1/2))*cot(f*x+e)+6*(-I*(I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(I*(-cot(
f*x+e)+csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/
2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*csc(f*x+e)-3*(-I*(I+cot(f*x
+e)-csc(f*x+e)))^(1/2)*(I*(-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(I-cot(f*x+e)
+csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1
/2))*csc(f*x+e)-3*2^(1/2)*csc(f*x+e)-2^(1/2)*cot(f*x+e)*csc(f*x+e)^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx =$$
$$-\frac{3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3}{}$$

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*(cos(f*x + e)^2 - 1)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(cos(f*x + e)^2 - 1)*sqrt
(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*s
in(f*x + e))) + 2*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(d/sin(f*x + e)))
/(d^2*f*cos(f*x + e)^2 - d^2*f)
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)**5/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**5/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)), x)
```

3.538 $\int (b\csc(e+fx))^n (a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (b\csc(e+fx))^n (a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(b\csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\sin^2(e+fx)\right)(a\sin(e+}{af(1+m-n)\sqrt{\cos^2(e+fx)}}$$

[Out] cos(f*x+e)*(b*csc(f*x+e))^n*hypergeom([1/2, 1/2+1/2*m-1/2*n],[3/2+1/2*m-1/2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^(1+m)/a/f/(1+m-n)/(cos(f*x+e)^2)^(1/2)

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2668, 2722}

$$\int (b\csc(e+fx))^n (a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m-n+1),\frac{1}{2}(m-n+3),\sin^2(e\right.}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

[In] Int[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]

[Out] (Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 + m - n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m - n)*Sqrt[Cos[e + f*x]^2])

Rule 2668

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
```

```
[e + f*x])^FracPart[n], Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b
, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \left((b\csc(e+fx))^n (a\sin(e+fx))^n\right)\int (a\sin(e+fx))^{m-n}\,dx$$

$$= \frac{\cos(e+fx)(b\csc(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\sin^2(e+fx)\right)(a$$
$$af(1+m-n)\sqrt{\cos^2(e+fx)}$$

Mathematica [A] (verified)

Time = 8.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int (b\csc(e+fx))^n (a\sin(e+fx))^m\,dx$$
$$= \frac{2(b\csc(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}(1+m-n),1+m-n,\frac{1}{2}(3+m-n),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2}{f(1+m-n)}$$

```
[In] Integrate[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
[Out] (2*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + m - n)/2, 1 + m - n, (3 + m -
n)/2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(m - n)*(a*Sin[e + f*x])^m*
Tan[(e + f*x)/2])/(f*(1 + m - n))
```

Maple [F]

$$\int (b\csc(fx+e))^n (a\sin(fx+e))^m\,dx$$

```
[In] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
[Out] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)

Sympy [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=\int (a\sin(e+fx))^m\,(b\csc(e+fx))^n\,dx$$

[In] integrate((b*csc(f*x+e))**n*(a*sin(f*x+e))**m,x)

[Out] Integral((a*sin(e + f*x))**m*(b*csc(e + f*x))**n, x)

Maxima [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)

Giac [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=\int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \int (a\,\sin(e+f\,x))^m\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

```
[In] int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```